APSU Math Problem of the Week

Problem #7: A Long Division Cryptarithm

Submission Deadline: 10/22/2021 by 12pm to
Dr. Brad Fox in **MMCS 109** or by email to **foxb@apsu.edu**

A cryptarithm is a puzzle featuring an arithmetic problem with the numbers replaced by letters. Each of the letters in the following long division cryptarithm represents a different digit. Determine the appropriate digits to reconstruct the division problem.

$$
\begin{array}{r|l}
 & \mathrm{RHR} \\
\hline
\mathrm{GR} & \mathrm{LMTTR} \\
 & \underline{\mathrm{LTR}} \\
 & \mathrm{VVT} \\
 & \underline{\mathrm{VLS}} \\
 & \mathrm{LTR} \\
 & \underline{\mathrm{LTR}}
\end{array}
\qquad
\begin{array}{r|l}
 & 565 \\
\hline
35 & 19775 \\
 & \underline{175} \\
 & 227 \\
 & \underline{210} \\
 & 175 \\
 & \underline{175}
\end{array}
$$

There are probably a lot of approaches to solve this, but this was my process, which may not be the shortest or easiest. One place to start is with the first multiplication of $R \cdot GR = LTR$. Since R is in the ones place of both factors and the product, it must be 1, 5, or 6 (In Abstract Algebra, we call these numbers idempotent (mod 10)). Then looking at the first subtraction step of $LMT - LTR = VV$, we know $M - T = V$ and $T - R = V$. This means $R < T < M$ and $V \leq 4$. The possibilities for R, T, M are $5, 6, 7$; $6, 7, 8$; $5, 7, 9$; $1, 3, 5$; $1, 4, 7$; and $1, 5, 9$. Looking at the next subtraction, we see that $T - S = T$, forcing S to be 0, and $V - L = L$, so $V = 2L$, making it an even digit. Knowing this about V eliminates $5, 6, 7$; $6, 7, 8$; and $1, 4, 7$ from consideration for R, T, M, particularly leaving R to be 1 or 5. Then to have $RH \cdot GR$ have 0 as its ones digit, $H \cdot R$ must be a multiple of 10. If $R = 1$ then $H \cdot R$ can't be a multiple of 10 unless $H = 0$, but $S \neq H$. Therefore, $R = 5$, implying $T = 7$, $M = 9$, and $V = 2$. Thus, $V = 2L$ tells us $L = 1$. Finally, $5 \cdot G5 = 175$ makes $G = 3$ and $H \cdot 35 = 210$ gives us $H = 6$, filling in the remaining digits.

Feel free to take this printout, or find each Problem of the Week by scanning this:

Complete the problem each week for a chance to win a prize