Austin Peay State University

CLARKSVILLE TENNESSEE

MATH TRAIL
Middle School

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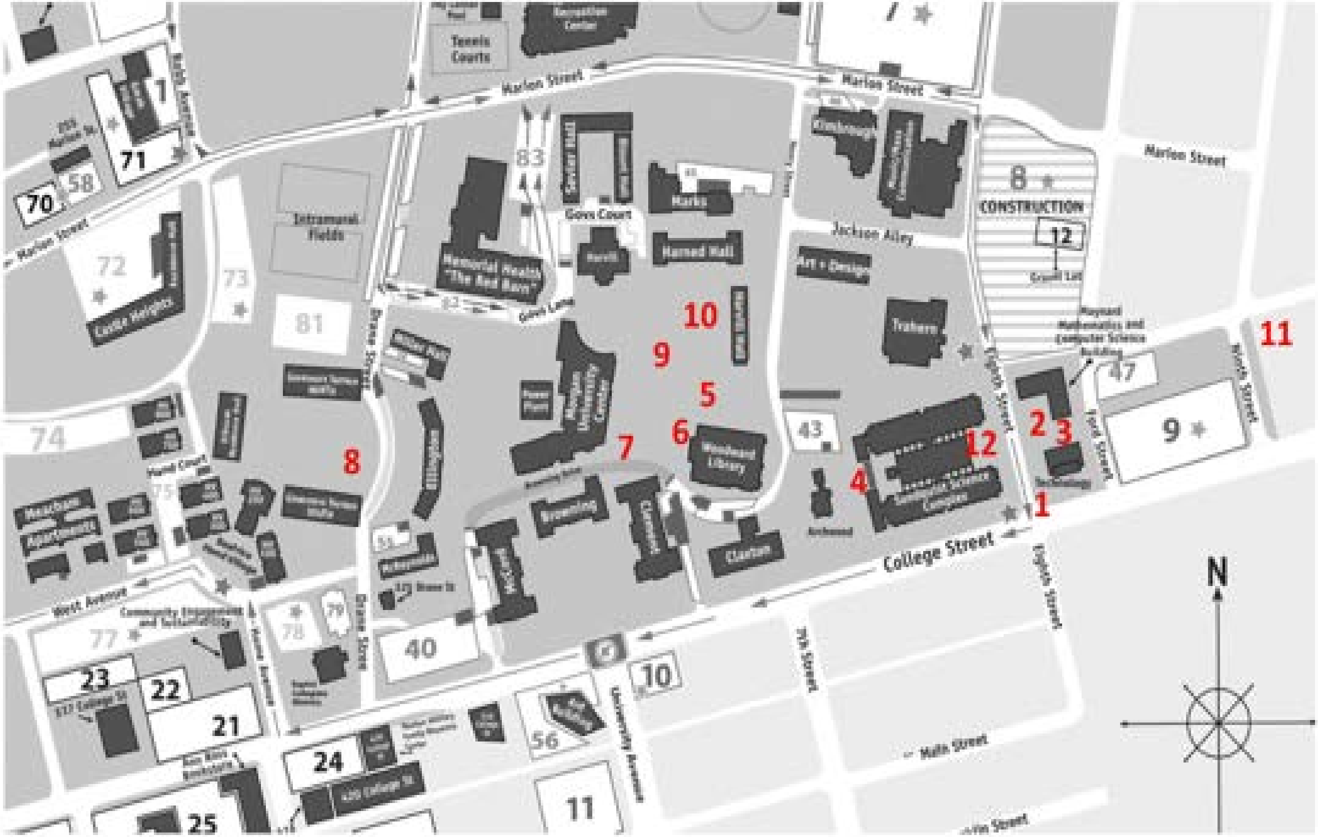
What is a Math Trail?

A math trail is a way to discover mathematics in the world around us. You can explore the trail on your own, but it's fun to go with other people so you can share the experience - collaborating in your exploration and sharing ideas!

A math trail is an activity in which the participants immerse themselves in mathematics around them, taking notice of the beauty and patterns of mathematics in everyday objects and occurrences. A math trail is a guide to observe, wonder, and engage at several stops along the way.

Some of the tasks on the math trail can be completed as you walk along the trail, while others can be completed after collecting some measurements and data. The tasks on the trail vary in math knowledge and ability level, but all of them are meant to be fun and engaging! Choose the tasks that interest you most, write things down or record data on your phone, and most importantly, have fun!

At each stop along the way, you will be directed to notice certain items around you. You may also find mathematics in things that are not mentioned in the trail guide, and we encourage you to record that information and share it with us!



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College Street
Eighth Street
Ninth Street
Ford Street
7th Street
Main Street
University Avenue
Drane Street
Home Avenue
West Avenue
Robb Avenue
Jackson Alley
Goins Court
CONSTRUCTION
Gravel Lot
Maynard Mathematics and Computer Science Building
Tennis Courts
Intramural Fields
Memorial Health "The Red Barn"
Harned Hall
Marks
Trahern
Art + Design
Kimbrough
Woodward Library
Morgan University Center
Clement
Browning
McCord
Clayton
Archwood
Sundquist Science Complex
Ellington
Castle Heights
Meacham Apartments
Community Engagement and Sustainability
Austin Peay Bookstore
420 College St.
527 College St.
255 Marion St.
325 Drane St.
1
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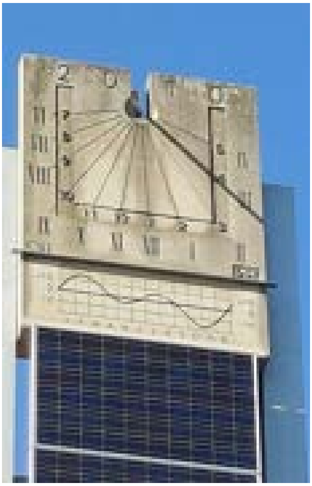
STOPS MENU

Stop 1: Ancient Technology on a Very Modern Technology Building

Stop 1 is a piece of ancient technology on top of the Austin Peay Technology Building – a **sundial**. Notice the CST in the lower left-hand corner and the DST in the lower right-hand corner. CST stands for Central Standard Time and DST stands for Daylight Savings Time.

We do not know when the first sundial was used, but a stone fragment exists of a sundial from 1500 B.C.



Look up at the sundial. What time does it show?

Does the time on the sundial match the time on your watch or device?

Why do you think the Roman numerals are always one less than the corresponding Arabic numbers (except for 1 and XII)?

This part of Tennessee is in the Central Time Zone. How many time zones are in the US?

Stop 1: Ancient Technology on a Very Modern Technology Building

Did you have trouble reading the Roman numerals? This chart may help.

One	I	Five	V	Nine	IX
Two	II	Six	VI	Ten	X
Three	III	Seven	VII	Eleven	XI
Four	IV	Eight	VIII	Twelve	XII

Did you notice the graph below the sundial?

What do you suppose the graph is showing?

The **equation of time** describes the discrepancy between two kinds of solar time. Time measured by the sun on the sundial is not the same as time measured by our clocks. The difference can be up to 22 seconds shorter or 29 seconds longer than a 24-hour day. You can read more about the equation of time here:

https://www.timeanddate.com/astronomy/equation-of-time.html

Next: *Stop 2 is the fountain in front of the Maynard Mathematics and Computer Science Building. Walk along 8th Street toward campus, and it is on your right.*

Stop 2: Fountain in the Ken and Amy Landrum Courtyard

Stop 2 is located between the Technology Building (TB) and the Maynard Mathematics and Computer Science Building (MMCS).

Notice the circle in the middle of the courtyard and prepare to take some measurements. If you do not have a measurement tool, you can improvise and use your own two feet! Have a friend help you to count your footsteps across the circle and then around the circle. Use the same feet for both measurements.

There are many circles in the fountain. For each one, use the same units to measure the distance across the center of the circle (diameter) and the distance around the circle (circumference) on its edge. Enter the values (whole numbers or decimals) in the table on the next page and calculate the ratio. What do you notice? Are the ratios close to a famous number?

Stop 2: Fountain in the Ken and Amy Landrum Courtyard

Circle	Circumference C	Diameter d	Ratio C/d
1			
2			
3			
4			

Greek mathematician Archimedes discovered the ratio of a circle's circumference to its diameter in the third century B.C. The ratio is represented by the Greek letter π. It is spelled **pi** and pronounced "pie."
You will see π in equations that calculate the circumference and area of a circle, as well as many other places in mathematics.

As of 2022, mathematicians have identified more than 62 million digits of pi, but for practical purposes, only a few are used. For example, a rocket scientist only needs about 12 digits of pi for their calculations.

$$3.14159265359$$

If your calculations gave you a number between 3.1 and 3.2, your measurements were fairly accurate. Good job!

Next: *Stop 3 is a set of numbers on the left wing of the Maynard Mathematics and Computer Science Building, only a few steps away.*

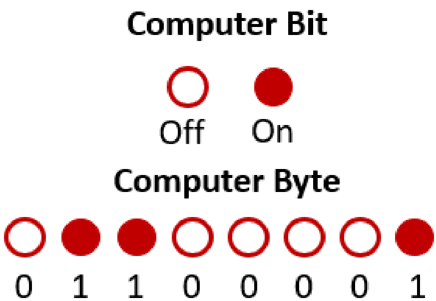
Stop 3: Binary Numbers on Maynard Mathematics and Computer Science Building

Look at the top of the left wing of the Maynard Mathematics and Computer Science Building.

Surely, they use more than just those two digits inside that building to learn mathematics! So, what could this be?

(Hint: look at the columns of numbers. There are four columns with eight numbers in each one.)

In computing, the most basic unit of information is a single circuit that can be either on or off, like a light switch. We call this a **bit**. The numbers 0 and 1 represent off and on for a bit. Eight bits together make a **byte**.



The byte shown here is from the first column of 0’s and 1’s and represents the binary number 01100001 in the base 2 number system. This corresponds to a lowercase “a” in ASCII code. Can you guess what the last three bytes represent?

Stop 3: Binary Numbers on Maynard Mathematics and Computer Science Building

If you couldn't guess the last three bytes, first convert the binary (base 2) numbers to base 10, using powers of two. Then, use the table in the appendix to match the decimal number (base 10) to its character.

0	1	1	0	0	0	0	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

The base 2 number 01100001 is equivalent to base 10 number 64 + 32 + 1 = 97, which corresponds to a lowercase "a."

Other fun facts about bits and bytes:

- John W. Tukey shortened "binary information digit" to "bit" in a memo written in 1947.
- A string of four bits is called a nibble.
- One megabyte is equivalent to 1,000,000 bytes.

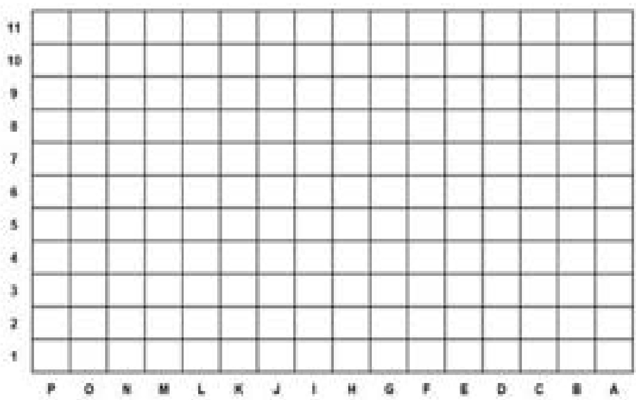
How many megabytes of information can be stored on your phone, tablet, or computer? That is a lot of zeros and ones!

Next: *Stop 4 is the garden behind the Sundquist Science Building. Cross 8th Street at the crosswalk. Walk to the back of the Sundquist Building (on your left) and you will find the Native Plant Teaching and Research Garden.*

Stop 4: Native Plant Teaching and Research Garden

Stop 4 is on the west side the Sundquist Science Complex, behind the building

The garden contains many species of plants and is constantly growing to include new varieties. The majority of the species are Tennessee natives, but there are some additional species native to the surrounding states in the southeast region of the US. You will find a grid similar to a coordinate plane, labeled like the one below. The labels are along the walkway around the garden.



Stop 4: Native Plant Teaching and Research Garden

Use the grid to find a marker for a plant in location M8.

____

Use the grid to find a marker for a plant in location O5.

____

The Native Plant Teaching and Research Garden contains over 300 species of native plants from grasslands, savannahs, rock outcrops, and woodlands of the <u>Interior Plateau Ecoregion</u> that is shown on the map below.

Next: *Stop 5 is near the Woodward Library at the Gazebo and the Certified Wildlife Habitat.*

Stop 5: Water Feature (A) at the Gazebo and the Certified Wildlife Habitat

This sculpture consists of two square basins. The top one has a side length of 15 inches and the bottom one has a side length of 30 inches. This means the ratio of the side lengths is 15:30, or 1:2 in lowest terms.

Consider the areas of the two square bottoms of the basins. The ratio of the side lengths is 1:2.

Is the ratio of the areas also 1:2? Why or why not?

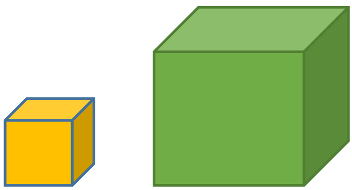
What is the ratio of the areas?

Suppose each basin has a depth of 5 inches. What is the ratio of the volumes?

Stop 5: Water Feature (A) at the Gazebo and the Certified Wildlife Habitat

If one side of the yellow square is twice as long as one side of the red square, the ratio of the side lengths is 1:2. However, you will notice that it takes four red squares to fill in the area of the yellow square. The ratio of the areas of the two squares is 1:4.

Look for the relationship of the volumes of two cubes using these same side lengths.



Next: *Turn toward the Woodward Library and walk uphill. You should be able to see Stop 6, the next water feature, from where you are standing.*

Stop 6: Water Feature (B) in front of Library

This water feature is near the Woodward Library entrance.

Draw a sketch of what you think it would look like from above.

Stop 6: Water Feature (B) in Front of Library

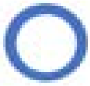
It would look like concentric circles – two or more circles with the same center but different radii. Test your knowledge of concentric circles with these questions.

Concentric circles have the same radii. True False

How many common points are shared by concentric circles?

All Some None

What do we call the area bounded by two concentric circles? (The blue area shown here.)

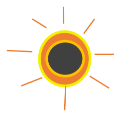


Annulus Intersection Ring

The word annulus is used in many different contexts.

Earthworms are made up of ring-like segments called annuli.

Solar eclipses may be classified as either total, in which the moon completely covers the sun, or annular, in which the moon obscures all but an outer ring of the sun.



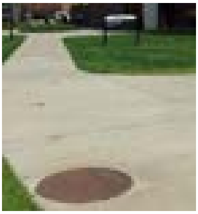
Your “ring finger” is known more formally as **digitus anularis**.

An annular lake is a ring-shaped lake caused by a meteor impact.

Next: *Stop 7 is on the sidewalk next to Browning Drive near the corner of the University Center, just a few steps away.*

Stop 7: Utility Hole (Manhole) Cover

Stop 7 is a utility hole cover near Browning Drive and the corner of the University Center. The Austin Peay Physical Plant maintains the utilities on campus, which keep our buildings warm in the winter and cool in the summer.



You will see many of these utility holes across campus, where the maintenance workers access the maze of underground pipes and wiring.

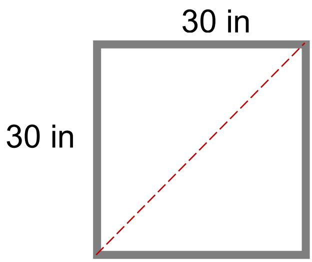
Why are utility hole (manhole) covers round?

The round shape makes it easier to put the covers back in place after the workers have used them to access passageways below the cover, since they don't need to be rotated to find the right fit. Round covers are also easier to manufacture since there are no angles to match up.

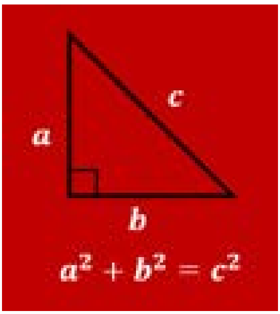
A very important safety reason why manhole covers are round, however, is that round covers will not accidentally fall into the manhole itself. With a round cover, no matter how you hold it, it will not fall through the hole. If it were square, someone could hold the cover diagonally over the hole and drop it in. This is hazardous to the workers below as well as people who travel over the manhole. Can you prove that this is true?

Stop 7: Utility Hole (Manhole) Cover

Let's use a square that measures 30 inches on each side. How long is the *diagonal* of the square hole? Could the cover be positioned in a way that it would fall through the opening?



If you are having trouble finding the diagonal of the square, here is a hint: The diagonal of the square is the hypotenuse of a right triangle. Use the Pythagorean Theorem for right triangles $a^2 + b^2 = c^2$ to find the length of the diagonal.



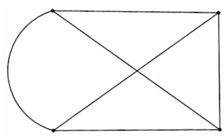
Next: *Stop 8 is the Quad between the two Governor's Terrace residential buildings for students.*

Stop 8: Is this an Euler Path?

Stop 8 is at the Quad between the two Governor's Terrace residential buildings for students (also known as "dorms").

Notice the shape formed by the intersecting sidewalks. Here is a picture from above that might help.

Can you walk a path along the sidewalks so that you only walk along each sidewalk exactly one time? You may cross a path that you have already traveled, but you may not walk along the same sidewalk twice.



In graph theory, a path is a connected sequence of edges (like the sidewalks) that starts at one point and ends at another. If a path returns you to the starting point, then it is called a circuit.

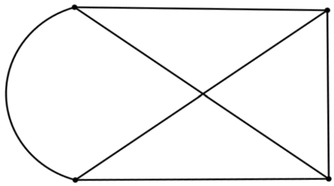
A path or circuit is an **Euler path** or **circuit** if it covers every edge only once.

Is the sidewalk structure in the courtyard an Euler path?

(Note: Euler is pronounced "oil-er".)

Stop 8: Is this an Euler Path?

Were you successful? Draw the path that you walked if you were able to complete this task.



If you were unable to complete the task, is there a way to add just one sidewalk path to make it work?

Euler paths and circuits are named for Leonhard Euler (1707 - 1783). He was a Swiss mathematician, physicist, astronomer, geographer, logician and engineer who founded the studies of graph theory and topology.

Next: *Stop 9 is between the University Center and the Woodward Library, so walk behind the Ellington Building down Browning Drive towards the main courtyard.*

Stop 9: How Did That Cupola Get Here?

Stop 9 is at the cupola located between the University Center and Woodward Library.

This is an odd sight! Doesn't it look like it belongs somewhere else, like on the top of one of the buildings? And, why is the top crooked? Find the plaque nearby and read about why this is here.

In what year did this cupola appear here?

Where did the cupola come from? Can you find that location?

What is your estimate of how far this is from its original location?

How do you think we might be able to find the exact distance? What information would you need?

Stop 9: How Did That Cupola Get Here?

This is the Clement Building topped with a replica of the original Cupola. The tornado caused $17 million in damages at Austin Peay, which was the largest property loss in Tennessee up to that time.

Fortunately, everyone on campus survived the F3 tornado with no serious injuries. The Austin Peay spirit was strong, and together with the Clarksville community, the University was able to reopen after just six days.

***Next:** Stop 10 is at the red and white AP Bowl just a few steps away.*

Stop 10: If I Have Told You Once, I Have Told You a Million Times (AP Bowl)

Stop 10 is at the AP Bowl between Harned and Harvill Halls.

Exactly how much is a million?

One million days is more than 2,737 years.

One million seconds is more than 11½ days.

If we want to count a million items and we could count one per second, it would take us 11 ½ days if we didn't stop at all, even to eat or sleep!

Is there a place on the Austin Peay Math Trail where you may have seen a million of something?

Do you think there might be 1,000,000 stones in the AP Bowl?

What would be a good way to estimate the number of stones in the AP Bowl?

Stop 10: If I Have Told You Once, I Have Told You a Million Times (AP Bowl)

10 zeros	100 zeros
0000000000	0000000000 0000000000 0000000000 0000000000 0000000000 0000000000 0000000000 0000000000 0000000000 0000000000

If I can fit 6,000 zeros on one 8.5 inches by 11 inches page, how many whole pages would it take to hold one million (1,000,000) zeros?

Here's a school legend about the AP Bowl: If a student steps on the red stones, it is said to be bad luck. If you stand on the red stones, you will not graduate this year. Seniors, beware!

What else do you recognize in the aerial photo? Does it remind you of something from Stop 6?

Next: *Stop 11 is The Austin Peay Governors Water Tower. It is located behind the parking lot next to the Technology Building.*

Stop 11: The Water Tower

Stop 11 is at the Austin Peay Governors Water Tower. It is located behind the parking lot next to the Technology Building.

If you have ever traveled to Austin Peay State University along Wilma Rudolph Boulevard to College Street, you may have noticed this water tower off in the distance and thought about how far away it looked and how tall it seemed.

Now that you are here, right next to it, can you estimate the height of the water tower?

You can use the length of shadows. This is a good method to use if it is a sunny day and it is not too close to noontime.

Have someone help you measure the length of your shadow and the shadow of the water tower.

Stop 11: The Water Tower

You can use a proportion to find the height of the tower.

$$\frac{your\ height}{your\ shadow's\ length} = \frac{tower\ height}{tower\ shadow's\ length}$$

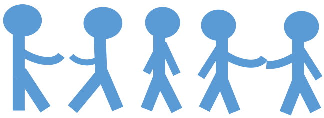
Enter the values you know and solve for the tower height.

Another way to estimate the water tower height is to take a selfie in front of the tower. Later, you can look at the picture and compare your height to the height of the tower.

Next: *Stop 12 is the Peay Pickup near the crosswalk from Sundquist to the Maynard Mathematics and Computer Science Building across 8th Street.*

Stop 12: The Peay Pickup

The Peay Pickup is a bus that provides transportation around the Austin Peay campus on two different routes, north and south. Marion Street is the dividing line between the north and south sides of campus. According to the Austin Peay website, each bus takes approximately 5 to 6 minutes to complete a route circuit.



Let's say that five friends decided to meet at the Peay Pickup stop so that they can travel to the other side of campus together. Each time someone arrives for the meetup, that person shakes hands with everyone else at the stop. By the time the bus arrives, all five friends have arrived, and each of them has shaken hands once (and only once) with each of their friends. How many handshakes would happen in total?

What if there were ten people in the group, how many handshakes would there be?

Write a method or draw a picture that can show how to find out how many handshakes there would be if there were twenty people in the group.

Answers

STOP 1:

The time on the sundial should match the time on your watch or device. Make sure you choose CST or DST. The Roman numerals represent Central Standard Time. The Arabic numerals show Daylight Savings Time. This part of Tennessee is in the Central Time Zone. How many time zones are in the US? Four: Eastern, Central, Mountain, and Pacific for mainland US. (Including Alaska and Hawaii, there are seven time zones!)

STOP 2:

The ratio of the circumference to the diameter of a circle is the definition of pi. Your ratios should be close to 3.14.

STOP 3:

The four binary numbers translate to **apsu**, a shortened name of the University.

STOP 4:

M8: Solidago Auriculata (Eared Goldenrod); O5: Phlox pilosa (Downy Phlox)

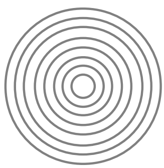
STOP 5:

Consider the areas of the two square bottoms of the basins. The ratio of the side lengths is 1:2. Is the ratio of the areas also 1:2? Why or Why not? What is the ratio of the areas? No. The area of the bottom of the smaller basin is 15 in x 15 in = 225 in^2 and the area of the bottom of the larger basin is 30 in x 30 in = 900 in^2. Then 225 in^2 / 900 in^2 = ¼. The ratio of the areas is 1:4. In general, if the ratio of the side lengths is a:b, then the ratio of the areas would be $a^2 : b^2$.

Suppose each basin has a depth of 5 inches. What is the ratio of the volumes? The volumes would be $5(225\ in^2)$ and $5(900\text{in}^2)$ and the ratio would be $\frac{1{,}125in^2}{4{,}500in^2} = \frac{1}{4}$. The ratio of the volumes is 1:4.

STOP 6:

Concentric circles have the same radii. False. They have the same center.



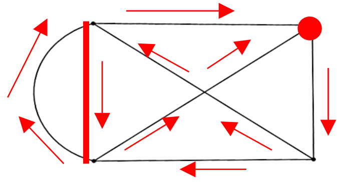
How many common points are shared by concentric circles? None. Concentric circles share no common points. They just have a common center.

STOP 7:

The diagonal of a square is the hypotenuse of a right triangle, and the length of the diagonal can be found using the Pythagorean Theorem $a^2 + b^2 = c^2$. So, $(30in)^2 \times (30in)^2 = c^2$ and then $c = \sqrt{1{,}800in^2} \approx 42.43\ in$. The diagonal is wider than the side length, so the cover could fall through.

STOP 8:

You cannot walk along each sidewalk exactly one time. It is not an Euler path. You can make it an Euler path by adding one line.



STOP 9:

The cupola appeared here on Jan. 22, 1999. It came from the top of the Clement Building. Ground distance is about 106 meters per Google Maps. The height of building is needed to find the actual distance travelled (again, the Pythagorean Theorem comes in handy).

STOP 10:

Do you think there might be 1,000,000 stones in the AP Bowl? Maybe! What would be a good way to estimate the number of stones in the AP Bowl? Choose a small area and count the stones. You can use this count as an estimate and multiply it by the number of equal-sized sections in the AP Bowl to get an estimate.

If I could fit 6,000 zeros on one 8.5 in by 11 in page, it would take 166 full pages and 2/3 of another page to hold one million zeros.

STOP 11:

A good estimate of the actual height of the water tower is between 150 and 160 feet tall.

STOP 12:

The first person to arrive makes no handshakes, the second to arrive makes one handshake, the second to arrive makes two, etc. 0 + 1 + 2 + 3 + 4 = 10 handshakes. 10 friends: Another way to think about it is each of the 10 friends will shake hands with 9 people, 10 x 9 = 90. But we have <u>double counted</u> since friend 1 with friend 2 is the same as friend 2 with friend 1, so we divide by 2. 90/2 = 45 handshakes. 20 friends: (20 x 19)/2 = 190 handshakes. Peay Pickup Route: Answers will vary.

Appendix

ASCII Table Decimal Number to Character

DEC	CHAR	DEC	CHAR	DEC	CHAR	ODEC	CHAR	DEC	CHAR	DEC	CHAR
		48	0	64	@	80	P	96	`	112	p
33	!	49	1	65	A	81	Q	97	a	113	q
34	"	50	2	66	B	82	R	98	b	114	r
35	#	51	3	67	C	83	S	99	c	115	s
36	$	52	4	68	D	84	T	100	d	116	t
37	%	53	5	69	E	85	U	101	e	117	u
38	&	54	6	70	F	86	V	102	f	118	v
39	'	55	7	71	G	87	W	103	g	119	w
40	(	56	8	72	H	88	X	104	h	120	x
41	)	57	9	73	I	89	Y	105	i	121	y
42	*	58	:	74	J	90	Z	106	j	122	z
43	+	59	;	75	K	91	[	107	k	123	{
44	,	60	<	76	L	92	\	108	l	124	|
45	-	61	=	77	M	93	]	109	m	125	}
46	.	62	>	78	N	94	^	110	n	126	~
47	/	63	?	79	O	95	_	111	o	127	DEL

Congratulations!

You have reached the end of the

Austin Peay Middle School Math Trail.

If you enjoyed thinking mathematically and you wonder how you can become a mathematician, you can visit:

Department of Mathematics and Statistics
Maynard Mathematics and Computer Science Building Room 205
931-221-7833

You can also use this QR code:

We would love to talk to you about degree options and careers in mathematics!

AP136/10-23/250