TENNESSEE MATHEMATICS TEACHERS ASSOCIATION

SIXTY-SEVENTH ANNUAL MATHEMATICS CONTEST

2025

Statistics

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Scoring Formula: $4 \times$ (Number Right) $-$ (Number Wrong) $+ 40$

Directions:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. Suppose that event A and event B are independent. Let event C be the intersection of event A and event B. Which of the following may not be true?

   a. $P(A) = P(A|B)$
   b. $P(A) = P(B|A)$
   c. $P(C) = P(A)P(B|A)$
   d. $P(C) = P(B)P(A|B)$
   e. $P(C) = P(A)P(B)$

2. Which of the following is the best classification of the variable "hourly temperature in Memphis, TN"?

   a. Categorical
   b. Qualitative
   c. Ordinal
   d. Continuous
   e. Discrete

3. The following data set is provided:

   $$8, 7, 17, 18, 15, 16, 14, 10, 14, 6$$

   What is the z-score of the third data value ($x_3 = 17$)?

   a. 1.03
   b. 4.38
   c. 7.25
   d. 8.50
   e. 12.50

4. What is the interquartile range of the data in the previous question?

   a. 6
   b. 7
   c. 8
   d. 9
   e. 12

5. Suppose that Q is a χ^2 random variable with 1 degree of freedom. Which of the following is correct regarding its density curve?

   a. It is skewed to the left.
   b. It is skewed to the right.
   c. It is unskewed.
   d. It extends indefinitely in both positive and negative directions.
   e. Its peak occurs at $q = 1$.

6. In a sample from a field sobriety test, 437 and 521 people were correctly labeled as drunk and sober respectively by the test, whereas 15 and 27 people were incorrectly labeled as drunk and sober respectively by the test. Based on this set of data, what is an estimate for the false positive rate of the test?

   a. $\frac{27}{437}$
   b. $\frac{27}{464}$
   c. $\frac{27}{548}$
   d. $\frac{15}{521}$
   e. $\frac{15}{536}$

7. Suppose that Devin were to guess the answer of two questions on a quiz. The first question was a multiple choice where 1 out of 4 answers was correct, and the second one was a True/False question. What is the probability that Devin will answer both questions correctly?

   a. 0
   b. $\frac{1}{2}$
   c. $\frac{1}{4}$
   d. $\frac{1}{8}$
   e. $\frac{1}{16}$

8. Consider randomly drawing 2 marbles (without replacement) from a jar that contains 4 blue, 5 red, and 6 yellow marbles. What is the probability that both marbles drawn will be red?

   a. $\frac{1}{3}$
   b. $\frac{1}{7}$
   c. $\frac{1}{9}$
   d. $\frac{2}{21}$
   e. $\frac{4}{21}$

9. Suppose that Prof. Charlie has a loaded 4-sided game piece with the following probability distribution:

Number	Probability
1	0.2
2	x
3	x
4	0.1

Given that the probabilities of rolling a "2" and rolling a "3" are equal, what is the value of the missing probability x.

a. 0.15
b. 0.25
c. 0.35
d. 0.45
e. 0.55

10. Consider rolling a fair 20-sided game piece that is numbered 1–20. What is the probability of rolling a prime number?

a. 0.35
b. 0.4
c. 0.45
d. 0.5
e. 0.55

11. Let X denote the sum of the numbers that you get from rolling a standard fair cube with sides labeled 1–6 a total of 6 times. What is the expected value of X?

a. 21
b. 18
c. 12
d. 6
e. 3

12. A local coffee shop did a survey asking 200 college students about their school year and whether they liked green tea lattes. Among the respondents, 35 were freshmen, with 20 of the freshmen stating that they liked green tea lattes. In total, 50 students reported that they liked green tea lattes. How many students are either freshman or like green tea lattes?

    a. 20
    b. 35
    c. 50
    d. 65
    e. 85

13. In a mobile game, a player can spend 900 gems (the in-game currency) to summon 10 heroes. For each independent summon, there is a 70% chance of obtaining a bronze-tier hero, a 20% chance of obtaining a silver-tier hero and a 10% chance of obtaining a gold-tier hero. Let X denote the number of bronze-tier heroes obtained within the 10 summons. Calculate $P(4 \leq X \leq 8)$.

    a. 0.2335
    b. 0.7004
    c. 0.8033
    d. 0.8401
    e. 0.8507

14. For the game mentioned in the previous question, alternatively, a player can spend 100 gems to summon 1 hero. A player is addicted to the game and will not stop spending gems summoning until they obtain a gold-tier hero. What is the probability that they will be spending at most 600 gems?

    a. 0.0531
    b. 0.3504
    c. 0.4686
    d. 0.4783
    e. 0.5217

15. In DNA sequencing, there are 4 bases: adenine (A), guanine (G), cytosine (C) and thymine (T). Suppose we have a sequence consisting of 5 bases, where each base can be used multiple times. How many different possible ordered sequences can be formed? (For example, ACTAG and CATAG are considered two different sequences.)

    a. 56
    b. 96
    c. 120
    d. 480
    e. 1024

16. Suppose that the Pikachu Power Company (PPC) supplies electricity with voltage levels uniformly distributed between 110 and 120 volts. What is the probability that a randomly selected voltage level from PPC will be below 114.1 volts?

    a. 0.31
    b. 0.41
    c. 0.49
    d. 0.51
    e. 0.59

17. In the previous question, what is the median voltage level from PPC (in volts)?

    a. 105
    b. 110
    c. 115
    d. 120
    e. 125

18. Let Z denote a standard normal random variable, which of the following statements is NOT correct?

    a. The density curve of Z is centered at 0.
    b. The variance of Z is 1.
    c. The height of the density curve of Z at 0 is 0.5.
    d. The area under the density curve of Z between -1 and 1 is around 68.27%.
    e. The density curve of Z is symmetric and bell-shaped.

19. Suppose that the heights of Lilliputian people are normally distributed with a mean of 6 inches and a standard deviation of 1 inch. Approximately what percentage of Lilliputians are shorter than a Lilliputian who is 6.84 inches tall?

    a. 65%
    b. 70%
    c. 75%
    d. 80%
    e. 85%

20. In the previous question, what is the lower quartile of heights of all Lilliputian people?

    a. 5.326 inches
    b. 5.615 inches
    c. 5.874 inches
    d. 6.385 inches
    e. 6.674 inches

21. Suppose that the random variable T follows a Student's t-distribution with 5 degrees of freedom. Which of the following is NOT correct?

    a. Its density curve is centered at 0.

    b. Compared to the standard normal distribution, its tails are lighter.

    c. Compared to the standard normal distribution, its peak is lower.

    d. Compared to that of a standard normal distribution, its upper quartile is larger.

    e. $P(T < 1) \approx 0.8184$.

22. Assuming that 26% of all citizens in Pallet Town (PT) prefer wooden pallets, a simple random sample of 100 PT citizens is taken. Let $\widehat{P}$ represent the proportion of the selected citizens who prefer wooden pallets. What is the sampling distribution of $\widehat{P}$?

    a. $\widehat{P} \sim \mathcal{N}ormal(\mu = 0.26, \sigma = 0.04386)$

    b. $\widehat{P} \sim \mathcal{N}ormal(\mu = 0.26, \sigma = 0.4386)$

    c. $\widehat{P} \sim \mathcal{N}ormal(\mu = 0.04386, \sigma = 0.26)$

    d. $\widehat{P} \sim \mathcal{N}ormal(\mu = 0.0386, \sigma = 0.26)$

    e. $\widehat{P} \sim \mathcal{B}inomial(n = 1000, p = 0.26)$

23. In a hypothesis testing scenario, suppose that you rejected the null hypothesis at the significance level of 5%. What would the correct decision be if the significance level was lowered to 1%?

    a. Reject the null hypothesis

    b. Fail to reject the null hypothesis

    c. Reject the alternative hypothesis

    d. Fail to reject the alternative hypothesis

    e. Cannot determine based on the given information

24. A research group from a pet shop is interested in constructing a 95% confidence interval of the proportion of all Tennessee citizens who have or would like to own a pet. Suppose they have no prior information about what this proportion might be. What is the minimum sample size n if they wanted the margin of error to be no more than 0.1?

    a. $n \geq 67$

    b. $n \geq 77$

    c. $n \geq 87$

    d. $n \geq 97$

    e. $n \geq 107$

25. Dr. Bob was manufacturing his own brand of light bulbs, claiming that 80% of them last more than 500 hours. A researcher suspected that this proportion was overestimated. To test this claim, a random sample of 479 light bulbs was examined and 363 were found to last more than 500 hours. What is the p-value for the appropriate hypothesis test?

    a. 0.0105
    b. 0.0210
    c. 0.0315
    d. 0.0420
    e. 0.0525

26. From the data in the previous question, what would be 95% two-sided confidence interval for the true proportion of Dr. Bob's brand of light bulbs that last more than 500 hours?

    a. (0.718, 0.794)
    b. (0.726,0.790)
    c. (0.707,0.808)
    d. (0.719,0.796)
    e. (0.733,0.783)

27. The math learning center presented students' math score before and after their tutoring sessions to see if there has been an improvement. Below are the data that they provided:

Before	67	45	59	66	56	43	74	80	56	66
After	64	69	61	69	62	62	65	77	60	66

    Assume that the population is normally distributed. At the significance level of $\alpha = 5\%$, which of the following is the best conclusion?

    a. There is enough evidence to conclude an improvement because p-value is larger than α.
    b. There is not enough evidence to conclude an improvement because p-value is larger than α.
    c. There is enough evidence to conclude an improvement because p-value is smaller than α.
    d. There is not enough evidence to conclude an improvement because p-value is smaller than α.
    e. A conclusion cannot be drawn based on the given information.

28. A classroom has 4 rows with 10 students sitting in each row. An instructor wanted to select a sample of 10 students so he picked 1 of the 4 rows at random. Which of the following best describes this sample?

    a. This is a random sample but not a simple random sample.
    b. This is a simple random sample but not a random sample.
    c. This is both a random sample and a simple random sample.
    d. This is neither a random sample nor a simple random sample.
    e. This is a cross-sectional sample.

29. Prof. Ann believes that students who perform well in biology also tend to perform well in chemistry. Below is a set of data showing the biology and chemistry final exam scores of 10 students:

Biology	80	74	82	84	79	92	84	83	81	77
Chemistry	76	72	76	77	80	93	83	84	76	78

    What is the value of the correlation coefficient r of the two variables?

    a. -0.6447
    b. -0.5233
    c. -0.1413
    d. 0.7137
    e. 0.8448

30. In the previous question, let x and y represent the biology and chemistry final exam scores, respectively. What is the equation of the best-fitting line?

    1. $\widehat{y} = -8.045 + 3.061x$
    2. $\widehat{y} = -4.058 + 6.031x$
    3. $\widehat{y} = -5.048 + 1.036x$
    4. $\widehat{y} = 0.548 - 3.016x$
    5. $\widehat{y} = 4.085 - 0.136x$

31. A researcher is investigating whether there is a relationship between vaping and asthma. The data that they collected have been summarized in the following contingency table:

	Vape	Do not Vape
Asthma	301	207
No Asthma	103	246

If a χ^2-test performed, what is the value of the test statistic; and if the critical value is $q^* = 10.83$, what should your decision be?

a. The decision is to reject the null hypothesis since the test statistic is 73.43, which is larger than the critical value provided.

b. The decision is to not reject the null hypothesis since the test statistic is 73.43, which is larger than the critical value provided.

c. The decision is to reject the null hypothesis since the test statistic is 7.343, which is smaller than the critical value provided.

d. The decision is to not reject the null hypothesis since the test statistic is 7.343, which is smaller than the critical value provided.

e. A decision cannot be made since the test statistic is 10.83, which is exactly the same as the critical value provided.

32. Knowing that a χ^2 distribution with 1 degree of freedom is the square of a standard normal distribution, suppose that Q follows a χ^2 distribution with 1 degree of freedom. What is the 98^{th} percentile of Q? (Hint: Use and manipulate the standard normal percentile.)

a. 2.054

b. 2.706

c. 3.841

d. 4.218

e. 5.412

33. Thomas was recently introduced to a game that charges a \$1 entrance fee per round. In each round, the player rolls a standard fair 12-sided game piece once. They win \$2 if the number rolled is a prime number, otherwise they lose and receive nothing. What would be the expected net earnings of the game if Thomas were to play 5 rounds?

a. $-\frac{25}{6}$

b. $-\frac{5}{6}$

c. 0

d. $\frac{5}{6}$

e. $\frac{25}{6}$

34. Consider randomly drawing 3 marbles (without replacement) from a jar that contains 4 blue, 5 red and 6 yellow marbles. What is the probability that the 3 marbles all have different colors?

a. $\frac{99}{455}$
b. $\frac{123}{455}$
c. $\frac{16}{91}$
d. $\frac{20}{91}$
e. $\frac{24}{91}$

35. Below is a probability distribution table for a discrete random variable W:

w	$P(W = w)$
-2	0.2
-1	0.3
0	x
1	0.1
2	y

Given that the variance of the random variable W is 1, what are the approximate values of x and y, respectively?

a. $x \approx 0.0254, y \approx 0.3746$
b. $x \approx 0.3746, y \approx 0.0254$
c. $x \approx 0.1432, y \approx 0.2568$
d. $x \approx 0.2568, y \approx 0.1432$
e. $x \approx -1.1746, y \approx 1.5746$

36. Marcus has been using his old PC for about a decade, and it occasionally experiences blue screen errors. According to Marcus, the PC encounters approximately 12 blue screens per year. Let the random variable X count the number of blue screens in a month. What is the variance of X?

    a. 1
    b. 3.46
    c. 6.92
    d. 9.27
    e. 12

37. In the previous question, what is the probability that the PC will encounter exactly 12 blue screens in a certain year?

    a. 0.1144
    b. 0.4240
    c. 0.4616
    d. 0.5384
    e. 0.5760

38. In the previous question, suppose that Marcus will become enraged and destroy the PC if it experiences at least 15 blue screens in a given calendar year. What is the probability that his PC will survive for two years?

    a. 0.2280
    b. 0.5960
    c. 0.7130
    d. 0.7720
    e. 0.8444

39. Consider tossing a standard fair coin 16 times. Let X denote the random variable that counts the number of tails from the 16 tosses. What is the expected value of X^2?

    a. 68
    b. 64
    c. 16
    d. 8
    e. 4

40. Consider rolling a standard fair cube twice. Let X be the random variable that represents the outcome of the first roll, and Y be the random variable that represents the outcome of the second roll. What is the expected value of X/Y?

    a. 0.70
    b. 1.00
    c. 1.43
    d. 2.19
    e. 3.52