# SIXTY-FIFTH ANNUAL MATHEMATICS CONTEST 

2023

Statistics

Prepared by:
Stacy Brown
Tennessee Tech University
Cookeville, TN

Scoring Formula: $4 \times($ Number Right $)-($ Number Wrong $)+40$

## Directions:

Do not open this booklet until you are told to do so.
This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. Which of the following measures of central tendency is least affected by outliers?
(a) Midrange
(b) Range
(c) Mean
(d) Median
(e) Standard Deviation
2. Julie had a z score of 0.80 on a test that had a mean of 75 and a standard deviation of 5. What was Julie's raw score on the test?
(a) 79
(b) 80
(c) 81
(d) 72
(e) 71
3. Principal Jones of Oak Grove High School is seeking student body opinion regarding the school's dress code policy. He plans on surveying the opinions of 200 randomly selected students- 50 Freshmen, 50 Sophomores, 50 Juniors and 50 Seniors. What type of sampling method is Principal Jones using?
(a) Cluster
(b) Stratified
(c) Systematic
(d) Convenience
(e) Snowball
4. An amateur meteorologist records temperatures daily during the month of June. She determines the average temperature is $82^{\circ} \mathrm{F}$ and the standard deviation is $4^{\circ} \mathrm{F}$. Unfortunately, it was discovered that the thermometer was calibrated incorrectly which resulted in all temperatures being $5^{\circ} \mathrm{F}$ higher than reported. What are the mean and standard deviation of the corrected temperature readings?
(a) $\bar{x}=87^{\circ} \mathrm{F}$ and $\mathrm{s}=9^{\circ} \mathrm{F}$
(b) $\bar{x}=87^{\circ} \mathrm{F}$ and $\mathrm{s}=4^{\circ} \mathrm{F}$
(c) $\bar{x}=77^{\circ} \mathrm{F}$ and $\mathrm{s}=9^{\circ} \mathrm{F}$
(d) $\bar{x}=77^{\circ} \mathrm{F}$ and $\mathrm{s}=4^{\circ} \mathrm{F}$
(e) $\bar{x}=82^{\circ} \mathrm{F}$ and $\mathrm{s}=9{ }^{\circ} \mathrm{F}$
5. The five-number summary of the distribution of scores on a Statistics final exam was $(18,39,62,76,100)$. What can be said about the $90^{\text {th }}$ percentile?
(a) It is 76 .
(b) It is between 18 and 39 .
(c) It is between 62 and 76.
(d) It is between 76 and 100 .
(e) It is between 39 and 76 .
6. For a sample of 400 blood pressure values, the mean is 120 and the standard deviation is 10 . Assuming a bell-shaped curve, which interval is likely to contain almost all (over $99 \%$ ) of the blood pressure values in the sample?
(a) 119 to 121
(b) 110 to 130
(c) 100 to 140
(d) 90 to 150
(e) 85 to 125
7. Mr. Adams gives a 20-question multiple choice history test. Each question has 5 possible answer choices. Assume there is only 1 correct answer per question. Tommy is not prepared for the test. He guesses at each question. How many questions can Tommy expect to answer correctly?
(a) 3
(b) 4
(c) 5
(d) 6
(e) 10
8. The prices, in thousands of dollars, of 35 vacation packages offered by a local travel agency are shown in the table below:

| Price <br> (in thousands) | $\$ 7$ | $\$ 8$ | $\$ 9$ | $\$ 10$ | $\$ 11$ | $\$ 12$ | $\$ 13$ | $\$ 14$ | $\$ 15$ | $\$ 16$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 4 | 6 | 7 | 6 | 4 | 2 | 2 | 2 | 1 | 1 |

Which of the following best describes the shape of the distribution of vacation packages offered by the travel agency?
(a) Skewed to the left (negatively skewed)
(b) Skewed to the right (positively skewed)
(c) Bimodal
(d) Uniform
(e) Approximately normal
9. Which of the following is not a discrete random variable:
(a) The lifetime of a light bulb
(b) The number of checkout lines operating at a large grocery store
(c) The number of defective tires on a truck
(d) The number of pages in a book
(e) The number of puppies in a litter
10. If $40 \%$ of all women are employed outside the home, what is the probability that in a random sample of 10 women at most 3 are employed outside the home?
(a) 0.215
(b) 0.382
(c) 0.055
(d) 0.167
(e) 0.026
11. The commuting time for a student to travel from home to a college campus is normally distributed with a mean of 30 minutes and a standard deviation of 5 minutes. If the student leaves home at 8:25 A.M., what is the probability that the student will arrive at the college campus after 9 A.M?
(a) 0.16
(b) 0.32
(c) 0.50
(d) 0.84
(e) 0.68
12. What is the probability of an impossible event?
(a) 1
(b) -1
(c) 0.5
(d) -0.5
(e) 0
13. A study was conducted to determine the number of years of experience at a small college. The results follow:

| Experience | Instructors | Asst. Professors | Professors |
| :---: | :---: | :---: | :---: |
| $1-5$ years | 8 | 3 | 2 |
| $6-10$ years | 4 | 15 | 10 |
| $11-15$ years | 1 | 12 | 8 |

If a person is selected at random, what is the probability the person is an Assistant Professor given that he or she has 11-15 years of experience?
(a) $4 / 21$
(b) $2 / 5$
(c) $3 / 5$
(d) $4 / 7$
(e) $1 / 7$
14. A prom committee is formed of 15 seniors ( 8 of whom are male) and 10 juniors ( 5 of whom are male). A chairperson is chosen at random. What is the probability that the chairperson will be a male or a senior?
(a) 1.12
(b) 0.8
(c) 0.875
(d) 0.312
(e) 0.125
15. What is the mean of the following list? $78,70,47,54,46,64,69,52$
(a) 60
(b) 59
(c) 61
(d) 68.6
(e) 53.3
16. What is the median of the following list? $78,70,47,54,46,64,69,52$
(a) 60
(b) 59
(c) 50
(d) 47
(e) 69
17. What value would be needed to complete the following probability distribution?

| x | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{x})$ | $1 / 3$ | $1 / 8$ | $?$ | $1 / 4$ | $1 / 6$ |

(a) $1 / 5$
(b) $1 / 12$
(c) $1 / 24$
(d) $1 / 8$
(e) 1
18. A finance company rates applicants for credit. The ratings are normally distributed with a mean of 200 and a standard deviation of 50 . If 40 different applicants are randomly selected, what is the probability their mean is above 215?
(a) 0.1179
(b) 0.0289
(c) 0.3821
(d) 0.4713
(e) 0.9711
19. What z value corresponds to the $63^{\text {rd }}$ percentile?
(a) 1.13
(b) -0.33
(c) -0.24
(d) 0.24
(e) 0.33
20. A box contains 9 red marbles, 8 green marbles, and 4 white marbles. If a marble is selected at random, what is the probability that it is not green?
(a) $8 / 21$
(b) $4 / 21$
(c) $13 / 21$
(d) $9 / 21$
(e) $1 / 7$
21. A box contains 9 red marbles, 8 green marbles, and 4 white marbles. If two marbles are randomly selected (without replacement), what is the probability that both marbles are white?
(a) $1 / 35$
(b) $16 / 441$
(c) $7 / 41$
(d) $8 / 21$
(e) $17 / 21$
22. Which is not part of the five number summary?
(a) Median
(b) Mean
(c) $Q_{1}$ and $Q_{3}$
(d) The smallest and largest values of the data set
(e) $\mathrm{P}_{50}$
23. What is the sample standard deviation for the following data set? $75,65,83,100,92$
(a) 35
(b) 12.3
(c) 13.8
(d) 151.6
(e) 189.5
24. A manager wishes to estimate the number of hours employees work each week. The population standard deviation is 2.6 hours. Assume the variable is normally distributed. What is the minimum sample needed to be $95 \%$ confident the true mean differs from the sample mean by 0.5 hours?
(a) 11
(b) 180
(c) 500
(d) 100
(e) 104
25. A recent survey of 1500 randomly selected adults found that 375 say access to high-speed internet is a "major problem" in their community. What is the $99 \%$ confidence interval of the true population proportion who say access to highspeed internet is a major problem?
(a) $(0.217,0.283)$
(b) $(0.228,0.272)$
(c) $(0.225,0.275)$
(d) $(0.221,0.279)$
(e) $(0.300,0.310)$
26. When a $99 \%$ confidence interval is calculated instead of a 95\% confidence interval with the sample size being the same, how will the margin of error will change?
(a) It will be smaller.
(b) It will be the same.
(c) It will be larger.
(d) It cannot be determined.
(e) It will be zero.
27. When the population standard deviation is unknown and the sample size is less than 30 , what table value should be used in computing a confidence interval for the mean?
(a) z
(b) t
(c) Chi-square
(d) Poisson
(e) Binomial
28. What are the degrees of freedom for the $t$ test for the mean when the population standard deviation is unknown and $n$ is the sample size?
(a) $n$
(b) $n-1$
(c) $n-2$
(d) 1
(e) $2 n$
29. If you wish to test a claim that the mean of a population is 100 , What would be the appropriate null hypothesis?
(a) $H_{0}: \bar{x}=100$
(b) $H_{0}: \mu \leq 100$
(c) $H_{0}: \mu \geq 100$
(d) $H_{0}: \mu=100$
(e) $H_{0}: \bar{x} \neq 100$

Use the information below to answer questions 30-33.
A survey conducted by Brides reports that brides who earned more than $\$ 50,000$ a year, $37 \%$ of them would give away a gift they did not like. To verify this claim, a sample of 600 brides who earn more than $\$ 50,000$ a year was questioned and 215 of them said they had given away wedding gifts they did not like.
30. What is the null hypothesis?
(a) $\mathrm{H}_{0}: \mathrm{p} \neq 0.37$
(b) $\mathrm{H}_{0}: \mathrm{p}=0.37$
(c) $\mathrm{H}_{0}: \mathrm{p}>0.37$
(d) $\mathrm{H}_{0}: \mathrm{p}<0.37$
(e) $\mathrm{H}_{0}: \mathrm{p}<0.73$
31. What is the alternative hypothesis?
(a) $\mathrm{H}_{1}: \mathrm{p} \neq 0.37$
(b) $\mathrm{H}_{1}: \mathrm{p}=0.37$
(c) $\mathrm{H}_{1}: \mathrm{p}>0.37$
(d) $\mathrm{H}_{1}: \mathrm{p}<0.37$
(e) $\mathrm{H}_{1}: \mathrm{p}=0.73$
32. For a level of significance $\alpha=0.01$, what are the critical value(s)?
(a) $\pm 2.05$
(b) $\pm 1.28$
(c) $\pm 1.68$
(d) $\pm 1.96$
(e) $\pm 2.58$
33. For a level of significance $\alpha=0.01$, what are the test statistic(s)?
(a) -0.59
(b) 0.59
(c) -0.50
(d) -0.99
(e) -1.58
34. What is the best point estimate of the mean of a population?
(a) Median of the sample
(b) Mode of the sample
(c) Mean of the sample
(d) Midrange of the sample
(e) Standard deviation of the sample
35. If the correlation coefficient is -0.84 , then what type of slope does the regression line have?
(a) Positive
(b) Negative
(c) Constant
(d) Zero
(e) None of these
36. What is the probability a 4 -digit ATM pin selected at random has no repeated digits?
(a) 0.504
(b) 0.0256
(c) 0.256
(d) 0.0504
(e) 0.33
37. A deck consists of 10 red cards numbered $0-9,8$ blue cards numbered $0-7$, and 5 yellow cards numbered $0-4$. If two cards are drawn without replacement, what is the probability that both cards are the same number?
(a) 0.0119
(b) 0.0455
(c) 0.1890
(d) 0.0711
(e) 0.1779
38. How many ways can a committee of 3 people be formed from a group of 10 people?
(a) 120
(b) 720
(c) 60
(d) 167
(e) 10
39. Which of the following is not a property of the standard normal distribution?
(a) It is bell shaped
(b) It is symmetric about the mean
(c) It is unimodal
(d) It is uniform
(e) The mean is 0 and the standard deviation is 1
40. Which of the following statements are true?
I. Categorical variables are the same as qualitative variables.
II. Categorical variables are the same as quantitative variables.
III. Quantitative variables can be continuous variables.
(a) I only
(b) II only
(c) III only
(d) I and II
(e) I and III

