

FIFTIETH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Statistics 2006

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

Dr. Hal Ramer, President, Volunteer State Community College, Gallatin, Tennessee
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TMTA STATISTICS 2006

Mathematics Division
Northeast State Technical Community College

1. Company A makes a product that has average sales of about 4.5 million items per year with a standard deviation of 96 thousand. Company B makes a similar product that yields average sales of about 4.5 million, but with a standard deviation of 126 thousand. Which one of the following is the best conclusion to draw regarding this scenario?
 - A. Company A makes a better product.
 - B. Company B sells more of the product than Company A.
 - C. Company B sells their product for more than Company A.
 - D. Company A has a wider range in the number of products sold.
 - E. Company B has a wider range in the number of products sold.

2. A newspaper article lists the median salary paid to major league baseball players as \$523,000 while another article reported the mean salary paid to major league baseball players as \$1,610,000. A correct conclusion would be:
 - A. impossible situation, one article must have an error
 - B. the distribution of salaries is skewed right
 - C. the distribution of salaries is skewed left
 - D. the distribution must be uniform
 - E. the distribution must be bimodal

3. 60% of students at a local university are female. You randomly choose 100 students. What is the probability that exactly 60 are female?
 - A. 0.004
 - B. 0.036
 - C. 0.060
 - D. 0.081
 - E. 0.136

4. Each value meal at Hamburger Heaven comes in three sizes. How many different ways can you order a value meal at Hamburger Heaven if there are 6 different sandwiches, 2 different side items and 5 different drinks?
 - A. 16
 - B. 48
 - C. 180
 - D. 480
 - E. 1800

5. The heights of Imperial storm troopers are normally distributed with a mean of 76 inches and a standard deviation of 3.1 inches. Princess Leia is looking for a storm trooper who is less than 69 inches. What proportion of storm troopers is this?
 - A. 0.012
 - B. 0.042
 - C. 0.044
 - D. 0.956
 - E. 0.988

6. Ten teams competed in this year's Community College Cheering Competition. How many ways can the judges choose first, second, and third place?
 - A. 6
 - B. 120
 - C. 240
 - D. 720
 - E. 840

7. In a hypothesis test the null hypothesis is that the mean July temperature in a certain location is at least 85 degrees. The test statistic is $z = -1.25$. What is the p-value?
- A. 0.015 B. 0.045 C. 0.089 D. 0.106 E. 0.167
8. Given a 95% confidence interval of $\$2803 < \mu < \3437 for the mean earnings of college students, find the point estimate for the population mean and the maximum error of the estimate of the population mean.
- A. $\bar{x} = \$1401.50$, $E = \$634$ B. $\bar{x} = \$1718.50$, $E = \$634$
 C. $\bar{x} = \$3120$, $E = \$634$ D. $\bar{x} = \$3120$, $E = \$317$
 E. $\bar{x} = \$3120$, $E = .95$
9. Vince Straub is the manager of a Las Vegas hotel-casino. One of the slot machines in the casino is supposed to pay off on 20% of the quarters played in the machine. Vince wonders whether the machine is out of adjustment. He therefore monitors the next 300 plays of the machine and records 69 payoffs. Vince uses this data to perform a hypothesis test with level of significance 0.01. Find the result of the hypothesis test and what that indicates he should do about the machine.
- A. Reject the null hypothesis, and recalibrate the machine.
 B. Reject the null hypothesis, and leave the machine alone.
 C. Reject the claim, and recalibrate the machine.
 D. Fail to reject the null hypothesis, and recalibrate the machine.
 E. Fail to reject the null hypothesis, and leave the machine alone.
10. Your calculator gives you the following for a 95% confidence interval: (2910, 3330). This is:
- A. An interval with a margin of error of give or take 5%.
 B. An interval with a margin of error of give or take 95%.
 C. An interval which will be correct 95% of the time.
 D. An interval on which 95% of your data lies.
 E. An interval which has a 95% chance of containing the true population mean.
11. From a group of 8 men and 7 women, a committee of 4 is chosen. What is the probability that 2 men and 2 women will be chosen to serve on the committee?
- A. $\frac{4}{1365}$ B. $\frac{2}{1365}$ C. $\frac{4}{15}$ D. $\frac{2}{15}$ E. $\frac{28}{65}$
12. From experience, you estimate that the probability that you will make a sale on any given phone call is 23%. Find the probability that your first sale of the day will occur on your fourth or fifth sales call.
- A. 0.105 B. 0.081 C. 0.186 D. 0.814 E. 0.014

13. A pizza shop offers a choice of 12 toppings. How many different three-topping pizzas can be made if no topping is used twice and you have the choice of thin or thick crust?

- A. 220 B. 1320 C. 440 D. 2640 E. 96

14. If 20 % of the loan applications received by a bank are refused, what is the probability that among 225 loan applications, at least 50 will be refused?

- A. 0.776 B. 0.224 C. 0.821 D. 0.179 E. 0.046

15. The mean length of a blade of grass after Edward mows the lawn is 2.6 inches with a standard deviation of 0.2 inches. Assuming the distribution of blade lengths is bell-shaped, what is the approximate proportion of blades between 2.2 and 3.0 inches long?

- A. 0.68 B. 0.85 C. 0.88 D. 0.95 E. 0.997

16. The Heritage Company produces luggage, portfolios and other leather goods. The company encloses a questionnaire with each product for the customer to complete and return. Recent results from 500 respondents on a question "Reason why you purchased a Heritage product" were:

Reason	# of Respondents	Reason	# of Respondents
Quality	126	Price	107
Recommendation	82	Style	53
Color	41	Function	91

Which of these presentations and/or analysis seems more appropriate to deal with answering this question?

- A. Histogram and calculate the mean
 B. Boxplot
 C. Stem-and-leaf display and calculate the median
 D. Table with frequencies and % and/or a pie chart
 E. Calculate the mean and standard deviation

17. Please find the IQR of the set of data represented by the stem-and-leaf display.

```

0|7
1|2 2 5 8
2|1 2 6 7 9
3|3 6 7
4|1
  
```

- A. 7 B. 15 C. 18 D. 34 E. 41

18. Find the difference between the mean and the median for the following set of data:

28 21 23 24 23 18 31

- A. 0 B. 1 C. 2 D. 3 E. 4

19. The variance of a binomial distribution is 16. If $n=100$, what are the possible values for the mean of the binomial distribution?
- A. 20 or 80 B. 20 only C. 80 only D. 16 E. 100
20. An instructor at BTU teaches a very large section of statistics. Each student in her section was asked to guess her age at the beginning of the semester. The results were normally distributed with a mean of 33.3 and a standard deviation of 2.6. If 30 students are selected at random, what is the probability that the average guess is less than 32.3?
- A. 0.018 B. 0.030 C. 0.035 D. 0.078 E. 0.130
21. From a sample of 36 randomly chosen days, a 95% confidence interval for the mean number of cups of coffee sold per day at Joe Mugs was calculated to be (160.1, 169.9). What is the value of the sample mean divided by the standard deviation?
- A. 6 B. 7 C. 10 D. 11 E. 16
22. In testing the claim that the average number of hours of television that children watch per day is more than 4, with a significance level of 5%, which one of the following is correct?
- A. A p-value of 0.01 indicates that there is insufficient evidence to support the claim.
B. A p-value of 0.03 indicates that there is insufficient evidence to support the claim.
C. A p-value of 0.07 indicates that there is sufficient evidence to support the claim.
D. A p-value of 0.09 indicates that there is insufficient evidence to support the claim.
E. A p-value of 0.11 indicates that there is sufficient evidence to reject the claim.
23. The coefficient of determination, r^2 , for a set of data is determined to be 0.81. Which of the following must be true?
- A. 81% of the variation in the response variable can be explained by the regression line.
B. The correlation coefficient is 0.9.
C. There is a weak relationship between the two variables.
D. There is a positive relationship between the two variables.
E. The correlation coefficient is 0.81
24. A researcher calculated the population variance of a set of 25 data to be 24. He later realized that the data was a sample and not a population. What is the sample variance?
- A. 23 B. 24 C. 25 D. 26 E. 27
25. One out of every ten soda bottle caps is a winning game piece. If Greg buys five sodas, what is the probability that at least one is a winner?
- A. 0.410 B. 0.500 C. 0.590 D. 0.610 E. 0.990

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Notation:

$P(A)$ represents the probability of the event A .

The letter z always represents a quantity having a standard normal (i.e. Gaussian) distribution.

Some possibly useful formulas:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

$$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Only two statistical tables are needed and provided for this contest exam: the 'standard normal' table of probabilities and the t-table of critical values. The form of the standard normal table provided has probabilities of the form $P(0 < z < c)$, where c is a constant ranging from 0 to 3.49. *Each problem on this contest exam has an ordinary solution not requiring any other statistical tables.*

CRITICAL VALUES OF "STUDENT'S T" DISTRIBUTION

Critical values t_p satisfy $p = P(t \geq t_p)$.

d.f.	$t_{.250}$	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.0025}$	$t_{.001}$	$t_{.0005}$
1	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
inf.	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

26. What is the expected value of the probability distribution given at right?

x	P(x)
0	.10
10	.15
20	.50
30	.25

- A. 15 B. 16 C. 19 D. 20 E. 25

27. Scores on a new video game are normally distributed with a mean of 4,000,000 and a standard deviation of 1,000,000. What range of scores makes up the middle 50%?

- A. 3,000,000 – 5,000,000 B. 3,325,510 – 4,674,490
 C. 3,500,000 – 4,500,000 D. 3,950,000 – 4,050,000
 E. 4,000,000 – 5,000,000

28. Jerry wants to shorten the width of a confidence interval and increase the level of confidence at the same time. What is the only legitimate way he can do this?

- A. Increase the sample size
 B. Decrease the sample size
 C. Recalculate the mean
 D. Convenience sample
 E. Impossible

29. Recent newspaper articles state that the average waiting time in a certain hospital emergency room is at least 20 minutes. Researchers want to test to see if the mean waiting time is actually less than 20 minutes. This hypothesis test is:

- A. left-tailed B. right-tailed C. two-tailed D. uniform E. biased

30. Number of absences and final grade in a course are listed below. Using linear regression, what is the best prediction, if any, for a student's final grade if the student was absent 13 times.

Absences	1	0	3	6	6	9
Final Grade	93	96	83	72	84	62

- A. 22 B. 38 C. 48 D. 52 E. should not predict

31. The local fire department needs to borrow 2 boats from residents in the community for a search and rescue effort. The following table contains the number of boats at a residence, x , along with its probability. If the fire chief randomly asks three residents how many boats they own, what is the probability that he has located at least 2 boats?

x	P(x)
0	1/3
1	1/3
2	1/3

- A. 0.778 B. 0.852 C. 0.889 D. 0.963 E. 0.997

32. The least squares regression line that fits the data below is $\hat{y} = 0.87x + 5$. If the mean value for x is 12, what is the missing y -value?

$$Y = \{12.32, 14.54, 17.32, \text{____}, 16.77\}$$

- A. 10.44 B. 11.25 C. 15.44 D. 16.25 E. 17.44

33. An instructor at a local college wanted to estimate the mean amount a student spends on books per semester. After collecting data from 60 students the instructor concluded that he was within 10 dollars of the population mean. Assuming the standard deviation for the entire population is 47.09, what level of confidence did he use for his study?

- A. 0.90 B. 0.95 C. 0.98 D. 0.99 E. 0.80

34. The scores on a university examination are normally distributed with a mean of 62 and a standard deviation of 11. If the bottom 5% of the students will fail the course, what is the lowest mark that a student can have and still be awarded a passing grade?

- A. 62 B. 57 C. 44 D. 40 E. 80

35. You are tired of buying those big bags of chips that are labeled 6 ¼ oz but are half-empty. You decide to test the weights to see if you are consistently being cheated and receiving less than 6 ¼ oz or if that much “settling” really does occur. You buy 15 bags and find the weights of the chips are:

6.9	6.7	6.6	6.9	6.8
6.4	6.7	6.5	6.8	6.3
6.8	6.8	6.7	6.4	6.5

The p -value for this test is:

- A. 0.036 B. 0.072 C. 0.964 D. 0.928 E. -1.95

36. A soft-drink vending machine is set to dispense 6.0 oz per cup. The owner suspects it is overfilling the cups and really needs recalibration, so he writes the following hypotheses to conduct a test:

$$H_0 : \mu \leq 6 \quad H_a : \mu > 6 \text{ (claim)}$$

Testing sample cups nineteen times, he obtains a sample mean of 6.17 oz. with a standard deviation of 0.15 oz. He enters this data and receives a printout which states $t = 4.9400$ $p = 5.2857E-5$.

What should he do at the 0.05 level of significance? (Circle the correct choice)

- A. Reject the claim and leave the machine alone
- B. Reject the null hypothesis and leave the machine alone.
- C. Reject the null hypothesis and recalibrate the machine.
- D. Fail to reject the null hypothesis and recalibrate the machine.
- E. Fail to reject the null hypothesis and leave the machine alone.

37. The linear correlation r can be found for:

- A. any two variables
- B. any qualitative variable and any quantitative variable
- C. any two qualitative variables
- D. any two quantitative variables
- E. C & D

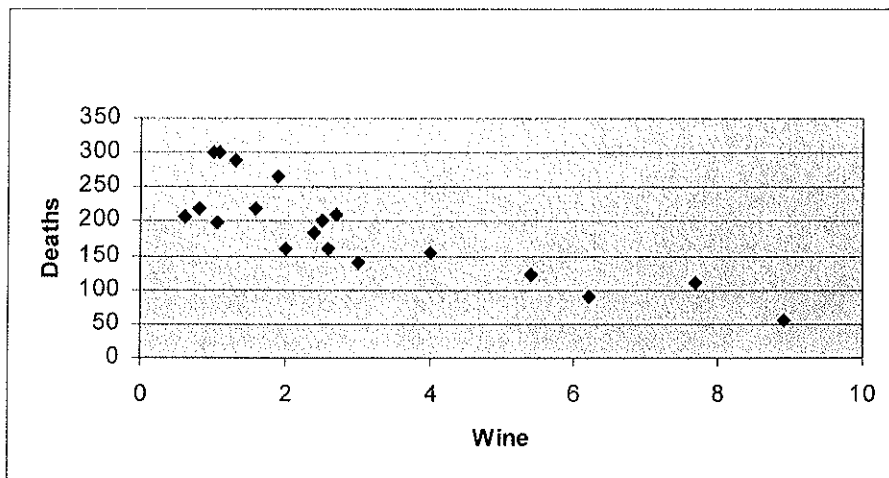
38. In a certain town, 50% of the households own a cellular phone, 40% own a pager, and 20% own both a cellular phone and a pager. The proportion of households that own neither a cellular phone nor a pager is

- A. 10%
- B. 80%
- C. 70%
- D. 90%
- E. 30%

39. If we reject the null hypothesis when, in fact, it is true, we have

- A. committed a Type I error
- B. committed a Type II error
- C. committed a Type III error
- D. a probability of being correct that is equal to the P-value
- E. a probability of being correct that is equal to $1 - P$ -value

40. For nineteen developed Western countries, the number of liters of wine consumed per person, and the heart disease rate per 100,000 people appear in the graph. $Y = 260.4 - 23.95 X$ is the least square regression line and the linear correlation coefficient is -0.83 where $Y =$ deaths and $X =$ liters of wine consumed. Do these data provide strong evidence that drinking wine actually causes a reduction in heart disease deaths?



- A. Yes, the strong straight line association in the plot shows that wine has a strong effect on heart disease deaths.
- B. No, countries that drink lots of wine may differ in other ways from countries that drink little

wine. We can't be sure the wine accounts for the difference in heart disease deaths.

- C. Yes, the data come from a randomized comparative experiment, and this is the best way to evidence for cause-and-effect
- D. No, the plot shows that deaths go up as more wine is consumed.
- E. No, since the correlation is negative.

