

FORTY-EIGHTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Statistics 2004

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

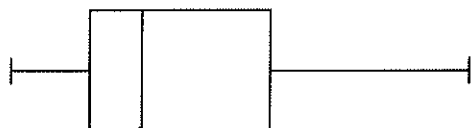
When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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TMTA Statistics Test 2004

1. The birth weights of babies born at City General Hospital have a normal distribution with mean weight 7.5 pounds and standard deviation 1.0 pound. Approximately 95% of the babies will weigh between
 - (a) 5.5 and 9.5 pounds
 - (b) 6.0 and 9.0 pounds
 - (c) 6.5 and 8.5 pounds
 - (d) 7.0 and 8.0 pounds
 - (e) 7.3 and 7.7 pounds
2. An observation that is extremely small or extremely large relative to the rest of the data in a data set is called
 - (a) A standard deviate
 - (b) A standard score
 - (c) An outlier
 - (d) A percentile
 - (e) An interquartile range observation
3. Suppose a set of data has boxplot shown below.



Consider the following three statements about the data set.

- I. The distribution of the data set is skewed.
- II. The mean is larger than the median.
- III. The standard deviation is positive.

Which of the above statements are true?

- (a) I and II only
- (b) I and III only
- (c) II and III only
- (d) I, II, and III
- (e) I only

4. In a group of teenagers, 20% brush their teeth after each meal, 40% drive safely, and 10% do both. What is the probability that a randomly selected teenager from the group has at least one of these good habits?
- (a) .30
 - (b) .40
 - (c) .50
 - (d) .60
 - (e) .70
5. In order to be accepted into a top-level university, applicants must score within the top 5% on the SAT exam. Given that scores on the exam are normally distributed with a mean of 1000 and a standard deviation of 200, what is the lowest possible score (approximately) a student needs to qualify for acceptance into this university?
- (a) 1600
 - (b) 1516
 - (c) 1389
 - (d) 1329
 - (e) 1257
6. The Academy of Orthopedic Surgeons states that the proportion of women who wear shoes that are too small for their feet is .80. An independent researcher wants to construct a 95% confidence interval for the true proportion p . What sample size is needed so that the researcher's estimate has an approximate margin of error of .03? (Use .80 as a preliminary estimate of p .)
- (a) 358
 - (b) 683
 - (c) 817
 - (d) 1067
 - (e) 1291
7. A student randomly guesses at 40 multiple-choice questions. Each question has 5 possible choices. What is the mean and the standard deviation for the number of questions the student answers correctly?
- (a) The mean is 8 and the standard deviation is 6.4.
 - (b) The mean is 8 and the standard deviation is $\sqrt{6.4}$.
 - (c) The mean is 20 and the standard deviation is 5.
 - (d) The mean is 20 and the standard deviation is $\sqrt{5}$.
 - (e) The mean is 40 and standard deviation is 10.

8. A machine that fills milk bottles is supposed to have a mean fill amount μ of 32 ounces. An inspector suspects that the true mean is less than 32 ounces. If a statistical hypothesis test is to be performed, how should the alternative hypothesis be stated?
- (a) $\mu = 32$
 - (b) $\mu > 32$
 - (c) $\mu < 32$
 - (d) $\mu \geq 32$
 - (e) $\mu \leq 32$

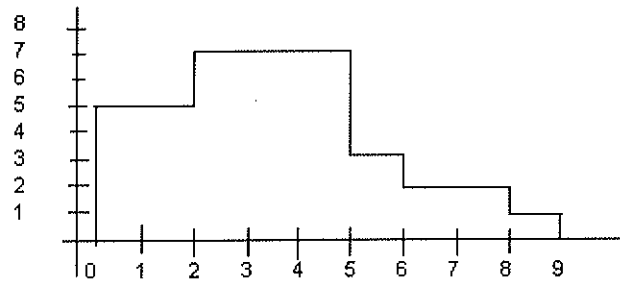
9. A random sample of Big State University students provided the data shown in the table below.

	Smoke	Don't Smoke
Men	60	120
Women	40	160

If smoking were independent of gender, what is the expected count for the table cell "women who don't smoke" assuming a chi square test for independence were performed? (Round your answer.)

- (a) 80
 - (b) 100
 - (c) 133
 - (d) 140
 - (e) 147
10. A shipment of 100 televisions contains 4 defective televisions. If a random sample (without replacement) of 3 televisions is taken from the shipment, what is the probability that the sample will contain no defective televisions? (Round to 3 decimal places.)
- (a) 0.531
 - (b) 0.644
 - (c) 0.739
 - (d) 0.884
 - (e) 0.970

11. Consider the following histogram of a data set. Approximately what percentage of the data is greater than 5?



- (a) 8.0%
(b) 10.5%
(c) 17.5%
(d) 20.5%
(e) 25.0%
12. Suppose a set of integer-valued data has the following frequency distribution.

Class	Frequency
70 – 77	25
78 – 85	20
86 – 93	9
94 – 101	6
102 – 109	3

Find the lowest and highest possible values for the mean of the data set, rounded to the nearest integer.

- (a) 70 and 85
(b) 78 and 93
(c) 79 and 86
(d) 81 and 88
(e) 86 and 93
13. Which one of the following confidence levels gives the widest confidence interval?
- (a) 90%
(b) 95%
(c) 97%
(d) 98%
(e) 99%

14. Suppose that some data have a mean of 20 and a standard deviation of 5. Replacing each original data item with the number resulting from multiplying the original data item by 3 and then subtracting 10 creates a new data set. (For example, if 12 were in the original data set, then it would be replaced by 26 since $3 \times 12 - 10 = 26$.) What are the mean and standard deviation of the new data?

- (a) 20 and 5
- (b) 50 and 5
- (c) 50 and 15
- (d) 60 and 5
- (e) 60 and 15

15. Consider the following four statements concerning hypothesis testing.

- I. We can decrease the probability of a Type I error by increasing the sample size.
- II. We can decrease the probability of a Type I error by decreasing the sample size.
- III. We can decrease the probability of a Type I error by increasing the level of significance α .
- IV. We can decrease the probability of a Type I error by decreasing the level of significance α .

Which of the above statements are true?

- (a) I only.
- (b) II only.
- (c) II and III only.
- (d) I and IV only.
- (e) None of the statements are true.

16. A machine that fills milk bottles is supposed to fill each bottle with a mean amount of milk equal to 64 ounces. An inspector suspects that the true mean is more than 64 ounces. Fifty bottles are randomly checked and their mean is found to be 64.1 ounces, with the sample standard deviation being 0.35 ounces. If a statistical hypothesis test is to be performed, using a significance level of 0.05, what is the proper conclusion?

- (a) There is sufficient evidence at the 0.05 level of significance to conclude that μ is less than 64 ounces.
- (b) There is sufficient evidence at the 0.05 level of significance to conclude that μ is more than 64 ounces.
- (c) There is not sufficient evidence at the 0.05 level of significance to conclude that μ is less than 64 ounces.
- (d) There is not sufficient evidence at the 0.05 level of significance to conclude that μ is more than 64 ounces.
- (e) A larger sample is needed if we want to perform this test.

17. Jane wants to find a 95% confidence interval for a population mean using the *t-distribution*. She has a sample of 15 observations. What is the critical value from the *t*-table used in constructing the confidence interval?
- (a) 1.645
 - (b) 1.960
 - (c) 2.131
 - (d) 2.145
 - (e) 2.576
18. A nutritionist wants to estimate the mean amount of cholesterol in a certain variety of chicken eggs. She wants to be within 10 mg with 99% confidence. It is believed that the standard deviation is 25 mg. How large a sample of eggs should she use?
- (a) 7
 - (b) 17
 - (c) 42
 - (d) 163
 - (e) 248
19. Assume the ACT Math section has a mean of 20 and a standard deviation of 5 points. Furthermore, assume that the SAT Math section has a mean of 500 and a standard deviation of 100 points. John scored 678 and David scored 688 on the Math SAT, and Sam scored 29 on the Math ACT. Which ordering of the students shown below indicates their relative standing on the standardized tests? The ordering should reflect the lowest to the highest relative test score.
- (a) John, David, Sam
 - (b) David, Sam, John
 - (c) Sam, John, David
 - (d) Sam, David, John
 - (e) John, Sam, David
20. In hypothesis testing, if the *p*-value (the observed level of significance) is less than or equal to α (the chosen level of significance), then
- (a) We have sufficient evidence to reject H_0 .
 - (b) We do not have sufficient evidence to reject H_0 .
 - (c) We have increased the power of the test.
 - (d) We have committed a Type I error.
 - (e) We have committed a Type II error.

21. Which of the following statements **best** describes the Central Limit Theorem?

- (a) When sampling from a normal population, the sample mean is approximately equal to the population mean.
- (b) When sampling from a population with standard deviation σ , the sample mean will have standard deviation $\frac{\sigma}{\sqrt{n}}$.
- (c) The sample mean is an unbiased estimator of the population mean.
- (d) When sampling from any population, the sample mean has an approximate normal distribution when the sample size is sufficiently large.
- (e) When sampling from a population with a normal distribution, the sample mean has a normal distribution when the sample size is sufficiently large.

22. Given the following stem and leaf plot of test scores, find the 5-number summary.

```
4 | 4 8
5 | 1 3 9
6 | 1 4 5 6
7 | 0 2 2 4 7 8 8
8 | 0 1 3 5 5 6 8 8 9
9 | 0 2 4 4 6 8
```

- (a) Minimum = 44, Q1 = 65, median = 78, Q3 = 88, maximum = 98
- (b) Minimum = 44, Q1 = 64, median = 78, Q3 = 89, maximum = 98
- (c) Minimum = 44, Q1 = 66, median = 78, Q3 = 77, maximum = 98
- (d) Minimum = 44, Q1 = 65, median = 80, Q3 = 85, maximum = 98
- (e) Minimum = 44, Q1 = 65, median = 80, Q3 = 88, maximum = 98

23. For the probability distribution given in the table below, find the mean.

x	1	2	3
$P(x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

- (a) $\mu = 0$
- (b) $\mu = \frac{2}{3}$
- (c) $\mu = 1$
- (d) $\mu = \frac{5}{3}$
- (e) $\mu = 2$

24. A political candidate wants to estimate his chances of winning the coming election for mayor. Out of a random sample of 1000 voters, 460 voters stated they supported the candidate. Find the 95% confidence interval for p , the true proportion of supporters. (Round to two decimal places.)

- (a) (.43, .49)
- (b) (.42, .50)
- (c) (.41, .51)
- (d) (.40, .52)
- (e) (.39, .53)

25. A random sample of the ACT scores of 100 students at State University provided a sample mean score of 22.0 with a sample standard deviation of 6.10. What is the 90% confidence interval for the mean score of all State University students who took the ACT?

- (a) (17.7, 26.3)
- (b) (18.1, 25.9)
- (c) (19.2, 24.8)
- (d) (20.8, 23.2)
- (e) (21.0, 23.0)

26. Four different brands of golf balls were tested for distance traveled when hit by a golf club. An incomplete analysis of variance table is given below.

Source of Variation	DF	Sum of Squares	Mean Square	F Value	P-value
Brand (treatment)	3	2794.389	931.463	???	0.000
Error	36	762.301	21.175		
Total	39	3556.690			

- (a) The F-statistic value is 3.67; we conclude that at least two means differ.
- (b) The F-statistic value is 3.67; we conclude that the means could very well be the same.
- (c) The F-statistic value is 43.99; we conclude that at least two means differ.
- (d) The F-statistic value is 43.99; we conclude that the means could very well be the same.
- (e) The F-statistic value cannot be determined with the information provided.

27. Suppose 4 dice are rolled. What is the probability that at least one die shows a "1"?

- (a) 1.000
- (b) 0.167
- (c) 0.385
- (d) 0.482
- (e) 0.518

28. A musician plans to perform 9 selections. In how many ways can she arrange the musical selections?

- (a) 387,420,489
- (b) 362,880
- (c) 1,068
- (d) 81
- (e) 9

29. Two methods of teaching reading to first graders are being compared. Independent random samples provided the following reading score data.

<u>Method 1</u>	<u>Method 2</u>
$\bar{x}_1 = 65.7$	$\bar{x}_2 = 72.5$
$s_1 = 10.3$	$s_2 = 12.6$
$n_1 = 33$	$n_2 = 45$

Let μ_1 = Method 1 population mean reading score and
 μ_2 = Method 2 population mean reading score.

Construct a 99% confidence interval for $\mu_2 - \mu_1$. (Round to one decimal place.)

- (a) 6.8 ± 4.3
- (b) 6.8 ± 4.7
- (c) 6.8 ± 5.1
- (d) 6.8 ± 5.4
- (e) 6.8 ± 6.7

30. Which of the following is more likely: getting exactly 8 heads when tossing a fair coin 10 times, getting exactly 80 heads when tossing a fair coin 100 times, or getting exactly 800 heads when tossing a fair coin 1000 times?

- (a) Getting 8 heads when tossing a fair coin 10 times.
- (b) Getting 80 heads when tossing a fair coin 100 times.
- (c) Getting 800 heads when tossing a fair coin 1000 times.
- (d) The events (choices) above are equally likely.
- (e) There is not enough information to determine.

31. Below is a set of paired data from a random sample of 15 recent fires in a certain suburban area. Distance (in miles) from a fire station is represented by x , and fire damage (in thousands of dollars) is represented by y . Summary statistics are given below.

x	3.4	1.8	4.6	2.3	3.1	5.5	0.7	3.0	2.6	4.3	2.1	1.1	6.1	4.8	3.8
y	26.2	17.8	31.3	23.1	27.5	36.0	14.1	22.3	19.6	31.3	24.0	17.3	43.2	36.4	26.1

Summary Statistics: $\bar{x}=3.28$, $\bar{y}=26.4133$, $\sum xy=1470.65$, $\sum x^2=196.16$,
 $\sum y^2=11,376.48$

The fitted least squares line is given by

- (a) $y = 9.6 + 5.34x$
 - (b) $y = 10.3 + 4.92x$
 - (c) $y = 25.8 + 0.19x$
 - (d) $y = 5.34 + 9.6x$
 - (e) $y = 4.92 + 10.3x$
32. Five U.S. Senators are selected at random from the 100 U.S. Senators. What is the probability that at least two of those selected will be from the same state? (Round to the nearest thousandth.)
- (a) .028
 - (b) .099
 - (c) .105
 - (d) .204
 - (e) .972
33. Sonny runs a carnival game in which he charges \$1 for a player to randomly select 4 cards from a 52 card deck. If the player gets 4 cards of the same color, the player is handed a \$5.00 bill. If 500 players play the game, how much should Sonny expect to profit? (Round to the nearest dollar.)
- (a) \$ 55
 - (b) \$188
 - (c) \$207
 - (d) \$224
 - (e) \$362

34. Consider the following three statements.

- I. If a sufficiently large random sample is taken, the sample mean will be very close to the population mean.
- II. If a fair coin is tossed 100,000 times, the proportion of heads will be very close to $\frac{1}{2}$.
- III. If a fair coin is tossed and the first five outcomes are all heads, then a tail is more likely to occur on the sixth toss.

Which of the above statements exemplify the law of large numbers?

- (a) I and II only
- (b) I and III only
- (c) II and III only
- (d) I, II, and III
- (e) None of the statements

35. A medication is claimed to lower blood pressure. A sample of eight patients was given this medication for two weeks. Their systolic blood pressure was taken before and after using this medication for a two-week period. If a hypothesis test is performed using a paired sample t -test (under the null hypothesis), what is the value of the test statistic?

Systolic Blood Pressure <u>Before</u> Taking the Medication	Systolic Blood Pressure <u>After</u> Taking the Medication For Two Weeks
135	124
120	118
155	148
137	128
115	112
187	172
173	161
160	151

- (a) 9.5×10^{-4}
- (b) 0.48
- (c) 0.72
- (d) 4.41
- (e) 5.45

36. A nationwide study of American homeowners estimated that 65% have one or more lawn mowers. A lawn equipment retailer in Atlanta feels the estimate is too low for households in Atlanta. Conducting its own study, the retailer randomly sampled 497 homes in Atlanta, and found that 340 had one or more lawn mowers. Find the approximate p-value for a test of the claim that the percentage of Atlanta homeowners with lawn mowers is higher than 65%.
- (a) .03
 - (b) .06
 - (c) .07
 - (d) .09
 - (e) .11
37. A regression equation is obtained and it is found that the total variation is 24.384, the explained variation is 18.199, and the unexplained variation is 6.185. Find the coefficient of determination.
- (a) 0.254
 - (b) 0.340
 - (c) 0.493
 - (d) 0.746
 - (e) 1.340
38. Releasing a guilty person for lack of evidence is analogous to
- (a) Committing a Type I error.
 - (b) Committing a Type II error.
 - (c) Increasing the level of significance.
 - (d) Increasing the power of the test.
 - (e) Decreasing the level of significance.
39. Suppose a 95% confidence interval for the mean heart rate (beats/minute) of a particular population is (69.7, 74.5). Which one of the following statements is correct?
- (a) We are 95% certain that average heart rate of the population is between 69.7 and 74.5.
 - (b) Ninety-five percent of the population has a heart rate somewhere between 69.7 and 74.5.
 - (c) Five percent of the population has a heart rate outside the range 69.7 to 74.5.
 - (d) Ninety-five percent of the time each person's heart has a rate somewhere between 69.7 and 74.5.
 - (e) Ninety-five percent of the population has an average heart rate and 5% does not.

40. Consider the data collected from Television Station XYZ's weather forecasts. The columns represent what the weather forecaster predicted. The rows represent what actually happened on that day. Use this data to determine the probability that it rains when rain is forecasted (rounded to three decimal places).

	Forecast Rain	Did NOT Forecast Rain
Rain	2 days	8 days
Not Rain	4 days	6 days

- (a) .100
- (b) .300
- (c) .333
- (d) .556
- (e) .700

**Forty-eighth Annual Mathematics Contest
Tennessee Mathematics Teachers Association
STATISTICS 2004**

Notation:

$P(A)$ represents the probability of the event A .

The letter z always represents a quantity having a standard normal (i.e. Gaussian) distribution.

Some possibly useful formulas:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

$$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Only two statistical tables are needed and provided for this contest exam: the 'standard normal' table of probabilities and the t-table of critical values. The form of the standard normal table provided has probabilities of the form $P(0 < z < c)$, where c is a constant ranging from 0 to 3.49. *Each problem on this contest exam has an ordinary solution not requiring any other statistical tables.*

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CRITICAL VALUES OF "STUDENT'S T" DISTRIBUTION

Critical values t_p satisfy $p = P(t \geq t_p)$.

d.f.	$t_{.250}$	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.0025}$	$t_{.001}$	$t_{.0005}$
1	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
inf.	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

