



# TENNESSEE MATHEMATICS TEACHERS ASSOCIATION

## SIXTY-SIXTH ANNUAL MATHEMATICS CONTEST

2024

Precalculus

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Scoring Formula:  $4 \times (\text{Number Right}) - (\text{Number Wrong}) + 40$

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Directions:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. Which of the following is equivalent to  $\left(\frac{2x^{-3}}{3y^{-1}}\right)^{-2}$  ?

- A.  $\frac{9x^6}{4y^2}$       B.  $\frac{2x^6}{3y^4}$       C.  $-\frac{2x^6}{3y^4}$       D.  $\frac{4y^2}{9x^6}$       E.  $\frac{2y^2}{3x^6}$

2. Which of the following is an equation of the line perpendicular to the line  $y = \frac{3}{2}x - 4$  that contains the point  $(2, -3)$ ?

- A.  $y = -\frac{2}{3}x - \frac{5}{3}$   
B.  $y = -\frac{3}{2}x$   
C.  $y = -\frac{3}{2}x - 6$   
D.  $y = \frac{3}{2}x - 6$   
E.  $y = -\frac{2}{3}x - \frac{13}{3}$

3. If six kids run a race, how many different ways can they finish if Rufus finishes 3<sup>rd</sup>?

- A. 120      B. 240      C. 720      D. 3125      E. 7776

4. Let  $f(x) = 3 - 2x$ , and  $g(x) = -x^2 + 1$ . What is  $(f \circ g)(2)$ ?

- A. -3      B. 0      C. 2      D. 3      E. 9

5. What is the period of  $4\cos\left(\frac{x-3\pi}{4}\right) + 12$  ?

- A.  $\frac{\pi}{4}$       B.  $\frac{4\pi}{3}$       C.  $2\pi$       D.  $4\pi$       E.  $8\pi$

6. What is the magnitude of the vector  $\langle 2, -4 \rangle$ ?

- A.  $4\sqrt{5}$       B.  $2\sqrt{5}$       C.  $2\sqrt{2}$       D.  $2\sqrt{5}$       E.  $\sqrt{2}$

7. What is the domain of  $f(x) = \ln(x^2 - 2x - 3)$ ?

- A.  $[-1, 3]$   
B.  $(-1, 3)$   
C.  $(-\infty, -1] \cup [3, \infty)$   
D.  $(-\infty, -1) \cup (3, \infty)$   
E. all real numbers

8. How many of the numbers in the set  $\left\{-8, -\frac{2}{3}, 0, \sqrt{2}, 0.5\bar{2}, \pi, 3.37, 5\right\}$  are rational?

- A. 4      B. 5      C. 6      D. 7      E. 8

9. At which values for  $x$  does the function  $f(x) = \frac{x^2 - 6x + 8}{x^2 + x - 6}$  have vertical asymptotes?

- A. -3      B. 4      C. 2 and 4      D. -3 and 2      E. -3, 2, and 4

10. A fifth-degree polynomial is known to have  $3 - 2i$  as one of its zeros. What are the possible number of distinct real zeros that it could have?

- A. 1 or 3
- B. 0, 1, 2, or 3
- C. 1, 2, or 3
- D. 0, 1, 2, 3, or 4
- E. 1, 2, 3, or 4

11. What is the maximum value of  $f(x) = -2(x+4)\left(x+\frac{3}{2}\right)$ ?

- A.  $-\frac{25}{8}$
- B.  $\frac{25}{8}$
- C. 3
- D.  $-\frac{11}{4}$
- E.  $\frac{11}{4}$

12. A radioactive substance decays exponentially with a half-life of 13 years. If we start with 1000 g, approximately how many years will it take for there to be 10 g?

- A. 87
- B. 86
- C. 85
- D. 84
- E. 83

13. What is the inverse function of  $f(x) = \frac{2x^3 + 4}{7}$  or is the inverse not a function?

A.  $f^{-1}(x) = \sqrt[3]{\frac{7x-4}{2}}$

B.  $f^{-1}(x) = \sqrt[3]{\frac{7x+4}{2}}$

C.  $f^{-1}(x) = \sqrt{\frac{7x+4}{2}}$

D.  $f^{-1}(x) = \frac{7x^3-4}{2}$

- E. The inverse is not a function.

14. If  $\log_b 2 \approx 0.344$ ,  $\log_b 3 \approx 0.545$ , and  $\log_b 5 \approx 0.799$ , then what is the approximate value of  $\log_b \frac{10}{9}$ ?

- A. 0.053      B. 0.504      C. 0.530      D. 0.598      E. 5.98

15. What value must  $c$  have for function  $f$  given below to be continuous?

$$f(x) = \begin{cases} -\frac{2}{c}x - 7 & \text{if } x < 3 \\ (x - 1)^2 + 4 & \text{if } x \geq 3 \end{cases}$$

- A.  $-\frac{5}{2}$       B.  $-\frac{3}{2}$       C.  $-\frac{2}{3}$       D.  $-\frac{2}{5}$       E.  $\frac{5}{2}$

16. What is the sixth term of the geometric sequence 12, 18, 27, ... ?

- A. 42      B. 54      C. 60.75      D. 72      E. 91.125

17. What is  $\sin^2 \theta \cos \theta \csc^5 \theta \tan^3 \theta$  when simplified?

- A. 1      B.  $\tan^2 \theta$       C.  $\cot^2 \theta$       D.  $\csc^2 \theta$       E.  $\sec^2 \theta$

18. The math club consists of 6 girls and 6 boys. If 3 members are chosen at random, what is the probability that they are all boys or all girls?

- A.  $\frac{1}{12}$       B.  $\frac{1}{11}$       C.  $\frac{1}{8}$       D.  $\frac{2}{11}$       E.  $\frac{1}{4}$

19. You invest \$2400 in an account that pays 8% interest compounded monthly. If you leave the money in the account for 6 years, how much interest will you have earned?

- A. \$3872.41      B. \$3808.50      C. \$1478.58      D. \$1472.41      E. \$1408.50

20. What is the complex number  $\frac{1+2i}{2-3i}$  when written in standard form?

- A.  $-\frac{4}{5} + \frac{7}{5}i$
- B.  $\frac{4}{5} + \frac{7}{5}i$
- C.  $-\frac{4}{13} + \frac{5}{13}i$
- D.  $\frac{8}{13} + \frac{7}{13}i$
- E.  $-\frac{4}{13} + \frac{7}{13}i$

21. Given  $a + b = 4$ ,  $a + c = -12$ , and  $b + c = 6$ , what is the value of  $a + b + c$ ?

- A. -2
- B. -1
- C. 0
- D. 1
- E. 2

22. What is the radius of the circle  $36x^2 - 36x + 36y^2 + 24y + 10 = 0$ ?

- A.  $\frac{\sqrt{2}}{6}$
- B.  $\frac{\sqrt{3}}{6}$
- C.  $\frac{1}{3}$
- D.  $\frac{\sqrt{5}}{6}$
- E.  $\frac{\sqrt{6}}{6}$

23. Two sides of a triangle are length 3 cm and 4 cm. If the angle between these sides is  $60^\circ$ , how long is the third side?

- A.  $\frac{\sqrt{13}}{2}$  cm
- B.  $\frac{5}{2}$  cm
- C.  $\sqrt{13}$  cm
- D.  $2\sqrt{5}$  cm
- E. 5 cm

24. If  $\tan \theta = -\frac{\sqrt{5}}{3}$  and  $\theta$  is in quadrant II, what is the value of  $\csc \theta$ ?

- A.  $-\frac{\sqrt{14}}{3}$
- B.  $\frac{\sqrt{70}}{5}$
- C.  $\frac{\sqrt{14}}{3}$
- D.  $\frac{\sqrt{42}}{14}$
- E.  $\frac{\sqrt{42}}{3}$

25. What is the determinant of the matrix  $\begin{bmatrix} a & -2 \\ 3 & 4 \end{bmatrix}$ ?

- A.  $4a+6$
- B.  $4a-6$
- C.  $a+5$
- D.  $a-5$
- E.  $-24a$

26. If you graph the polar equation  $r = \sin(4\theta)$  you will get a flower shape with how many petals?

- A. 2            B. 3            C. 4            D. 6            E. 8

27. What is the coefficient of  $x^4y^3$  in the expansion of  $\left(-2x + \frac{y}{4}\right)^7$ ?

- A.  $-\frac{35}{2}$         B.  $-\frac{35}{4}$         C.  $\frac{1}{4}$             D.  $\frac{35}{2}$             E.  $\frac{35}{4}$

28. The sum of 5 consecutive even integers is 4 less than the sum of the first 8 consecutive odd positive integers. What is the smallest of the even integers?

- A. 2            B. 4            C. 6            D. 8            E. 10

29. In a survey of 100 students, 50 like history, 45 like science, 40 like math, 20 like history and science, 15 like math and science, 20 like history and math, and 5 liked all three subjects. How many students do not like any of the three subjects?

- A. 5            B. 10            C. 15            D. 20            E. 25

30. If  $c > 8$ , what is the solution set for  $-3|x + 2| + c \geq 8$ ?

- A.  $\left(-\infty, \frac{14-c}{3}\right) \cup \left(\frac{c-2}{3}, \infty\right)$   
B.  $\left(-\infty, \frac{14-c}{3}\right] \cup \left[\frac{c-2}{3}, \infty\right)$   
C.  $\left(-\infty, \frac{2-c}{3}\right] \cup \left[\frac{c-14}{3}, \infty\right)$   
D.  $\left[\frac{14-c}{3}, \frac{c-2}{3}\right]$   
E.  $\left[\frac{2-c}{3}, \frac{c-14}{3}\right]$

31. What is the remainder when  $x^4 - 3x^3 + 2x - 5$  is divided by  $x^2 - x + 1$ ?

- A. -2            B. -5            C.  $x-5$         D.  $3x-5$         E.  $x-2$

32. For the polynomial  $4x^4 - 3x^2 + 8x + 6$ , what is the sum of all possible positive rational zeros given by the rational zeros theorem?

- A. 11      B. 12      C. 13      D. 14      E. 15

33. If  $\cos \theta = \frac{\sqrt{3}}{4}$ , what is the value of  $\cos(2\theta)$ ?

- A.  $-\frac{13}{16}$       B.  $-\frac{5}{8}$       C.  $-\frac{3}{8}$       D.  $\frac{3}{8}$       E.  $\frac{5}{8}$

34. A theater has 26 rows of seats. The first row has 20 seats, and each row after it has 2 more seats than the row before. How many total seats does the theater have?

- A. 1100      B. 1118      C. 1170      D. 1242      E. 1294

35. If  $\frac{2x-15}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3}$ , what is the value of  $A - B$ ?

- A. -4      B. -3      C. -2      D. 0      E. 1

36. The polynomial  $5x^3 - 3x^2 - \frac{9}{5}x + \frac{27}{5}$  can be factored to  $5(x-a)(x-b)(x-c)$ . What is the value of  $a+b+c$ ?

- A. -3      B.  $\frac{9}{25}$       C.  $\frac{3}{5}$       D.  $\frac{9}{5}$       E.  $\frac{27}{5}$



37. What is the polar equivalent to the equation  $2x^2 - 3x + 2y^2 + y = 0$ ?

A.  $r = 6\cos\theta - 2\sin\theta$

B.  $r = -\frac{3}{2}\cos\theta + \frac{1}{2}\sin\theta$

C.  $r = \frac{3}{2}\cos\theta - \frac{1}{2}\sin\theta$

D.  $r = 3\cos\theta - \sin\theta$

E.  $r = -3\cos\theta + \sin\theta$

38. A surveyor is trying to measure the height of a mountain. From his starting position, the angle of elevation to the top of the mountain is  $35^\circ$ . He moves 900 m closer to the base of the mountain and finds that the angle of elevation is now  $47^\circ$ . The height of the device used to measure the angle of elevation is 2 m. To the nearest meter, how tall is the mountain?

- A. 1814 m    B. 1818 m    C. 1822 m    D. 1826 m    E. 1830 m

39. For the system of equations  $\begin{cases} 2x - 3y = 8 \\ x + z = 10 \\ y + cz = 4 \end{cases}$ , what value of  $c$  will result in there being infinitely many solutions?

- A.  $\frac{4}{3}$     B.  $\frac{2}{3}$     C.  $\frac{1}{3}$     D.  $-\frac{1}{3}$     E.  $-\frac{2}{3}$

40. For  $-1 < a < 1$ ,  $\csc(\cos^{-1} a)$  is equivalent to which of the following?

A.  $a$

B.  $\frac{1}{a}$

C.  $\frac{\sqrt{1+a^2}}{1+a^2}$

D.  $\frac{\sqrt{1-a^2}}{1-a^2}$

E.  $\sqrt{1-a^2}$