TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

SIXTY-SEVENTH ANNUAL MATHEMATICS CONTEST

2025

Precalculus

Prepared by:

Scott Houston
University of Memphis
Memphis, TN

Scoring Formula: $4 \times (\text{Number Right}) - (\text{Number Wrong}) + 40$

Directions:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. What value(s) of p satisfy the equation $p^3 = 27$.
   a. $p = 3$
   b. $p = 3, \frac{-3 \pm 3i\sqrt{3}}{2}$
   c. $p = -3, \frac{-3 \pm 3i\sqrt{3}}{2}$
   d. $p = 3, \frac{-3 \pm 3i\sqrt{2}}{2}$
   e. $p = -3$

2. What of the following is equivalent to the expression $\left(\frac{2x^2y}{3x^3}\right)^{-2}$?
   a. $\frac{4x^2}{9y^2}$
   b. $\frac{4y^2}{9x^2}$
   c. $\frac{2x^2}{3y^2}$
   d. $\frac{9x^2}{4y^2}$
   e. $\frac{9y^2}{4x^2}$

3. What are the center and radius of the circle with equation $x^2 + 4x + y^2 + 8y = -17$?
   a. Center: $(2, 4)$; radius: 3
   b. Center: $(2, 4)$; radius: $\sqrt{3}$
   c. Center: $(-2, -4)$; radius: $\sqrt{3}$
   d. Center: $(-2, -4)$; radius: 3
   e. Center: $(2, 4)$; radius: 9

4. What is the coefficient on i, where $i = \sqrt{-1}$, in the expansion of $(-2 - 3i)^3$?
   a. 9
   b. -9
   c. 46
   d. -46
   e. 0

5. What is the domain of $(g \circ f)(x)$ if $f(x) = 2x + 1$ and $g(x) = \sqrt{3x + 2}$?
   a. $\left[-\frac{5}{6}, \infty\right)$
   b. $\left(-\frac{5}{6}, \infty\right)$
   c. $\left[\frac{5}{6}, \infty\right)$
   d. $\left(\frac{5}{6}, \infty\right)$
   e. $(-\infty, \infty)$

6. What are the zeros of the function $f(x) = x^4 - 2x^2 - 8$?
   a. $x = 2, -2, \sqrt{2}, -\sqrt{2}$
   b. $x = 2i, -2i, \sqrt{2}, -\sqrt{2}$
   c. $x = \sqrt{2}, -\sqrt{2}$
   d. $x = 2, -2$
   e. $x = 2, -2, \sqrt{2}i, -\sqrt{2}i$

7. What is the complex number $\frac{-2+3i}{4-i}$ written in standard form?
   a. $\frac{11}{17} - \frac{10}{17}i$
   b. $\frac{11}{17} + \frac{11}{17}i$
   c. $-\frac{11}{17} + \frac{10}{17}i$
   d. $-\frac{11}{13} + \frac{10}{13}i$
   e. $\frac{11}{13} - \frac{10}{13}i$

8. What is the equation of the horizontal asymptote for the curve $y = 3e^{x-2} - 4$?
   a. $y = 4$
   b. $y = 3$
   c. $x = -4$
   d. $y = -4$
   e. $y = 0$

9. What value(s) of x provide real solutions to the equation $\sqrt{x} = \sqrt[4]{x+12}$?
   a. $x = 4$
   b. $x = -3$
   c. $x = 4, -3$
   d. $x = -4, 3$
   e. $x = -4$

10. What value(s) of x satisfy the equation $\begin{vmatrix} x+2 & 3 \\ 4 & x-1 \end{vmatrix} = -8$?
   a. $x = 3, -2$
   b. $x = -3, 2$
   c. $x = 3$
   d. $x = 2$
   e. $x = 4, 3$

11. What is the value of $\sin\left(2\pi - \frac{\pi}{6}\right)$?
   a. 0
   b. $\frac{1}{2}$
   c. $\frac{\sqrt{3}}{2}$
   d. $-\frac{\sqrt{3}}{2}$
   e. $-\frac{1}{2}$

12. What is the difference quotient for the function $f(x) = 2x^3 - 5$?
   a. $6x + 6xh + 2h$
   b. $6x^2 + 6x + 2h^2$
   c. $6x^2 + 6xh + 2h^2$
   d. $6x^2 + 2h^2$
   e. $6x^2 - 6xh - 2h^2$

13. What is the remainder if $3x^3 + 2x^2 - x + 5$ is divided by $2x + 1$?
   a. $\frac{35}{7}$
   b. $-\frac{35}{7}$
   c. $\frac{45}{8}$
   d. $\frac{35}{8}$
   e. $-\frac{45}{8}$

14. For a right triangle, if $\sin\theta = \frac{5}{7}$, what is the value of $\cot\theta$?

a. $\frac{2\sqrt{6}}{5}$
b. $\frac{7}{5}$
c. $-\frac{2\sqrt{6}}{5}$
d. $\frac{\sqrt{6}}{5}$
e. $-\frac{7}{5}$

15. What is the solution set for the inequality $\frac{x+1}{x-1} < \frac{1}{3}$?

a. $[-2, 1]$
b. $(-\infty, -2) \cup (1, \infty)$
c. $(-\infty, -2] \cup [1, \infty)$
d. $(-2, 1)$
e. $(-\infty, \infty)$

16. What is the inverse of the function $f(x) = \log_8(x + 1) + 5$?

a. $f^{-1}(x) = 8^{x-5} + 1$
b. $f^{-1}(x) = 8^{x+5} + 1$
c. $f^{-1}(x) = 8^{x-6}$
d. $f^{-1}(x) = 8^{x-5} - 1$
e. $f^{-1}(x) = 8^{x} + 6$

17. What is the slope of the line that is perpendicular to the line containing the points (-6, 4) and (-2, -5)?

a. $-\frac{4}{9}$
b. $\frac{4}{9}$
c. $-\frac{9}{4}$
d. $\frac{9}{4}$
e. -9

18. What is the value of $A - B$ if A and B satisfy the equation $\frac{A}{x+3} + \frac{B}{x-3} = \frac{4x-6}{x^2-9}$?
    a. -2
    b. 4
    c. -4
    d. 0
    e. 2

19. The access code for a security system consists of six digits from the digits 0 - 9. The first digit cannot be 0, and the third digit cannot be 9. How many different possible codes are there if repetition is allowed?

    a. 1,000,000
    b. 890,000
    c. 810,000
    d. 100,000
    e. 1,250,000

20. If $\cos\theta = -\frac{\sqrt{6}}{3}$ for $\frac{\pi}{2} \leq \theta \leq \pi$, what is the value of $\sin(2\theta)$?
    a. $-\frac{2\sqrt{2}}{3}$
    b. $\frac{2\sqrt{2}}{3}$
    c. $-\frac{\sqrt{2}}{3}$
    d. $\frac{\sqrt{2}}{3}$
    e. $-\frac{3\sqrt{2}}{2}$

21. The point $\left(8, \frac{3\pi}{4}\right)$ in the polar coordinate system corresponds to which point in the Cartesian coordinate system?
    a. $(4\sqrt{2}, 4\sqrt{2})$
    b. $(4\sqrt{2}, -4\sqrt{2})$
    c. $(-4\sqrt{2}, -4\sqrt{2})$
    d. $(-4\sqrt{2}, 4\sqrt{2})$
    e. $(-\sqrt{2}, \sqrt{2})$

22. What is the range of $f^{-1}(x)$ if $f(x) = \frac{x+1}{x-2}$?

a. $(2, \infty)$
b. $(-\infty, \infty)$
c. $(-\infty, 2) \cup (2, \infty)$
d. $(-\infty, 2] \cup [2, \infty)$
e. $(-\infty, 2)$

23. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -3 \\ -2 & -1 \end{bmatrix}$, what is $(AB)^{-1}$?

a. $\begin{bmatrix} -13 & 5 \\ 20 & -8 \end{bmatrix}$
b. $\begin{bmatrix} 13 & 5 \\ 20 & 8 \end{bmatrix}$
c. $\begin{bmatrix} -8 & -5 \\ -20 & -13 \end{bmatrix}$
d. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
e. $\begin{bmatrix} -\frac{13}{4} & \frac{5}{4} \\ 5 & -2 \end{bmatrix}$

24. What is the sum of the first six terms of the sequence $1000, 100, 10, 1, \ldots$?

a. 1111.11
b. 1100.00
c. 10001.11
d. 1111.00
e. 11111.11

25. What are all solutions to the equation $2\sin(x)^2 - \sin(x) = 1$ on the interval $[0°, 360°]$?

a. $90°, 210°, 330°$
b. $90°, 210°$
c. $30°, 90°, 150°$
d. $210°, 270°, 330°$
e. $90°, 270°$

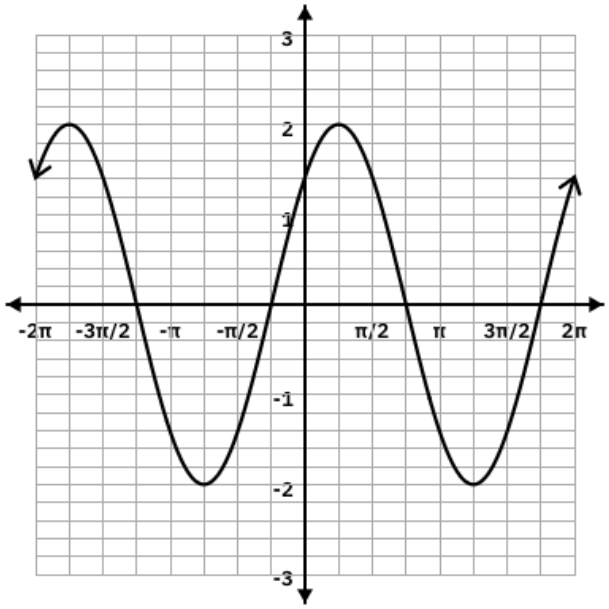
26. Suppose \$3000 is invested in an account earning 3.5% interest compounded semiannually. After t years, \$3694.32 is withdrawn from the account. What is the approximate value of t?
    a. 7 years
    b. 6 years
    c. 10 years
    d. 5 years
    e. 8 years

27. Which of the following polynomials have $3i$ and $-\sqrt{2}$ as zeros?

    a. $2x^4 + 14x^2 - 18$
    b. $x^4 - 7x^2 + 18$
    c. $x^4 + 7x^2 - 18$
    d. $x^4 - 7x^2 - 18$
    e. $2x^4 - 14x^2 - 18$

28. If $\log_a 2 = 1.652$ and $\log_a 5 = 1.903$, what is $\log_a 50$?
    a. 5.983
    b. 2.154
    c. 6.365
    d. 5.458
    e. 4.069

29. Which of the following functions have the graph below?



a. $y = 2\sin\left(x - \frac{\pi}{4}\right)$
b. $y = 2\sin\left(x + \frac{\pi}{4}\right)$
c. $y = 2\cos\left(x + \frac{\pi}{4}\right)$
d. $y = \sin\left(x - \frac{\pi}{4}\right)$
e. $y = \cos\left(x - \frac{\pi}{4}\right)$

30. What is the solution to the equation $2^{x+1} = 3^{2-x}$ rounded to the nearest hundredth?
a. 0.84
b. 0.64
c. -3.71
d. 1.59
e. 0.43

31. If $a = \log\frac{2}{3}, b = \log\frac{3}{5}$, and $c = \log\frac{5}{2}$, what is $a + b + c$?.
a. 1
b. -1
c. 0
d. $\frac{1}{2}$
e. -2

32. A 22-foot ladder reaches the top of a house, and the ladder forms an angle with the wall that measures $60°$. How tall is the house?
    a. 19.05 feet
    b. 38.11 feet
    c. 11.00 feet
    d. 15.00 feet
    e. 44.00 feet

33. What is the value of the sum
$$\sum_{i=1}^{5} 3^i?$$
    a. 365
    b. 363
    c. 425
    d. 373
    e. 14348907

34. What are the vertical asymptote(s) for the function $f(x) = \frac{x^2+2x-3}{x^2+2x+1}$?
    a. $x = -1$
    b. $x = -3$
    c. $x = 0$
    d. $x = 1$
    e. There are no vertical asymptotes.

35. An isosceles triangle has two congruent sides each of length 7 inches with an included angle measuring $74°$. What is the approximate length of the third side?
    a. 8.425 inches
    b. 4.212 inches
    c. 9.025 inches
    d. 6.958 inches
    e. 7 inches

36. What is the domain of $h(t) = \sqrt{t^2 + 5t + 4}$?
    a. $(-\infty, -4) \cup (-1, \infty)$
    b. $(-\infty, 4] \cup [1, \infty)$
    c. $(-\infty, -4] \cup [1, \infty)$
    d. $(-\infty, -4]$
    e. $(-\infty, -4] \cup [-1, \infty)$

37. What is the exact distance from the origin to the vertex of the parabola with equation $h(x) = -\frac{1}{2}x^2 + 4x - 4$?

a. 4 units
b. $\sqrt{2}$ units
c. $4\sqrt{2}$ units
d. $\sqrt{33}$ units
e. 8 units

38. Using a sum or difference formula, what is the exact value of $\cos\left(\frac{7\pi}{12}\right)$?

a. $\frac{\sqrt{2}+\sqrt{6}}{4}$
b. $\frac{\sqrt{2}-\sqrt{6}}{4}$
c. $\frac{1-\sqrt{3}}{2}$
d. $\frac{1+\sqrt{3}}{4}$
e. $\frac{3\sqrt{2}}{2}$

39. What is the equation describing the curve with the following pair of parametric equations?

$$x = 3\cos t + 2$$
$$y = 3\sin t - 5$$

a. $\frac{(x-2)^2}{9} + \frac{(y+5)^2}{9} = 1$
b. $\frac{(x-2)^2}{9} - \frac{(y+5)^2}{9} = 1$
c. $\frac{(x+2)^2}{9} + \frac{(y-5)^2}{9} = 1$
d. $\frac{(x-2)^2}{9} + \frac{(y+5)^2}{9} = 0$
e. $\frac{(x+2)^2}{9} + \frac{(y+5)^2}{9} = 81$

40. What are all zeros of the polynomial $p(x) = x^3 + 2x^2 - 9x - 18$?

a. $x = -3, 3, -2$
b. $x = -3, 3, 2$
c. $x = -3, 3$
d. $x = -3, -2$
e. $x = 3, -2$