TMTA Precalculus Test for 2010

1. Solve the inequality $\frac{2}{x+2} \geq \frac{3}{x-1}$

A. $[-8,\infty)$ B. $[-8,-2)\cup(1,\infty)$ C. $(-\infty,-8]\cup(-2,1)$

D. $(-\infty,-8]$ E. $-8,-2,1$

2. Solve the equation $\sqrt{x+\sqrt{x+4}}=4$

A. No solution B. $x=21$ and $x=12$ C. $x=21$

D. $x=12$ E. $x=0$

3. The area of a triangle is 26 square meters. The height is 1.5 meters greater than the base. Find the dimensions of the triangle.

A. 10 m × 2.6 m B. 8 m × 6.5 m C. 6 m × $4\frac{1}{3}$ m

D. 12.5 m × 4 m E. 4 m × 13 m

4. Evaluate $i^{35}\left(i^{17}-i^{41}\right)$, where $i=\sqrt{-1}$.

A. $-2i$ B. $2i$ C. $i-1$ D. $1-i$ E. 0

5. If $[f(x)]^2 - f(x^2) = 7$ and $f(2)=3$, what is the value of $f(16)$?

A. 0 B. 8 C. -3 D. -10 E. 12

6. Find the slope of the line passing through $(a-b,c)$ and $(a+b,b+c)$.

A. undefined B. $\frac{1}{2}$ C. 2 D. $\frac{2a}{b+2c}$ E. 0

7. A possible polynomial of degree 9 having zeros at 2 and 3, one of which having a multiplicity of 4 is:

A. $g(x)=5x(x-2)^3(x-3)^4$ B. $g(x)=5x(x-2)^5(x-3)^4$

C. $g(x)=5(x+2)^5(x+3)^4$ D. $g(x)=5(x-2)^4(x-3)^5$

E. $g(x)=5(x+2)^9(x+3)^9$

8. Determine the value of A such that the line whose equation is $Ax + y - 5 = 0$ is perpendicular to the line containing points (4, – 2) and (– 1, 5).

A. $\frac{-5}{7}$ B. $\frac{5}{7}$ C. 1 D. – 1 E. – 5

9. The value of $\csc 32°$ is equivalent to which of the following?

A. $\sin(1/32°)$ B. $1 - \cot^2 32°$ C. $\cos^2 58° - \sin^2 32°$

D. $\sec 58°$ E. $\cos 58° / \tan 58°$

10. Which of the following statements about the function $h(t) = 500 + e^{-30t}$ is true?

A. The function is always decreasing and h(t) > 500 for all t.
B. The function is always increasing and h(t) < 500 for all t.
C. The function is always increasing but does not have an upper bound.
D. The function is always decreasing but does not have a lower bound.
E. The function is sometimes increasing and sometimes decreasing.

11. What is the exact value of $\frac{1}{\log_2 36} + \frac{1}{\log_3 36}$?

A. 1/2 B. –2 C. $\ln 6$ D. 31 E. $e^{1/36}$

12. The equation sin(x) = x + 20 has how many solutions?

A. None B. One C. Two D. Four E. Infinitely Many

13. Given that $\tan^{-1}(15) = 1.5$, which of the following statements is valid?

A. The number 15 represents the angle and it is being measured in degrees.
B. The number 1.5 represents the angle and it is being measured in degrees.
C. The number 15 represents the angle and it is being measured in radians.
D. The number 1.5 represents the angle and it is being measured in radians.
E. cot(15) = 1/1.5.

14. The volume V of a gas is a function of its temperature T, namely Volume = V(T). Which of the following expressions would give the average rate of change in the gas' volume with respect to temperature on the interval [10, 18]?

A. $\frac{T(18) + T(10)}{2}$ B. $\frac{T(18) - T(10)}{8}$ C. $\frac{V(18) + V(10)}{2}$ D. $\frac{V(18) - V(10)}{8}$ E. $V(14)$

15. Given $f(x)=x^2-3x+7$ and $g(x)=\sqrt[3]{x+2}$, find $f[g^{-1}(x)]$.

A. x^6-7x^3+17 B. $x^5-3x^4+7x^3-2x^2+6x-14$

C. $\sqrt[3]{x^2-3x+9}$ D. x^6-3x^3+9

E. $x^6+36x^5+121x^4+90x^3+29x^2+189x+47$

16. Solve for x: $\log_8(x) = \frac{2}{3}\log_4\left(\frac{x}{40}\right)+\frac{1}{3}\log_2\left(\frac{x}{10}\right)$

A. 0.16 B 1 C 40 D 400 E No Solution

17. An initial deposit of $4000 is made into an account that earns interest that is continually compounded. The balance will triple in 15 years. What is the annual rate of interest for this account?

A. 6.2 % B. 6.5% C. 7.3% D. 7.9% E. 8.2%

18. What is the range of $f(x)=1+e^{-x}$?

A. $(-\infty,\infty)$ B. $(0,\infty)$ C. $(-1,\infty)$ D. $(1,\infty)$ E. $(2,\infty)$

19. The number of quervils on the planet Zeke at time $t \geq 0$ (t measured in days) is modeled by the function $P(t)=\frac{2000t+2400}{40t+30}$. Which of the following statements follows from this mathematical model?

A. While initially there are 2400 quervils, after a sufficiently long period of time we can expect the population of quervils to die out.

B. While initially there are 2400 quervils, after a sufficiently long period of time we can expect the population to stabilize at about 500 quervils.

C. While initially there are 80 quervils, after a sufficiently long period of time we can expect the population of quervils to die out.

D. While initially there are 80 quervils, after a sufficiently long period of time we can expect the population of quervils to stabilize at about 500 quervils.

E. While initially there are 80 quervils, after a sufficiently long period of time we can expect the population of quervils to stabilize at about 2000 quervils.

20. Suppose $\cos\alpha = -0.6$. Which of the following are true?

A. $\cos(\alpha+\pi) = -0.6$ and $\cos(-\alpha) = -0.6$

B. $\cos(\alpha+\pi) = -0.6$ and $\cos(-\alpha) = 0.6$

C. $\cos(\alpha+\pi) = 0.6$ and $\cos(-\alpha) = -0.6$

D. $\cos(\alpha+\pi) = 0.6$ and $\cos(-\alpha) = 0.6$

E. It is not possible to determine the value of $\cos(\alpha+\pi)$ from the above information.

21. The expression $2\sec^2 x - 2\sec^2 x \sin^2 x - \sin^2 x - \cos^2 x$ is equivalent to which of the following?

A. $2\sec^2 x - 1$ B. $\dfrac{2}{\sec^2 x}$ C. $\dfrac{2}{\sec^2 x} - 1$ D. -1 E. 1

22. The velocity v (m/hr) of a particle at time t (hr) is given by the function $v(t) = -150\sin(\pi t/100)$ for $0 \le t \le 200$. The particle moves backwards when its velocity is negative. For what time interval(s) will the particle be moving backwards?

A. (0,200) B. (0,100) C. (100,200) D. (0,π) E. (π,2π)

23. Find the number of points of intersection for the system $\begin{cases} x^2 + y^2 = 5 \\ x + 2y - 5 = 0 \end{cases}$

A. 4 B. 3 C. 2 D. 1 E. 0

24. Given $H(x) = x^2 - 3$ for $x \le 0$, determine the domain of $H^{-1}(x)$.

A. [0, ∞) B. (-∞, 0] C. [-3, ∞) D. (-∞, -3] E. (-∞, ∞)

25. The average of two positive integers is 66. One of the integers is the square root of the other. What is the difference between the two integers?

A. 110 B. 90 C. 30 D. 29 E. 20

26. Which of the following statements is true of the angle α = 6.

A. The angle rotates in the counterclockwise direction and terminates in quadrant IV.

B. The angle rotates in the clockwise direction and terminates in quadrant I.

C. The angle rotates in the counterclockwise direction and terminates in quadrant I.

D. The angle rotates in the clockwise direction and terminates in quadrant IV.

E. The angle rotates in the counterclockwise direction and terminates on the y-axis.

27. A function F is symmetric with respect to the origin. F is also periodic with a period of length 10. Suppose that $F(3) = -2$. Which of the following statements are true?

A. $F(-13) = 2$ and $F(-3) = 2$ B. $F(-13) = -2$ and $F(-3) = 2$

C. $F(-13) = -2$ and $F(-3) = -2$ D. $F(-13) = 2$ and $F(-3) = -2$

E. It is not possible to determine $F(-13)$ from the above information.

28. Find all real and imaginary roots of $3x^4 - x^3 + 4x^2 - 2x - 4 = 0$

A. $\frac{-2}{3}, 1, \pm i\sqrt{2}$ B. $\frac{-2}{3}, 1, \pm\sqrt{2}$ C. $\frac{-3}{2}, 1, \pm i\sqrt{2}$ D. $\frac{-3}{2}, 1, \pm\sqrt{2}$ E. $\frac{-4}{3}, \pm i$

29. Solve for x: $3^{1-x} = 5$

A. $\log_3(5)$ B. $\log_5(5/3)$ C. $\log_3(3/5)$ D. $\log_3(5/3)$ E. $-1 + \log_3(5/3)$

30. A hiker walks one-quarter of the way around a circular trail of radius 2 miles. Which of the following numbers gives the closest estimate to how far the hiker walked?

A. 1/2 mile B. 1 mile C. 2 miles D. 3 miles E. 4 miles

31. The height s (cm) of an object at time t (sec) is given by the function $h(t) = 300 - 15\cos(\pi t)$, $0 \le t \le 4$. Which of the following statements is true?

A. The object's initial height is 300 cm and its maximum height is 300 cm.

B. The object's initial height is 285 cm and its maximum height is 315 cm.

C. The object's initial height is 300 cm and its maximum height is 315 cm.

D. The object's initial height is 285 cm and its maximum height is 300

E. The object's initial height is 285 cm. It is not possible to determine its maximum height.

32. What is the inverse of the function $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$?

A. $f^{-1}(x) = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$ B. $f^{-1}(x) = \dfrac{1}{2}\ln\left(\dfrac{x+1}{1-x}\right)$ C. $f^{-1}(x) = \dfrac{2\ln x}{\ln(x+1)}$

D. $f^{-1}(x) = \dfrac{e^x}{\ln(1-x)}$ E. This function has no inverse.

33. What is the coefficient of the 'x^4y^2' term in the binomial expansion $(2x - 3y)^6$?

A. 720 B. 864 C. 1440 D. 2160 E. 2880

34. For any value a such that $-1 \le a \le 1$, what is the value of $\sin^{-1}(a) + \cos^{-1}(a)$?

A. 0 B. π C. $\pi/2$ D. $\pi/4$ E. Undefined.

35. The polar coordinate $(6, 120°)$ is equivalent to all of the following except which one?

A. $(-6, -120°)$ B. $(-6, -60°)$ C. $(6, -240°)$ D. $(6, 480°)$ E. $(-6, 300°)$

36. If it takes 20 minutes for a population of bacteria to double, what is the time it takes for the population to triple?

A. 30 minutes B. 31.7 minutes C. 38.9 minutes D. 40 minutes E. 43.6 minutes

37. A utility pole is anchored to level ground by two 53 ft wires, one on each side of the pole. The wires make a $65°$ angle with the ground below. How far apart along the ground are the wires?

A. 59.6 feet B. 51.2 feet C. 44.8 feet D. 42.6 feet E. 38.7 feet

38. What is an algebraic expression equivalent to $\tan(\cos^{-1} b)$?

A. $\sqrt{1-b^2}$ B. $b\sqrt{1-b^2}$ C. $\dfrac{\sqrt{b^2-1}}{b^2}$ D. $\dfrac{\sqrt{1-b^2}}{b}$ E. $\dfrac{\sqrt{b^2+1}}{b^2+1}$

39. Find the value of $(i-1)^{10}$.

A. 32 B. -32 C. $32i$ D. $-32i$ E. 0

40. What is the horizontal component of a vector having a magnitude of 80 units and a heading of N 40° W?

A. 74.4 B. -74.4 C. -29.4 D. -85.6 E. -56.7

KEY

1. C
2. D
3. B
4. E
5. C
6. B
7. D
8. B
9. D
10. A
11. A
12. B
13. C
14. D
15. A
16. D
17. C
18. D
19. D
20. C
21. E
22. B
23. D
24. C
25. A
26. A
27. B
28. A
29. C
30. D
31. B
32. B
33. D
34. C
35. A
36. B
37. C
38. D
39. D
40. C