

FORTY-EIGHTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Precalculus 2004

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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TMTA Precalculus

1. Find the equation of the line that is perpendicular to a line that passes through the points (2, 5) and (4, -3) and that passes through the point (2, 5).

- a) $y = -4x + 13$ b) $y = 0.25x + 4.5$ c) $y = -4x + 3$
 d) $y = 0.25x - 5.5$ e) $y = -4x - 3$

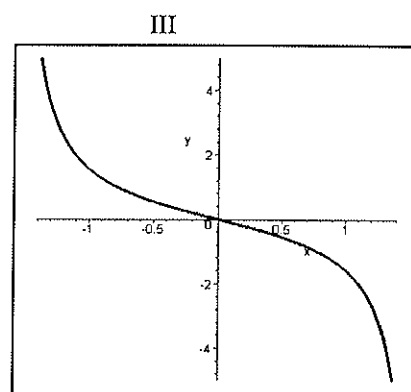
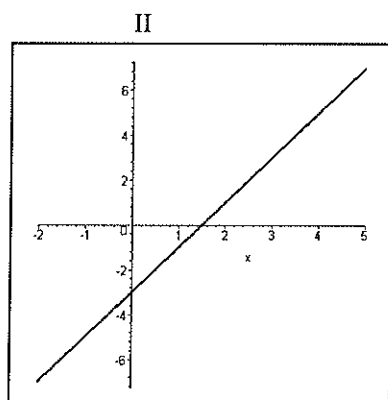
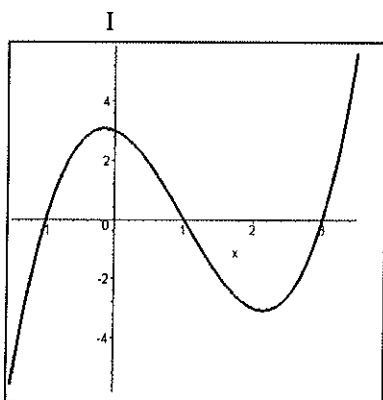
2. Find the domain of the function $h(x) = \frac{3x}{\sqrt{x^2 - 25}}$.

- a) $[-5, 5]$ b) $(-\infty, -5] \cup [5, \infty)$ c) $(-5, 5)$
 d) $(-\infty, -5) \cup (5, \infty)$ e) $(-\infty, -25) \cup (25, \infty)$

3. Find the remainder of $4x^3 + 3x + 5$ divided by $x - 2$.

- a) 0 b) -2 c) 33 d) 43 e) 21

4. Which functions have inverses that are functions?



- a) I only b) II only c) III only d) I and II e) II and III

5. Given $f(x) = 2x^2 - 5$ and $g(x) = x + 6$, find $f(g(-2))$.

- a) 59 b) 28 c) 123 d) -28 e) 27

6. $y = \frac{-2x+1}{1-x}$ has a horizontal asymptote of:

- a) $x = 1$ b) $y = 2$ c) $y = 1$ d) $x = 2$ e) $x = -2$

7. Find the values of k such that $k^2x^2 + 6x + 3 = 0$ has imaginary roots.
 a) $k < \sqrt{3}$ b) $-\sqrt{3} < k < \sqrt{3}$ c) $|k| \leq \sqrt{3}$ d) $k > \sqrt{3}$ e) $|k| > \sqrt{3}$
8. Determine the amplitude of the function $f(t) = 3\cos(4t - \pi) + 5$.
 a) 5 b) $\pi/4$ c) 4π d) 3 e) $\pi/2$
9. Evaluate the cosine of the angle in standard position whose terminal side contains the point $(-4, -3)$.
 a) $-\frac{3}{5}$ b) $-\frac{1}{\sqrt{5}}$ c) $-\frac{4}{5}$ d) $-\frac{4}{3}$ e) $\frac{4}{3}$
10. Suppose x is in the first and y is in the second quadrant, $\sin x = 4/5$ and $\cos y = -1/4$. Determine the exact value of $\sin(x + y)$.
 a) $\frac{4}{5} + \frac{\sqrt{15}}{4}$ b) $\frac{11}{20}$ c) $\frac{-4 - 3\sqrt{15}}{20}$ d) $\frac{4}{5} - \frac{\sqrt{15}}{4}$ e) $\frac{-4 + 3\sqrt{15}}{20}$
11. Determine the value of $\cos 2x$ given that $\cos x = -\frac{3}{5}$ and $\pi < x < \frac{3\pi}{2}$.
 a) $\frac{24}{25}$ b) $\frac{7}{25}$ c) $\frac{1}{5}$ d) $-\frac{7}{25}$ e) 1.
12. The expression $(\csc x + \cot x)(1 - \cos x)$ simplifies to
 a) $\sin 2x$ b) $\cos 2x$ c) $\sin x$ d) $\cos x$ e) none of these
13. $\tan(\sin^{-1} v) =$
 a) $\frac{1}{\sqrt{1+v^2}}$ b) $\frac{\sqrt{1-v^2}}{v}$ c) $\frac{v}{\sqrt{1-v^2}}$ d) $\frac{1}{v^2-1}$ e) $\frac{v}{\sqrt{v^2-1}}$
14. Simplify $\frac{8}{x^2 + 14x + 49} - \frac{4}{x^2 - 49}$.
 a) $\frac{4}{(x+7)^2}$ b) $\frac{4}{(x+7)^2(x-7)}$ c) $\frac{4(x-21)}{(x+7)(x-7)}$ d) $\frac{4x}{(x+7)^2}$ e) $\frac{4(x-21)}{(x+7)^2(x-7)}$

15. The graph of $y = |x|$ is reflected through the x-axis, moved 2 units up and 3 units to the left. What is the equation of the resulting graph?

- a) $y = -|x - 3| + 2$ b) $y = |x - 2| + 3$ c) $y = -|x + 3| + 2$
 d) $y = -|x + 3| - 2$ e) $y = |x - 3| + 2$

16. $\left(\frac{3x^2y^{-3}}{z^3}\right)^{-2}\left(\frac{z^2}{x^{-3}y}\right) =$

- a) $\frac{y^5z^8}{9x}$ b) $\frac{-6z}{x^3y^6}$ c) $\frac{-6y^5z^8}{x}$ d) $\frac{9y^5z^8}{x}$ e) $\frac{-9z}{x^3y^6}$

17. If all denominators are non-zero, $\frac{\frac{x}{y^2} - \frac{y}{x^2}}{\frac{1}{y^2} - \frac{1}{x^2}} =$

- a) $x - y$ b) $\frac{x^2 - xy + y^2}{x - y}$ c) $\frac{x^2 + xy + y^2}{x + y}$ d) $\frac{x - y}{x^2y^2}$ e) $\frac{x + y}{x - y}$

18. Given $f(x) = x^2 + 2x - 3$, find $\frac{f(x + h) - f(x)}{h}$.

- a) $h + 2xh + 2$ b) $h + 2$ c) $\frac{h^2 + 2h + 2xh + 4x - 6}{h}$ d) 1 e) $2x + h + 2$

19. Aircrafts require longer takeoff distances, called takeoff rolls, at high altitude airports because of diminished air density. (Takeoff rolls are also strongly influenced by air temperature; the data shown assume a temperature of 0° C.) If the takeoff roll is 734 feet at an elevation of 1000 feet and 805 feet at 2000 ft, what is the average rate of change of takeoff roll between 1000 and 2000 feet?

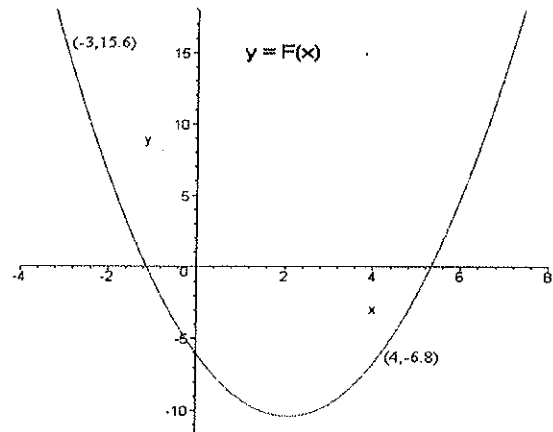
- a) 0.71 b) 71 ft. c) 35.5 d) 35.5 ft. e) 0.071

20. The sides of a right triangle have lengths in inches of three consecutive multiples of 4. Find the area of the triangle.

- a) 48 sq units b) 96 sq units c) 400 sq units d) 24 sq units e) 160 sq units

21. The equation for $y = F(x)$ in the graph is given by:

- a) $y = x^2 - 4.2x - 6$
- b) $y = x^2 + 4.2x + 6$
- c) $y = x^2 - 4.2x + 6$
- d) $y = x^2 + 4.2x - 6$
- e) $y = -x^2 + 4.2x - 6$



22. Which of the following statements are true for $g(x) = \frac{x + 3}{x^3 - 5x}$?

- I. $g(x)$ is positive when $x < -\sqrt{5}$
- II. $g(x)$ is positive when $-\sqrt{5} < x < 0$
- III. $g(x)$ is positive when $0 < x < \sqrt{5}$
- IV. $g(x)$ is positive when $x > \sqrt{5}$

- a) I and III b) I, III and IV c) III only d) I only e) II and IV

23. Which system has infinitely many solutions?

- a) $\begin{cases} x + 3y = 5 \\ 2x + 6y = 5 \end{cases}$ b) $\begin{cases} x + 3y = 5 \\ x - 3y = 5 \end{cases}$ c) $\begin{cases} x + 3y = 5 \\ -x - 3y = 5 \end{cases}$ d) $\begin{cases} x + 3y = 5 \\ -x + 3y = 5 \end{cases}$ e) $\begin{cases} x + 3y = 5 \\ 2x + 6y = 10 \end{cases}$

24. The set of x value(s) that satisfy the system $\begin{cases} y = x^2 + 3x - 3 \\ y = x^3 \end{cases}$ is

- a) $\{1\}$ b) $\{-1\}$ c) $\{\sqrt{3}\}$ d) $\{-\sqrt{3}, 1, \sqrt{3}\}$ e) $\{\}$, the empty set

25. $4x^3\sqrt{2x} + x^2\sqrt{2x^3} - \sqrt{18x^7} =$

- a) $2x^8\sqrt{2x}$ b) $2x^5\sqrt{22x^{11}}$ c) $(4x^3 + x^2 - 1)\sqrt{22x^{11}}$ d) $2x^3\sqrt{2x}$ e) does not simplify

26. Solve $\ln(x + 4) = 3$ for x .

- a) -1 b) $\ln 3 - 4$ c) $\frac{e^3}{4}$ d) $3\ln 4$ e) $e^3 - 4$

27. Solve $\log(x^3 + 3x) - \log x = 1$ for x .

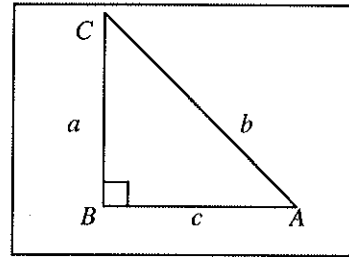
- a) $\{0, \sqrt{3}\}$ b) $\{-\sqrt{7}, \sqrt{7}\}$ c) $\{1, 10\}$ d) $\{0, \sqrt{2}\}$ e) $\{\sqrt{7}\}$

28. The number of solutions of the equation $\tan(x/2) = 5\cos x$ on the interval $0 \leq x \leq 4\pi$ is

- a) 6 b) 4 c) 8 d) 3 e) 5

29. In the figure on the right, given that $\csc A = 3$ and $b = 12$, find the length of c .

- a) $24\sqrt{2}$ b) 128 c) $2\sqrt{2}$ d) 8 e) $8\sqrt{2}$



30. If $\left| \cos\left(2k - \frac{\pi}{4}\right) \right| = \frac{4}{5}$ with $\frac{5\pi}{8} < k < \frac{7\pi}{8}$, what is the $\tan\left(2k - \frac{\pi}{4}\right)$?

- a) $\frac{3}{4}$ b) $\frac{4}{3}$ c) $-\frac{4}{3}$ d) $-\frac{3}{4}$ e) $-\frac{5}{4}$

31. Possible solutions to $2\sin^2 x + \sin x - 1 = 0$ are:

- a) $\frac{\pi}{2}$ and $-\frac{\pi}{6}$ b) $\frac{3\pi}{2}$ and $\frac{5\pi}{6}$ c) $-\frac{\pi}{2}$ and $-\frac{\pi}{6}$ d) $-\frac{\pi}{2}$ and $\frac{\pi}{3}$ e) no solutions

32. A wheel is rotating on its axle at 20 revolutions per minute (rpm). Find the angle in radians through which the wheel turns in 48 seconds.

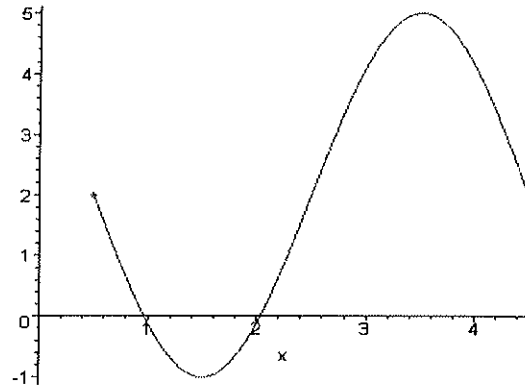
- a) 16π b) 40π c) 16 d) 32π e) 32

33. A riding lawn mower has wheels that are 15 inches in diameter. If the wheels are making 2.5 revolutions per second, how fast is the lawn mower traveling in miles per hour?

- a) 5π b) 6.69 c) 8.23 d) 3π e) 3.17

34. The graph shown in the figure on the right represents a certain sine function. Its equation is

- a) $y = -3\sin\left(\frac{\pi}{4}x + \frac{\pi}{2}\right) + 2$
 b) $y = 2\sin\left(\frac{\pi}{4}x - \frac{\pi}{2}\right) - 3$
 c) $y = -3\sin\left(\frac{\pi}{2}x - \frac{\pi}{4}\right) + 2$
 d) $y = -3\sin\left(\frac{\pi}{4}x - \frac{\pi}{2}\right) + 2$
 e) $y = 2\sin\left(\frac{\pi}{2}x - \frac{\pi}{4}\right) + 3$



35. Let $f(x) = \frac{x-3}{2x+5}$, then $f^{-1}(3) =$

- a) 0 b) -6/11 c) -18/5 d) -12/5 e) -1

36. What is the sum of the infinite series $3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \dots$?

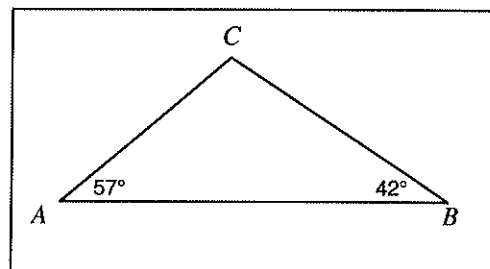
- a) 5.8125 b) 93/16 c) 6 d) 12 e) ∞

37. A certain culture of bacteria doubles every 7 hours. If 1000 bacteria are initially present, approximately how many are present one day later?

- a) 8000 b) 24000 c) 10767 d) 12542 e) 3428

38. A surveyor marks points A and B on one bank of a river so that they are 200 meters apart. He sights a point C on the opposite bank and determines the angles in the figure on the right. What is the approximate distance in meters from A to C ?

- a) 108.9 b) 135.5 c) 141.4 d) 169.8 e) 291.0



39. A rectangular corner lot has dimensions 25 by 40 yards. The city plans to take a strip of uniform width along the two sides bordering the streets in order to widen these roads. Determine how wide the strip should be if the remainder of the lot is to have an area of 814 square yards.

- a) 3 yds. b) 62 yds. c) 5 yds. d) 19.47 yds. e) 2.7 yds.

40. A swimming pool is 3 feet deep in the shallow end. The angle of depression of the bottom of the pool is 12° . The length of the pool is 60 feet. The depth at the deep end is approximately

- a) 61.69 b) 15.75 c) 51.96 d) 12.75 e) 54.96