

FORTY-SEVENTH ANNUAL MATHEMATICS CONTEST  
sponsored by  
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Precalculus 2003

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Scoring formula:  $4R - W + 40$

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**DIRECTIONS:**

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

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Contributors to TMTA for the Annual Mathematics Contest:

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1. Convert  $400^\circ$  to radians.

- A)  $\frac{20\pi}{9}$       B)  $\frac{4\pi}{9}$       C)  $\frac{5\pi}{9}$       D)  $\frac{9\pi}{10}$       E)  $\frac{10\pi}{9}$

2. What is the center of the circle  $x^2 - 2x + y^2 = 3$ ?

- A) (1, 0)      B) (-1, 0)      C) (1, 1)      D) (-2, 1)      E) (-2, 0)

3. The point  $(\frac{8}{3}, k)$  is on the line that passes through (-1, 2) and (2, 4). What is  $k$ ?

- A)  $\frac{13}{3}$       B)  $\frac{14}{3}$       C)  $\frac{25}{6}$       D)  $\frac{27}{6}$       E)  $\frac{40}{9}$

4. If  $x-3$  is divided into  $x^4 - 6x^2 - 5x - 5$ , the quotient is  $x^3 + 3x^2 + 3x + 4 + \frac{n}{x-3}$ . What is  $n$ ?

- A) -47      B) -25      C) 7      D) 15      E) 27

5. The equation  $x^3 - x = 2 - 2x^2$  has three solutions. One solution is -1. What is the sum of the other two?

- A) -1      B) 2      C) 3      D) -4      E) 5

6. When the radius of the base of a cylinder was doubled, the total surface area of the new cylinder was three times as large as the original. What was the ratio of the radius to the height of the original cylinder?

- A) 1:3      B) 1:2      C) 1:1      D) 2:1      E) 3:1

7. What is the domain of this function?  $f(x) = \frac{x-2}{x^2+x-6}$

- A) (-3, 2)      B)  $(-\infty, \infty)$       C)  $(-\infty, -3) \cup (-3, \infty)$   
D)  $(-\infty, -3) \cup (2, \infty)$       E)  $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

8. Solve the equation  $3^x = 2^{x-1}$  for  $x$ . Round your answer to the nearest thousandth.

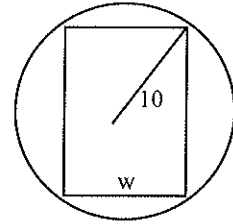
- A) -7.962      B) -1.710      C) -1.050      D) -0.566      E) -0.153

9. The midpoint of the segment connecting (-2, 3) and (5,  $n$ ) lies on the graph of the parabola  $y = 2x^2 - 6$ . What is  $n$ ?

- A) -6      B) -5      C)  $-\frac{3}{2}$       D)  $\frac{5}{6}$       E) 2

10. A rectangle is inscribed in a circle of radius 10. What is the width of the rectangle that will produce the maximum area? Round to the nearest thousandth (if necessary).

- A) 10      B) 11.823      C) 12.113      D) 14.142      E) 15



11. What is the smallest possible area of a right triangle, two of whose sides are 6 and 8?

- A) 12      B)  $2\sqrt{6}$       C)  $4\sqrt{6}$       D)  $6\sqrt{7}$       E) 24

12. The equation  $|2x + 2| = x + 4$  has two solutions. What is their sum?

- A) -2      B) -1      C)  $\frac{1}{2}$       D) 0      E) 2

13. Multiply:  $(4 + i)(3 - i)$

- A)  $12 + 2i$       B)  $13 - i$       C)  $13 + 2i$       D)  $14 - i$       E)  $14 + 2i$

14. A piggy bank contains only dimes and quarters. The total number of coins is  $n$  and the total value of the coins is  $v$ . Find the number of quarters in the bank.

- A)  $\frac{v}{15} + \frac{2n}{3}$       B)  $\frac{v}{15} - \frac{2n}{3}$       C)  $\frac{2n}{3} - \frac{v}{15}$       D)  $\frac{2n}{3} + \frac{v}{25}$       E)  $\frac{v}{25} - \frac{2n}{3}$

15. If  $5 + \sqrt{6 - x} = x + 1$  then the solution set is

- A) {2}      B) {4}      C) {5}      D) {2, 5}      E) {3, 5}

16. Simplify:  $\frac{2}{2 - \sqrt{2}} =$

- A)  $-\sqrt{2}$       B)  $-\frac{\sqrt{2}}{2}$       C)  $1 - \sqrt{2}$       D)  $2 + \sqrt{2}$       E)  $\frac{1}{1 - \sqrt{2}}$

17. Two pulleys, 50 cm and 30 cm in radius, respectively, are connected by a belt. The larger pulley makes 12 revolutions per minute. Find the angular speed of the smaller pulley in radians per second. (Round to the nearest thousandth.)



- A) .754      B) 1.264      C) 2.094      D) 3.490      E) 6.283

18. The formula  $C = \frac{5}{9}(F - 32)$  is used to change Fahrenheit temperatures to Celsius. At what temperature will Fahrenheit and Celsius thermometers read the same?

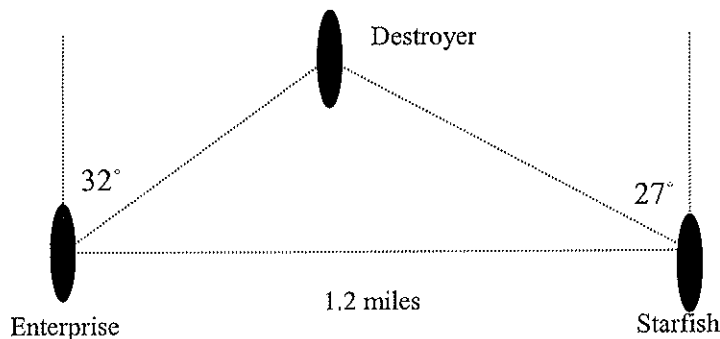
Ans

- A)  $-40^\circ$       B)  $-20^\circ$       C)  $-18^\circ$       D)  $0^\circ$       E)  $18^\circ$

19. Solve the inequality:  $\frac{(x-2)(x+4)}{(x-4)} > 0$

- A)  $(-4, 2)$       B)  $(-\infty, -4) \cup (2, \infty)$       C)  $(-4, -2) \cup (4, \infty)$   
 D)  $(-4, 2) \cup (4, \infty)$       E)  $(-\infty, -4) \cup (2, 4)$

20. The U.S.S. Enterprise spots a destroyer  $32^\circ$  off its starboard bow (i.e.  $32^\circ$  to its right). The U.S.S. Starfish is traveling in the same direction as the Enterprise and is 1.2 miles directly off its starboard. The Starfish spots the destroyer  $27^\circ$  off its port bow (i.e.  $27^\circ$  to its left.). How far is the destroyer from the Enterprise? Round your answer to the nearest tenth of a mile. (Picture not drawn to scale.)

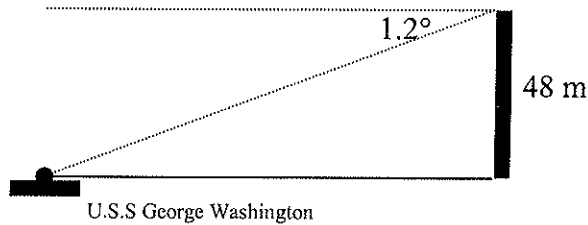


- A.) 1.1 miles    B.) 1.2 miles    C.) 1.3 miles    D.) 1.4 miles    E.) 1.5 miles

21. Solve  $8\cos^2 x = 3 - 2\cos x$  for  $x$  in  $[0^\circ, 360^\circ)$ . Round your answer to the nearest tenth of a degree.

- A.)  $41.4^\circ, 90^\circ, 180^\circ, 318.6^\circ$       B.)  $30^\circ, 41.4^\circ, 60^\circ, 138.6^\circ$   
 C.)  $60^\circ, 138.6^\circ, 221.4^\circ, 330^\circ$       D.)  $41.4^\circ, 60^\circ, 221.4^\circ, 300^\circ$   
 E)  $60^\circ, 138.6^\circ, 221.4^\circ, 300^\circ$

22. From the top of a particular lighthouse 48 meters above sea level, the angle of depression to the aircraft carrier U.S.S. George Washington is  $1.2^\circ$ . How far from the foot of the lighthouse is the carrier? Round to the nearest meter. (Picture not drawn to scale.)



- A.) 1005 meters      B.) 1904 meters      C.) 2180 meters  
 D.) 2291 meters      E.) 2409 meters

23) Find the inverse of the function  $f(x) = x^3 + 1$  if it exists.  $f^{-1}(x) =$

- A)  $\sqrt[3]{x} - 1$       B)  $\sqrt[3]{x-1}$       C)  $1 - \sqrt[3]{x}$       D)  $\sqrt[3]{1-x}$       E) The inverse does not exist.

24) Find a cubic equation that has solutions of  $-3$  and  $(3 - i)$ .

- A)  $x^3 - 3x^2 + 8x - 30 = 0$       B)  $x^3 - 9x^2 - 8x + 30 = 0$       C)  $x^3 - 3x^2 + 28x + 30 = 0$   
 D)  $x^3 - 9x^2 + 28x - 30 = 0$       E)  $x^3 - 3x^2 - 8x + 30 = 0$

25) What is the  $n^{\text{th}}$  term of the arithmetic sequence whose first two terms are  $-1$  and  $1/2$ ?

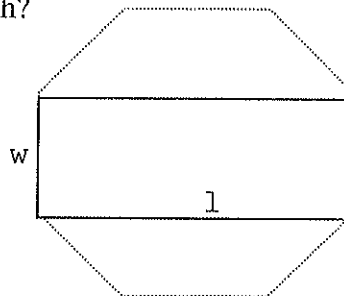
- A)  $n - \frac{1}{2}$       B)  $n - \frac{3}{2}$       C)  $\frac{3}{2}n - \frac{5}{2}$       D)  $1 - \frac{5}{2}n$       E)  $\frac{3}{2}n - 1$

26) Find the sum of the infinite geometric series  $\sum_{k=0}^{\infty} 3\left(\frac{1}{2}\right)^k$ .

- A) 4.5      B) 6      C) 9      D) 18      E)  $\infty$

27. If a rectangle is formed by connecting a pair of opposite sides of a regular octagon as shown below, what is the ratio of the width to the length?

- A)  $(1 + \sqrt{2}) : 2$       B)  $(\sqrt{2} - 1) : 1$   
 C)  $(2 - \sqrt{2}) : 1$       D)  $(2 - \sqrt{2}) : 2$   
 E)  $(2 + \sqrt{2}) : 2$



28. Find the value of  $a$  so the point  $(-2, -5)$  lies on the graph of  $y = ax^2 - 2x - 1$

- A)  $-2$       B)  $-1$       C)  $1$       D)  $2$       E)  $3$

29. A certain parabola has  $x$ -intercepts  $1$  and  $2$  and  $y$ -intercept  $-6$ . If it is expressed in the form  $y = ax^2 + bx + c$ , what is the sum of  $a$ ,  $b$ , and  $c$ ?

- A)  $-2$       B)  $-1$       C)  $0$       D)  $1$       E)  $2$

30. What is the inverse of  $\begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ ?

- A)  $\begin{bmatrix} 3 & -2 & -4 \\ 3 & 2 & -1 \\ -1 & -1 & 0 \end{bmatrix}$       B)  $\begin{bmatrix} 3 & 2 & 0 \\ 3 & -2 & -5 \\ -1 & -1 & 0 \end{bmatrix}$       C)  $\begin{bmatrix} 3 & -2 & 0 \\ 3 & 2 & -5 \\ -1 & -1 & 2 \end{bmatrix}$   
D)  $\begin{bmatrix} 3 & 2 & -4 \\ 3 & -2 & -1 \\ -1 & 1 & 2 \end{bmatrix}$       E)  $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

31. Given the vectors  $\vec{u} = \vec{i} - 2\vec{j}$  and  $\vec{v} = 2\vec{i} + \vec{j}$ , find  $2\vec{u} - 3\vec{v}$ .

- A)  $-4\vec{i} + 7\vec{j}$       B)  $4\vec{i} - 7\vec{j}$       C)  $-4\vec{i} - 7\vec{j}$   
D)  $4\vec{i} - \vec{j}$       E)  $-4\vec{i} + \vec{j}$

32. Find the fourth term of the expansion of  $(x - 2)^5$

- A)  $-20x^2$       B)  $20x^2$       C)  $40x^2$       D)  $80x^2$       E)  $-80x^2$

33. The ordered pair  $(1, \sqrt{3})$  in rectangular coordinates corresponds to which of the following ordered pairs in polar coordinates?

- A)  $(-2, -\frac{\pi}{3})$       B)  $(-2, \frac{\pi}{3})$       C)  $(2, -\frac{\pi}{3})$       D)  $(2, \frac{4\pi}{3})$       E)  $(-2, \frac{4\pi}{3})$

34. Evaluate  $\begin{vmatrix} 2 & 1 & -4 \\ -2 & -3 & 4 \\ 2 & 1 & 4 \end{vmatrix}$

- A)  $-32$       B)  $-16$       C)  $8$       D)  $16$       E)  $32$

35. If the ordered pair (2, 3) is contained in the function  $f(x)$ , which of the following must be contained in  $-2f(x+3)+2$ ?

- a) (-1, -4)    b) (5, -6)    c) (-1, 5)    d) (5, -4)    e) (2,5)

36. A can of coffee that is in the shape of a right circular cylinder 16 cm high and 10 cm in diameter sells for \$8.00. If you assume that cost is directly proportional to volume, how much in dollars should a can that is  $H$  cm high and  $D$  cm in diameter cost?

- a)  $40DH$     b)  $\frac{DH}{40}$     c)  $200D^2H$     d)  $\frac{D^2H}{200}$     e)  $\frac{D^2H}{4}$

37. Let  $z = a + bi$  and  $w = a - bi$ . What is  $\frac{1}{z} + \frac{1}{w}$ ?

- a) 0    b)  $\frac{1}{2a}$     c)  $a^2 + b^2$     d)  $\frac{2a}{a^2 + b^2}$     e)  $a^2 - b^2$

38. Which of the following inequalities is the solution of  $\left|1 + \frac{1}{x}\right| < 1$ ?

- a)  $x < -\frac{1}{2}$     b)  $-\frac{1}{2} < x < 0$     c)  $-1 < x < -\frac{1}{2}$     d)  $-2 < x < 0$     e) No solution.

39. The seats on a ferris wheel are 4 ft above the ground at the bottom and 77 ft. at the top. How high are they when they have completed one third of a revolution?

- a)  $48\frac{2}{3}$  ft.    b)  $52\frac{2}{3}$  ft.    c)  $58\frac{3}{4}$  ft.    d)  $62\frac{1}{2}$  ft.    e)  $54\frac{3}{4}$  ft.

40. The length of the diagonal of a rectangle is  $\sqrt{18.5}$  and the perimeter is 12. What is its area?

- a) 5    b) 6.25    c) 8.75    d) 9    e) 16

