'72 KEY

1. How many subsets of the set {2, 5, 9, 11} have exactly two elements?

(1). 3

(2). 6

(3). 8

(4). 4

(5). 16

$\binom{4}{2} = \frac{4!}{2!2!} = \frac{4 \cdot 3 \cdot 2!}{2 \cdot 2!} = 6$

2. $\frac{2 \pm 4\sqrt{3}}{2}$ is equal to:

(1). $1 \pm 4\sqrt{3}$

(2). $1 \pm 2\sqrt{3}$

(3). $2 \pm 4\sqrt{3}$

(4). $2 \pm 2\sqrt{3}$

(5). $\pm 3\sqrt{3}$

3. $-(3xy^2 z^3)^4$ is equivalent to:

(1). $81x^4y^8z^{12}$

(2). $81x^5y^6z^7$

(3). $-12x^4y^8z^{12}$

(4). $-81x^4y^8z^{12}$

(5). none of these

4. If $(3x/2) + 6 = 0$, then $2x - 1$ is equal to:

(1). -4

(2). 4

(3). -9

(4). 9

(5). -4.5

$\frac{3x}{2} + 6 = 0$

$3x + 12 = 0$

$x = -4$

$2x - 1 = -9$

5. The expression $2x - 3[5 - 2(x - 4) + 7]$ is equal to

(1). $16 - 4x$

(2). $4 - x$

(3). $8x - 43$

(4). $8x - 60$

(5). $4x - 6$

$2x - 15 + 6(x+4) - 21 =$

$2x - 15 + 6x - 24 - 21 =$

$8x - 60$

6. What is the value of $f(x) = x^2 - 3x^1 + 4x^0 - 5x^{-2} + x^{-3}$ when $x = 1/2$?

(1). $-9\ 1/4$

(2). $8\ 1/2$

(3). $-10\ 1/4$

(4). $11\ 3/4$

(5). $-9\ 1/2$

$\left(\frac{1}{2}\right)^2 - \frac{3}{2} + 4 - 5(2)^2 + (2)^3 =$

$\frac{1}{4} - \frac{3}{2} + 4 - 20 + 8 =$

$-8 - \frac{6}{4} + \frac{1}{4} =$

$-8 - \frac{5}{4} = -8 - 1\frac{1}{4}$

7. The simplest form of $(a^2b + ab^2)/(2a^2b - 2ab^2)$ is:

(1). $(ab)/2$

(2). $(a + b)/[2(a - b)]$

(3). $(a + b)/(2a - b)$

(4). $(a + b)/[2(b - a)]$

(5). none of these

$\frac{a^2b + ab^2}{2a^2b - 2ab^2} = \frac{ab(a+b)}{2ab(a-b)}$

$= \frac{a+b}{2(a-b)}$

8. An equivalent expression for $(1/x + 1/y)/(x + y)$ is:

(1). $(x + y)^2/(xy)$

(2). $(x + y)^2$

(3). $x + y$

(4). xy

(5). $1/(xy)$

$\frac{\frac{1}{x} + \frac{1}{y}}{x + y} = \frac{\frac{x+y}{xy}}{x+y} = \frac{\cancel{x+y}}{xy(\cancel{x+y})} = \frac{1}{xy}$

9. If four times a number is added to three times its square the sum is 175. Find the number.

(1). 25/3

(2). -7

(3). 7

(4). 5

(5). none of these

$4x + 3x^2 = 175$

$3x^2 + 4x - 175 = 0$

$x = \frac{-4 \pm \sqrt{16 - 4(3)(-175)}}{6}$

10. $[(a^2 - 4)/(2a + 2)] \div [(2 - a)/(1 + a)]$, $a \neq -1$, and $a \neq 2$, is equivalent to:

$\frac{(a+2)\cancel{(a-2)}}{2\cancel{(a+1)}} \cdot \frac{\cancel{a+1}}{-1\cancel{(a-2)}}$

(1). (a - 2)/2

(2). -(a + 2)/2

(3). (a + 2)/2

(4). (a/2) - 1

(5). (2 - a)/2

11. The solution set for the system of equations, $2x + 2y = 7$, $4x - y = -6$, is:

$8x - 2y = -12$

$10x = -5$

$\boxed{x = -\tfrac{1}{2}}$

(1). {(1,-5)}

(2). {(1/2,3)}

(3). {(-1/2,4)}

(4). {(0,7/2)}

(5). none of these

12. When each side of a given square is increased by 4 ft., the area is increased by 64 sq. ft. The dimensions of the orginal square are:

(1). 4 ft. by 4 ft.

(2). 5 ft. by 5 ft.

(3). 6 ft. by 6 ft.

(4). 8 ft. by 8 ft.

(5). none of these

$(x+4)^2 - x^2 = 64$

$x^2 + 8x + 16 - x^2 = 64$

$8x = 48$

$x = 6$

13. A straight line passing through the point (0,4) is perpendicular to the line $x - 3y - 7 = 0$. Its equation is:

(1). $y + 3x - 4 = 0$

(2). $3y + x - 12 = 0$

(3). $y + 3x + 4 = 0$

(4). $3y - x - 12 = 0$

(5). $y - 3x - 4 = 0$

$x - 3y - 7 = 0$
$-3y = -x + 7$
$y = \frac{x}{3} - \frac{7}{3}$
$y = -3x + b$
$4 = +b \Rightarrow b = +4$
$y = -3x + 4$
$y + 3x - 4 = 0$

14. Factor completely: $4x^2 - 4xy - 36 + y^2$

(1). $(2x - y + 6)(2x - y - 6)$

(2). $(2x + y - 6)(2x - y - 6)$

(3). $(2x - y + 6)(2x + y + 6)$

(4). $(2x + y + 6)(2x + y + 6)$

(5). $(2x - y - 6)(2x - y - 6)$

$2x - y - 6$
$2x - y + 6$
$12x - 6y - 36$
$+6y \quad +y^2 - 2xy$
$-12x \quad -2xy + 4x^2$
$-36 + y^2 - 4xy + 4x^2$

15. If $[(x + 1)(x - 3)] / (x + 2) < 0$, then

$\frac{(x+1)(x-3)}{x+2} < 0$

(1). $x < -1$

(2). $3 < x$

(3). $x < -2$ or $-1 < x < 3$

(4). x can be any real number

(5). $x = -2$

16. If x and y are non-zero numbers such that $x = 1 + (1/y)$ and $y = 1 + (1/x)$, then y equals

(1). $x - 1$

(2). $1 - x$

(3). $1 + x$

(4). $-x$

(5). x

17. In a certain puzzle, the larger of two numbers must exceed four times the smaller by exactly 5, and their difference must be at least 20. Find the least possible value of the smaller number.

(1). 50

(2). 4

(3). 5

(4). no solution exists

(5). there is no least value

X = LARGER
Y = SMALLER

$4Y + 5 = X$

$X - Y \geq 20$

$4y + 5 - Y \geq 20$

$3y + 5 \geq 20$

$3Y \geq 15$

$Y \geq 5$

18. The expression $(x^{-2} - y^{-2})/(x^{-2}y^{-2})$ can be simplified to:

(1). $(y - x)(y + x)$

(2). x^2y^2

(3). $(1/y^2) - (1/x^2)$

(4). $x^2 + y^2$

(5). none of the above

$$\frac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{x^2y^2}} = \frac{\frac{y^2 - x^2}{x^2y^2}}{\frac{1}{x^2y^2}} = \frac{\frac{(y-x)(y+x)}{x^2y^2}}{\frac{1}{x^2y^2}} = \frac{\cancel{x^2y^2}\,(y-x)(y+x)}{\cancel{x^2y^2}}$$

19. If $x < -1$, which of the following is equal to $|1 + x|$?

(1). $-1 - x$

(2). $-1 + x$

(3). $1 - x$

(4). $1 + x$

(5). none of the above

$1+x$ OR $-(1+x)$

20. Simplify the following complex fraction: $$\cfrac{1}{1 - \cfrac{1}{1 - \cfrac{1}{a}}}$$

(1). $1 - a$

(2). $(a - 1)/(2a - 1)$

(3). $a - 1$

(4). $(a - 1)/(1 - 2a)$

(5). none of the above

$1 - \frac{1}{a} = \frac{a-1}{a}$

$1 - \frac{a}{a-1} = \frac{a-1}{a-1} - \frac{a}{a-1} = \frac{-1}{a-1}$

$-(a-1)$

21. If these two linear equations are plotted on the same graph, what value of a will give an ordinate value of 4 for the point of intersection?

$(x, 4)$

$$6ax + y = 1$$

$$2ax + 2y = 7$$

(1). any real number

(2). only the number 1

(3). no value of a will work

? (4). any non-zero real number

(5). none of these

22. $(x^3 - y^3)/(x - y)$ reduced to lowest terms is:

$\frac{x^3 - y^3}{x - y} =$

(1). $(x - y)^2$

(2). $x^2 - y^2$

(3). $x^2 + xy + y^2$

(4). $x^2 - 2xy - y^2$

(5). $x^2 - xy + y^2$

2 The equation $[4/(3 - x)] + [2x/(5 + x)] = 1$ has

$\frac{4}{3-x} + \frac{2x}{5+x} = 1$

$\frac{20+4x+6x-2x^2}{15-2x-x^2} = 1$

$\frac{20+10x-2x^2}{15-2x-x^2} = 1$

$20+10x-2x^2 = 15-2x-x^2$

$5+12x-x^2 = 0$

$-x^2+12x+5=0$

$12^2 - 4(-1)(5)$

$144+20 = 164$

$\frac{-12 \pm \sqrt{164}}{-2} = 6 \pm \sqrt{164}$

(1). only one root

(2). one real root and one imaginary root

(3). no root

(4). two rational roots

(5). two real roots

24. Solve the following equation for x: $8/(x^2 - 1) = [4/(x - 1)] - [4/(x + 2)]$

(1). $x = 1$

(2). $x = -3$

(3). $x = -2,\ x = 1$

(4). $x = -2,\ x = -1$

(5). none of the above

25. The three digit number 2a3 is added to the number 326 to give the three digit number 5b9. If 5b9 is divisible by 9, then a + b equals:

(1). 2

(2). 4

(3). 6

(4). 8

(5). 9

26. If the arithmetic mean of a and b is double their geometric mean, with a > b > 0, then a possible value for the ratio a/b, to the nearest integer, is:

(1). 5

(2). 8

(3). 11

(4). 14

(5). none of these

$\frac{a+b}{2} = 2\sqrt{ab}$

$\frac{a+b}{4} = \sqrt{ab}$

$\left(\frac{a+b}{4}\right)^2 = ab$

$\frac{a^2+2ab+b^2}{16} = ab$

$\frac{a^2+2ab+b^2}{16ab} = 1$

$\frac{a}{16b} + \frac{1}{8} + \frac{b}{16a} = 1 \Rightarrow \frac{a}{b} = 14 - \frac{b}{a}$ [$a >> b$

27. The illumination from a source of light is inversely proportional to the square of the distance from the source. If a book is 8 feet from a lamp, how many feet from the lamp must it be placed in order that the illumination may be doubled?

(1). $6\sqrt{3}$

(2). $3\sqrt{2}$

(3). $4\sqrt{2}$

(4). $5\sqrt{3}$

(5). $8\sqrt{2}$

28. A tractor radiator contains 12 quarts of a 10% soution of alcohol and water. How many quarts must be replaced by pure alcohol to make a 20% solution of alcohol and water in the radiator?

(1). 1 and 1/2 quarts

(2). 2 quarts

(3). 1 and 1/3 quarts

(4). 4 quarts

(5). none of these

29. What is the solution set for the inequality $x^2 - 3x + 2 > 0$?

(1). $\{x;\ 1 < x < 2\}$

(2). $\{x;\ x > 2\}$

(3). $\{x;\ x < 1\}$

(4). $\{x;\ x < 1 \text{ or } x > 2\}$

(5). none of the above

$(x-2)(x-1) > 0$

$x-2>0 \wedge x-1<0 \quad \vee \quad x-2<0 \wedge x-1>0$

$x>2 \wedge x<1 \quad \vee \quad x<2 \wedge x>1$

$\phi \qquad 1<x<2$

30. The complement of a set A relative to a universe S is the set whose elements belong to S but not to A. Suppose A is the set of non-negative integer pairs (x,y) satisfying $x^2 + y^2 < 4$, and S is the set of none-negative integer pairs (x,y) where x = 0, 1,2, or 3, and y = 0, 1, 2, or 3. How many elements are in the complement of A?

(1). 4

(2). 10

(3). 12

(4). infinitely many

(5). none of the above

31 A collection of coins consists of nickels, dimes, and quarters. There are 12 more nickels than dimes and 3 less nickels than twice the number of quarters. The total value is \$6.05. The number of nickels in this collection is:

(1). 21

(2). 19

(3). 13

(4). 25

(5). none of the above

x = # DIMES

x+12 = # NICKELS

32. Which of the following is equivalent to $(7x + 3)^{-1/2}(x + 2)^{3/2}$?

~~(1)~~. $(7x^{-1/2} + 3^{-1/2})(x^{3/2} + 2^{3/2})$

(2). $(x + 2)\sqrt{7x^2 + 17x + 6}\,/(7x + 3)$

~~(3)~~. $\sqrt[3]{(x + 2)^2} \,/\, \sqrt{(7x + 3)}$

(4). $\sqrt{(x + 2)^3\ (7x + 3)}$

(5). $(-7x/2 - 3/2)(3x/2 + 3/2)$

$$\frac{\sqrt{(x+2)^3}}{\sqrt{7x+3}} = \sqrt{\frac{(x+2)^3}{7x+3}\cdot\frac{7x+3}{7x+3}} = \frac{(x+2)\sqrt{(x+2)(7x+3)}}{7x+3} = \frac{(x+2)\sqrt{7x^2+17x+6}}{7x+3}$$

33. Tom could read the proof for the college paper in 3 hours; Dick could read it in 2 hours, and Sam in 1 1/2 hours. How long would it take the three boys to read the proof together?

(1). 1 hour

(2). 1/2 hour

(3). 5/6 hour

(4). 3/4 hour

(5). 2/3 hour

34. y varies directly as the square of x. If x is made half as large, then y is:

$y = kx^2$

$(\frac{1}{2}x)^2$

(1). doubled

(2). made half as large

(3). quadrupled

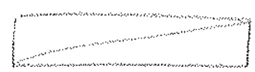
(4). unchanged

(5). none of these

35. If the perimeter of rectangle ABCD is 20 inches, what is the least value the diagonal AC can be?

$2x + 2y = 20$

$x^2 + y^2 = ?$



(1). 0 in.

(2). $\sqrt{50}$ in.

(3). 10 in.

(4). $\sqrt{200}$ in.

(5). none of these

36. If we assume that (a) all flowers are plants, and (b) no plants are mortal, we may conclude that:

$(\forall x)(F(x) \Rightarrow P(x))$

$(\forall x)(P(x) \Rightarrow {\sim}M(x))$

(1). All plants are mortal.

(2). All plants are flowers.

(3). Some flowers are mortal.

? (4). No flowers are mortal.

(5). None of these.

GLB

37. The greatest lower bound of $\{5/4, 7/8, 17/16, 31/32, \ldots\}$ is

(1). 1

(2). 5/4

(3). infinite

(4). does not exist

(5). none of these

38. Suppose $x_1, x_2, \ldots, x_n$ are real numbers and $\overline{x} = (1/n)\sum_{i=1}^{n} x_i$ $= (1/n)(x_1 + x_2 + \ldots + x_n)$. Then $\sum_{i=1}^{n}(x_i - \overline{x})$

(1). can range from 0 to $\sum_{i=1}^{n} x_i$

(2). is always positive

(3). is always zero

(4). can range from $-\sum_{i=1}^{n} x_i$ to $+\sum_{i=1}^{n} x_i$

(5). none of the above

39. Consider the equation $2x^2 - bx - 5 = 0$ with zeros x_1 and x_2. Which of the following is true?

(1). If b is an odd integer, then $x_1 = x_2$.

(2). If $b = 3$, then $x_1 \div x_2 = 5/2$.

(3). If $x_1 + x_3 = 3$, then $b = 4$.

(4). If $x_1 = x_2 = b$, then $b = \sqrt{5}$.

(5). If b is any real or complex number, then $(x_1)(x_2) = -5/2$.

40. Suppose T is a function from the real numbers to the real numbers satisfying $T(cx) = cT(x)$, for all numbers c and x. Which of the following is true?

(1). $T(x) = 0$ for all x

(2). $T(x^2) = cT(x)$ for all x

(3). $T(x) = T(cx)/c$ for all c

(4). $T(x) = xT(1)$ for all x

(5). none of the above