



TENNESSEE MATHEMATICS TEACHERS ASSOCIATION

SIXTY-SIXTH ANNUAL MATHEMATICS CONTEST

2024

Geometry/Integrated Math II

Scoring Formula: $4 \times (\text{Number Right}) - (\text{Number Wrong}) + 40$

Directions:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

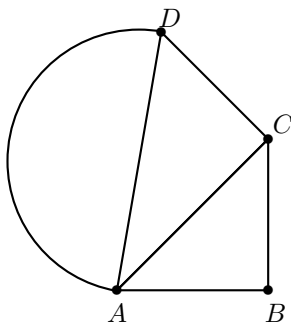
If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. A right triangle has one leg of length 10 cm and hypotenuse of length 15 cm. What is the area of the triangle to the nearest tenth of a square centimeter?
- (a) 75 cm^2 (b) 80.4 cm^2 (c) 126.2 cm^2 (d) 55.9 cm^2 (e) 35.7 cm^2
2. How many faces does a regular icosahedron have?
- (a) 12 (b) 8 (c) 20 (d) 16 (e) 6
3. The sum of the measures of the interior angles of a convex n -gon equals 900° . What is the value of n ?
- (a) 6 (b) 7 (c) 8 (d) 9 (e) 10
4. What is the minimum value of the function $f(x) = x^2 + 6x + 1$?
- (a) -3 (b) 3 (c) 0 (d) 8 (e) -8
5. A silo is the shape of a circular cylinder. If the cylinder has diameter 10 feet and height 20 feet, what is the volume of the silo to the nearest cubic foot?
- (a) 157 cubic feet (b) 628 cubic feet (c) 1257 cubic feet (d) 1571 cubic feet (e) 6283 cubic feet
6. In Square $ABCD$, vertex A is located at $(1, 1)$ and vertex C is located at $(4, 2)$. Which of the following is a possible location for vertex B ?
- (a) $(2, 0)$ (b) $(2, 3)$ (c) $(3, 2)$ (d) $(0, 3)$ (e) $(0, 2)$
7. A trapezoid has an area of 60 cm^2 . If the height of the trapezoid is 5 cm and one of the bases has length 10 cm, what is the length of the other base of the trapezoid?
- (a) 6 cm (b) 7 cm (c) 12 cm (d) 14 cm (e) 18 cm

8. An isosceles right triangle has a hypotenuse of length 20 cm. What is the perimeter of the triangle?
- (a) $10\sqrt{2}$ cm (b) $20 + 10\sqrt{2}$ cm (c) 100 cm (d) $20 + 20\sqrt{2}$ cm (e) $10 + 10\sqrt{2}$ cm
9. Which of the following statements concerning similar triangles is NOT necessarily true?
- (a) Congruent triangles are similar.
(b) Corresponding angles in similar triangle are congruent.
(c) All isosceles right triangles are similar.
(d) All equilateral triangles are similar.
(e) If two similar triangles have a scaling factor of k , then the scaling factor for their areas is also k .
10. Points A and B have coordinates $(-2, 3)$ and $(4, 5)$, respectively. What is the area of the circle with diameter \overline{AB} ?
- (a) 10π square units
(b) 17π square units
(c) 8π square units
(d) 16π square units
(e) 25π square units
11. A supplementary pair of angles have measures $(5x - 40)^\circ$ and $(3x - 32)^\circ$. What is the value of x ?
- (a) $x = 4$
(b) $x = 20.25$
(c) $x = 31.5$
(d) $x = 54$
(e) $x = 21.5$
12. A line segment has endpoints at coordinates $(2a + b, b - a)$ and $(2b - a, a + b)$. What are the coordinates of the midpoint of the segment in terms of a and b ?
- (a) $(a + 3b, 2a)$ (b) $\left(\frac{a + 3b}{2}, b\right)$ (c) $\left(\frac{3a - b}{2}, b\right)$ (d) $\left(\frac{a + 3b}{2}, a\right)$ (e) $\left(\frac{3a - b}{2}, a\right)$

13. In the figure below, $\angle ABC$ and $\angle ACD$ are right angles and $m\overline{AB} = m\overline{BC} = m\overline{CD} = 1$ cm. What is the area of the semicircle with diameter \overline{AD} ?



- (a) $\frac{\pi\sqrt{2}}{4}$ cm² (b) $\frac{\pi}{4}$ cm² (c) $\frac{3\pi}{8}$ cm² (d) $\frac{\pi}{2}$ cm² (e) $\frac{\pi}{3}$ cm²
14. The ratio between the measures of the smallest and largest interior angles of a triangle is 1 : 6. If the third interior angle measures x degrees, what is the measure of the smallest interior angle of the triangle in terms of x ?
- (a) $\frac{x}{6}$ degrees
 (b) $\frac{180 - x}{6}$ degrees
 (c) $\frac{x}{7}$ degrees
 (d) $\frac{180 - x}{7}$ degrees
 (e) $\frac{6(180 - x)}{7}$ degrees
15. A right circular cone has a height of 8 cm and a diameter of 10 cm. What is the lateral surface area of the cone?
- (a) 40π cm² (b) $2\pi\sqrt{41}$ cm² (c) $\frac{200\pi}{3}$ cm² (d) $5\pi\sqrt{89}$ cm² (e) $\frac{100\pi}{3}$ cm²
16. What is the volume of a regular tetrahedron with edge length 6 cm?
- (a) $18\sqrt{2}$ cm³ (b) $18\sqrt{3}$ cm³ (c) $36\sqrt{2}$ cm³ (d) $36\sqrt{3}$ cm³ (e) $12\sqrt{3}$ cm³

17. There are two similar polygons where the larger polygon has 6 times the area of the smaller polygon. How much larger is the perimeter of the larger polygon than the smaller polygon?

- (a) 6 times larger (b) 36 times larger (c) $\sqrt{6}$ times larger (d) 12 times larger (e) 3 times larger

18. A rectangular prism has edges of length 3 cm, 4 cm, and 12 cm. What is the length of a diagonal of the prism?

- (a) 13 cm (b) 19 cm (c) $12\sqrt{2}$ cm (d) $12\sqrt{3}$ cm (e) 24 cm

19. Consider a rhombus that has diagonals with lengths that total 8 cm. What is the largest area that such a rhombus can have?

- (a) 16 cm^2 (b) 12 cm^2 (c) 8 cm^2 (d) 4 cm^2 (e) $4\sqrt{2} \text{ cm}^2$

20. Which of the following is the equation of the circle with center $(4, 3)$ that is tangent to the x -axis?

- (a) $(x - 4)^2 + (y - 3)^2 = 16$
(b) $(x + 4)^2 + (y + 3)^2 = 16$
(c) $(x - 4)^2 + (y - 3)^2 = 9$
(d) $(x + 4)^2 + (y + 3)^2 = 9$
(e) $(x - 4)^2 + (y - 3)^2 = 25$

21. How many edges does a dodecagonal prism have?

- (a) 24 (b) 12 (c) 40 (d) 36 (e) 20

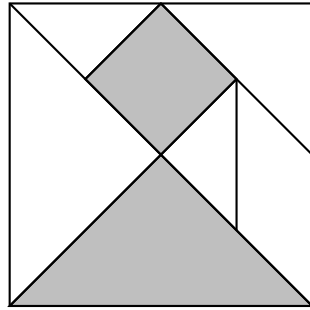
22. Consider three non-collinear points in three dimensional space. What geometric object describes the set of all points that are equidistant from the three points?

- (a) A point (b) A line (c) A circle (d) A triangle (e) A plane

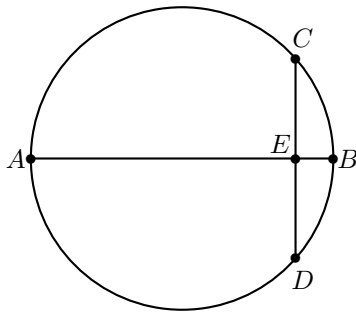
23. A pizza parlor wants to make a rectangular shaped pizza that has the same area as its circular pizza with a 16 inch diameter. The length of the rectangular pizza is to be twice as long as the width. What would be the width of the rectangular pizza to the nearest inch?

- (a) 9 inches (b) 10 inches (c) 11 inches (d) 12 inches (e) 13 inches

24. In the tangram puzzle below, all the triangular pieces are isosceles right triangles. If the shaded square has an area of 1 cm^2 , what is the area of the shaded triangle?



- (a) 2 cm^2 (b) 4 cm^2 (c) $\sqrt{2} \text{ cm}^2$ (d) 1 cm^2 (e) $2\sqrt{2} \text{ cm}^2$
25. In the picture below, \overline{AB} is a diameter of the circle, $\overline{AB} \perp \overline{CD}$, $m\overline{CD} = 50 \text{ cm}$, and $m\overline{BE} = 10 \text{ cm}$. What is $m\overline{AB}$?

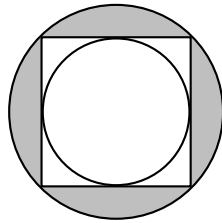


- (a) 62.5 cm (b) 72.5 cm (c) 87.5 cm (d) 100 cm (e) 80 cm
26. The circumference of a great circle of a spherically-shaped NBA basketball is 29.5 inches. What is the volume of a NBA basketball to the nearest cubic inch?
- (a) 277 cubic inches
 (b) 369 cubic inches
 (c) 371 cubic inches
 (d) 434 cubic inches
 (e) 870 cubic inches

27. What is the distance between the parallel lines with equations $2x - y = 5$ and $2x - y = 10$?
- (a) 5 units (b) $\frac{5\sqrt{2}}{2}$ units (c) $\sqrt{5}$ units (d) $\sqrt{3}$ units (e) 2.5 units

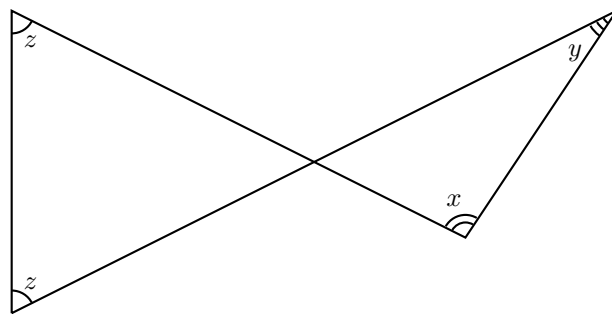
28. What is the measure of an exterior angle of a regular octagon?
- (a) 45 degrees (b) 135 degrees (c) 22.5 degrees (d) 157.5 degrees (e) 90 degrees

29. In the picture below, the smaller circle has a radius of length 1 cm. What is the total area of the shaded regions?



- (a) $\pi - 1 \text{ cm}^2$ (b) $2\pi - 2 \text{ cm}^2$ (c) $4\pi - 4 \text{ cm}^2$ (d) $2\pi - 4 \text{ cm}^2$ (e) $4\pi - 2 \text{ cm}^2$

30. The values of x , y , and z are the measures (in degrees) of the angles in the shown below. What is the value of $(z - x) + (z - y)$?



- (a) 0° (b) 45° (c) 90° (d) 135° (e) 180°

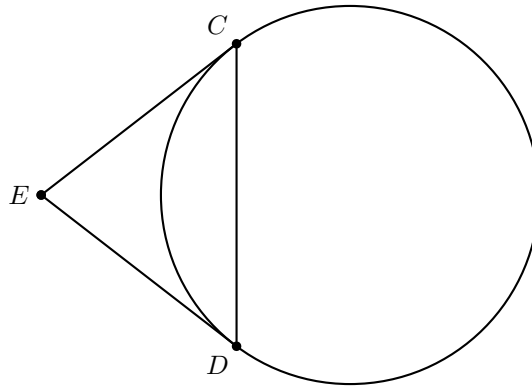
31. An n -gonal pyramid has 30 edges. What is the value of n ?

- (a) $n = 10$ (b) $n = 15$ (c) $n = 16$ (d) $n = 29$ (e) $n = 30$

32. How many diagonals are in a regular decagon?

- (a) 45 (b) 35 (c) 30 (d) 25 (e) 20

33. The circle in the diagram below has a diameter of length 20 cm. Segments \overline{DE} and \overline{CE} are congruent and are also tangent to the circle. If $m\widehat{CD} = 16$ cm, what is the area of $\triangle CDE$?



- (a) $\frac{128}{3}$ cm² (b) 64 cm² (c) $\frac{256}{3}$ cm² (d) 128 cm² (e) $\frac{512}{3}$ cm²

34. A sphere is inscribed in a cube with volume 18 cm³. What is the volume of the sphere?

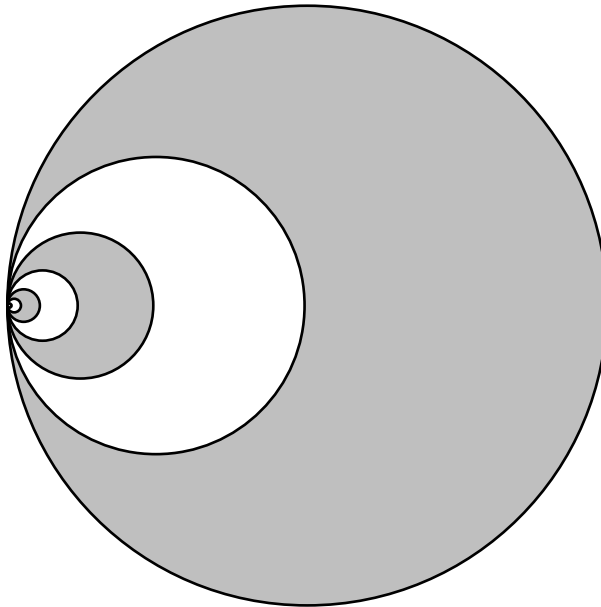
- (a) 2π cm³ (b) 4π cm³ (c) $\frac{9\pi}{2}$ cm³ (d) 3π cm³ (e) $\frac{3\pi}{2}$ cm³

35. A regular octagon has sides of length 4 cm. What is the length of the longest diagonal in the octagon?

- (a) 16 cm (b) $4\sqrt{4+2\sqrt{2}}$ cm (c) $6 + \sqrt{2}$ cm (d) $8\sqrt{2}$ cm (e) $4 + 4\sqrt{2}$ cm

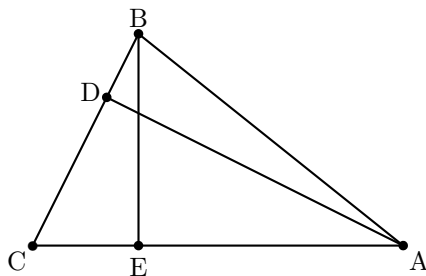
36. A pyramid with a square base has equilateral triangles as its other 4 faces. If the pyramid has a surface area of 36 cm^2 , what is the area of the square base to the nearest tenth of a square centimeter?
- (a) 14.7 cm^2 (b) 14.9 cm^2 (c) 12 cm^2 (d) 7.2 cm^2 (e) 13.2 cm^2

37. Consider the infinite collection of tangent circles in the diagram below. The largest circle has radius 1 cm. Each smaller circle has a radius that is half the length of the radius of the next larger circle. What is the total shaded area?



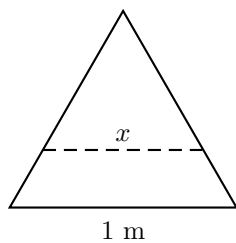
- (a) $\frac{\pi}{2} \text{ cm}^2$ (b) $\frac{2\pi}{3} \text{ cm}^2$ (c) $\frac{3\pi}{4} \text{ cm}^2$ (d) $\frac{4\pi}{5} \text{ cm}^2$ (e) $\frac{5\pi}{6} \text{ cm}^2$

38. In the diagram shown below, $\overline{AD} \perp \overline{BC}$ and $\overline{BE} \perp \overline{AC}$. If $m\overline{AD} = 25 \text{ cm}$, $m\overline{BE} = 16 \text{ cm}$, and $m\overline{AC} = 28 \text{ cm}$, what is $m\overline{BC}$?



- (a) 17.92 cm (b) 14.29 cm (c) 21.88 cm (d) 20.08 cm (e) 19.67 cm

39. An equilateral triangle with side length 1 meter is cut into two pieces—an equilateral triangle and a trapezoid—so that both pieces have the same area. What is the length (to the nearest centimeter) of the side of the equilateral triangular piece labelled as x in the picture below?



- (a) 50 cm (b) 58 cm (c) 65 cm (d) 71 cm (e) 87 cm
40. Rounded the nearest tenth, what is the area of a regular 12-gon with side length 1 cm?
- (a) 11.4 cm^2 (b) 11.5 cm^2 (c) 11.3 cm^2 (d) 11.1 cm^2 (e) 11.2 cm^2

