TENNESSEE MATHEMATICS TEACHERS ASSOCIATION

SIXTY-SEVENTH ANNUAL MATHEMATICS CONTEST

2025

Geometry/Integrated Math II

Prepared by:

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Knoxville, TN

Scoring Formula: $4 \times$ (Number Right) $-$ (Number Wrong) $+ 40$

Directions:

Do not open this booklet until you are told to do so.

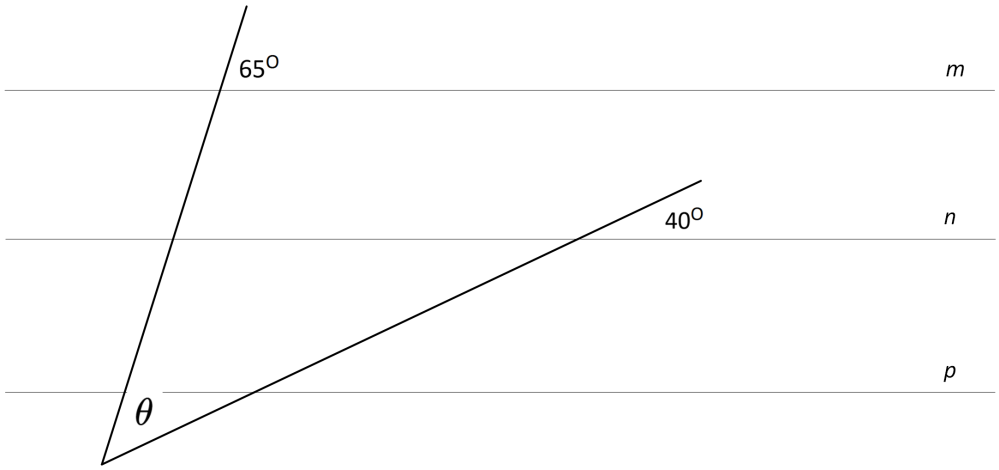
This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. Lines m, n, and p are parallel. Determine the measure of θ, the angle at which the two shown transversals intersect each other. Not drawn to scale.



(a) 20°
(b) 25°
(c) 30°
(d) 35°
(e) 40°

2. How many edges does a pentagonal prism have?

(a) 5
(b) 10
(c) 12
(d) 15
(e) 20

3. Which of the following has the least value?

(a) the number of faces an icosahedron has
(b) the number of edges an octahedron has
(c) the number of edges a hexahedron has
(d) the number of vertices an octahedron has
(e) the number of faces a dodecahedron has

4. Find the area of the triangle with vertices at points $(-4, 1), (2, 0), (-4, -3)$.

   (a) 6 square units
   (b) 12 square units
   (c) $7\sqrt{2}$ square units
   (d) $10\sqrt{7}$ square units
   (e) 11.5 square units

5. If the diameter of a sphere doubles, how will its volume change?

   (a) It does not change.
   (b) It doubles.
   (c) It is multiplied by $\frac{4}{3}$.
   (d) It is multiplied by 4.
   (e) It is multiplied by 8.

6. Consider exactly two fixed points in three-dimensional space. What geometric object describes the set of all points that are equidistant from both fixed points?

   (a) point
   (b) line
   (c) plane
   (d) circle
   (e) sphere

7. The interior of a regular polygon is divided into exactly n triangles by drawing all the possible diagonals from one vertex. How many edges must this polygon have?

   (a) n
   (b) $2n$
   (c) $n - 2$
   (d) $n + 2$
   (e) n^2

8. The Cartesian coordinates for four points are provided: $V(-7,5), R(3,0), S(-5,-1), T(4,-3)$. Which of the following inequalities is correctly ordered from smallest to largest?

   (a) $SR < RT < VT$
   (b) $VT < SR < RT$
   (c) $ST < RT < VT$
   (d) $RV < VT < ST$
   (e) $RT < SR < VT$

9. Which of the following points does **not** lie on the circle given by equation $(x-2)^2+(y+3)^2 = 9$?

   (a) $(2,-3)$
   (b) $(2,0)$
   (c) $(2,-6)$
   (d) $(5,-3)$
   (e) $(-1,-3)$

10. One interior angle of a triangle has a measure of $60°$. The two sides of the triangle that are adjacent to this angle have lengths of 4 and 5 units. What is the length of the edge that is opposite the $60°$ angle?

    (a) $\sqrt{41}$ units
    (b) 1 unit
    (c) $2\sqrt{3}$ units
    (d) 3 units
    (e) $\sqrt{21}$ units

11. An electric fence is stretched across a field in a perfectly straight line. One scarecrow is also positioned in the field, not on the fence line. A group of crows arrives. They are scared of both the electrical fence and the scarecrow, and they are trying to figure out where they can safely land in the field. Suppose these crows decide to land on the field such that each one is equidistant from the fence and from the scarecrow. When these crows do this, what shape is created by the crows?

    (a) straight line
    (b) ellipse
    (c) parabola
    (d) hyperbola
    (e) circle

12. Which of the following is **not** equal to π?

    (a) the volume of a sphere with radius $\frac{1}{3}$
    (b) the surface area of a sphere of radius $\frac{1}{2}$
    (c) the area of a circle of radius 1
    (d) the volume of a cylinder of radius 1 and height 1
    (e) the circumference of a circle of radius $\frac{1}{2}$

13. Which of the following is **not** a net for a triangular prism?

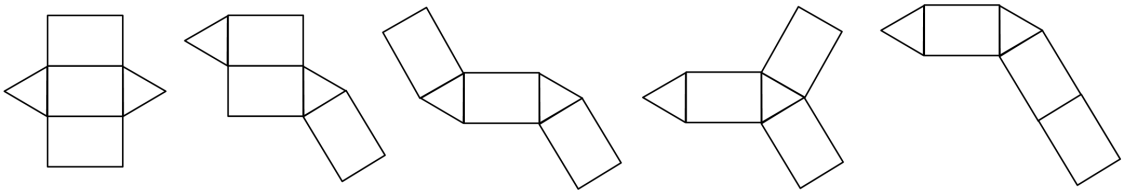


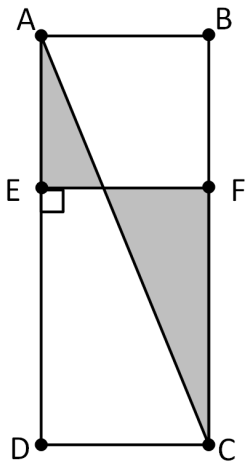
Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

    (a) Figure 1
    (b) Figure 2
    (c) Figure 3
    (d) Figure 4
    (e) Figure 5

14. A point P is exactly k units away from a plane. The set of all points that are in the plane and are exactly 10 units from point P constitutes a circle with a radius of 6. What is the value of k?

    (a) 3
    (b) 4
    (c) 5
    (d) 8
    (e) 10

15. If $AE = 12$, $FC = 24$, and $EF = 15$, what is the area of the total shaded region?



(a) 150 square units
(b) 360 square units
(c) 180 square units
(d) 90 square units
(e) 120 square units

16. Given that $0 < a < 90$, $0 < b < 90$, $\sin a^\circ = \cos b^\circ$, and $b = 2a$; what is the value of b?

(a) 15
(b) 30
(c) 45
(d) 60
(e) 75

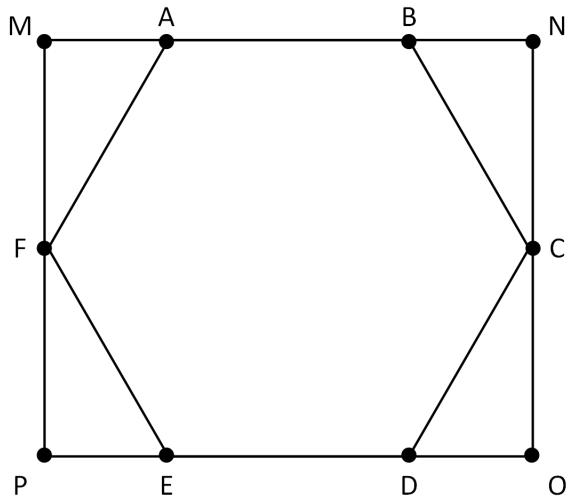
17. I am a circle. My circumference is C units, and my area is A square units. If $C = A$, how long is my radius?

(a) 4π units
(b) $\frac{\pi}{2}$ units
(c) π units
(d) 1 unit
(e) 2 units

18. In a triangle, the side opposite a $30°$ angle has a length of A units, and the side opposite a $45°$ angle has a length of B units. Which of the following is equal to $\frac{B}{A}$?

(a) $\frac{\sqrt{2}}{2}$
(b) 1
(c) 2
(d) $\sqrt{2}$
(e) $\sqrt{3}$

19. A swimming pool is shaped like a regular hexagon, shown below as $ABCDEF$, with a side length of 10 feet. The swimming pool is inscribed in a rectangular courtyard, shown below as $MNOP$. What is the area of rectangle $MNOP$?

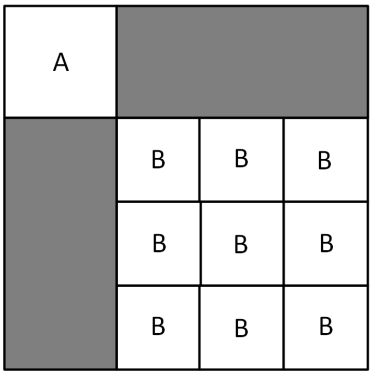


(a) $200\sqrt{3}$ ft^2
(b) 400 ft^2
(c) $\left(100 + 100\sqrt{3}\right)$ ft^2
(d) 300 ft^2
(e) $\left(300 + 200\sqrt{2}\right)$ ft^2

20. Suppose you are in a regular classroom shaped like a rectangular prism. You have a small speaker that plays music outward five feet in all directions. If you place this speaker into the corner of your classroom, where the floor meets two walls, what is the volume in cubic feet of space inside your classroom where music can be heard from the speaker?

(a) $\frac{125\pi}{3}$ ft^3
(b) $\frac{25\pi}{6}$ ft^3
(c) $\frac{125\pi}{6}$ ft^3
(d) $\frac{25\pi}{2}$ ft^3
(e) 25π ft^3

21. Each square labeled A has an area of 3 units. Each square labeled B has an area of 2 units. What is the total area of the shaded region in the diagram below?



(a) $3\sqrt{6}$
(b) $6\sqrt{6}$
(c) 12
(d) 18
(e) 16

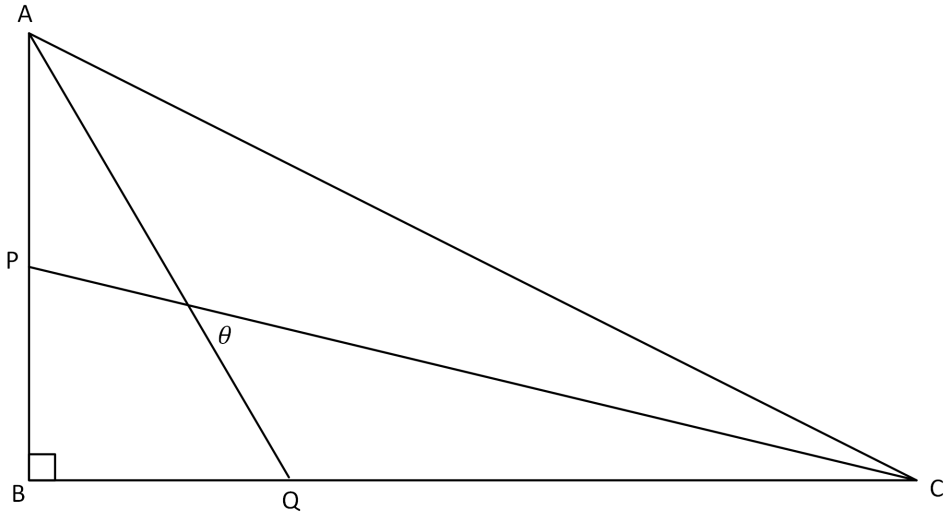
22. Two different great circles on a unit sphere intersect each other at point A. The supplementary angles they form at point A are θ and ϕ. If $\phi = 2\theta$, then what is the surface area of the smallest region enclosed by the two great circles?

(a) 2π square units
(b) $\frac{2\pi}{3}$ square units
(c) $\frac{2\pi}{9}$ square units
(d) $\frac{\pi}{3}$ square units
(e) 3π square units

23. You have a triangular pyramid. Two of the pyramid's faces are 3-4-5 right triangles. One of its faces is an isosceles right triangle with a hypotenuse of length $3\sqrt{2}$ units. What is the volume of the pyramid?

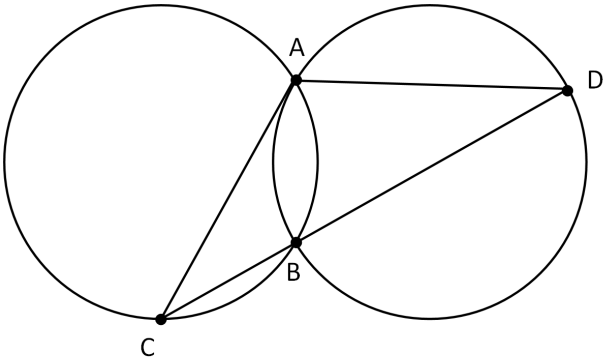
(a) 9 cubic units
(b) 15 cubic units
(c) 6 cubic units
(d) 20 cubic units
(e) 12 cubic units

24. In the image below, $m\angle ABC = 90°$, $\angle BAQ \cong \angle QAC$, and $\angle ACP \cong \angle PCB$. What is the measure of angle θ?



(a) $\frac{\pi}{4}$
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{5}$
(d) $\frac{2\pi}{5}$
(e) $\frac{5\pi}{18}$

25. The two circles below have the same radius and intersect each other at exactly two points. Points C and D each lie on one circle, and points A and B lies on both circles. Point B lies on $\triangle ACD$.



Is $\triangle ACD$ isosceles? How do you know? Choose the best response.

(a) No. $\triangle ACD$ overlaps with the right circle more than the left circle.

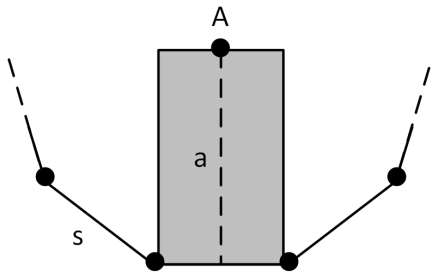
(b) Yes. If the right circle pivoted around point A until the two circles are concentric, then points C and D would coincide.

(c) Yes. The two circles are congruent. Thus, the two chords AC and AD are congruent.

(d) Yes. $m\angle ACB = m\angle ADB$ because they sweep out the same arclength. Thus, $AD = AC$.

(e) There is not sufficient evidence to determine that $\triangle ACD$ is isosceles.

26. A regular polygon, centered at Point A, is partially shown below. Suppose the polygon has 12 sides. The rectangle shares one edge with the polygon, which has sidelength s, and has height a, the distance from the polygon's center to its side. What is the ratio of the area of the polygon to the area of the shaded rectangle?



(a) 3 : 1

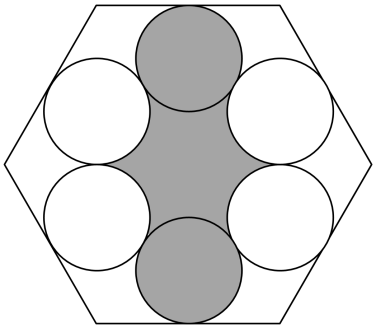
(b) 6 : 1

(c) 12 : 1

(d) 5 : 2

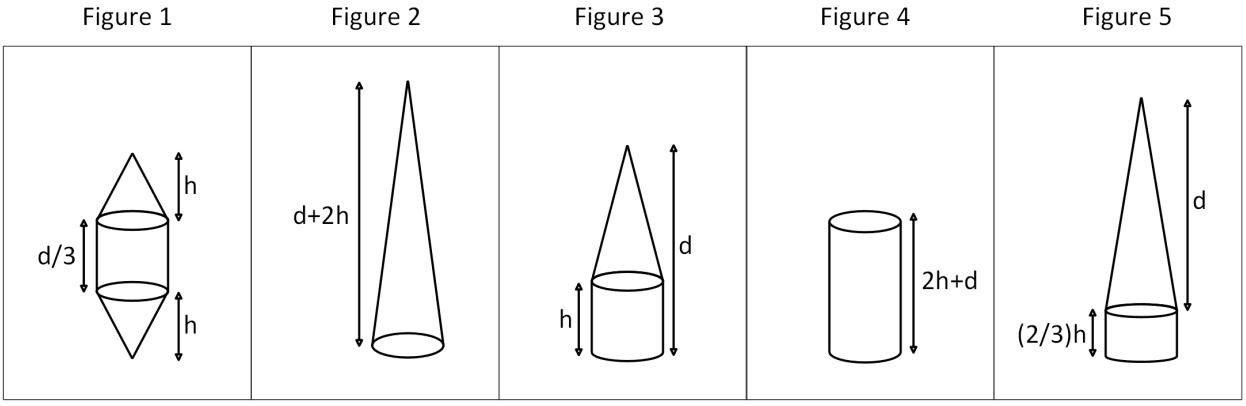
(e) 10 : 2

27. A regular hexagon has six congruent circles inside it. Each circle is tangent to two others. Each circle is also tangent to one of the hexagon's edges at the edge's midpoint. What is the ratio of the shaded area to the area of the hexagon?



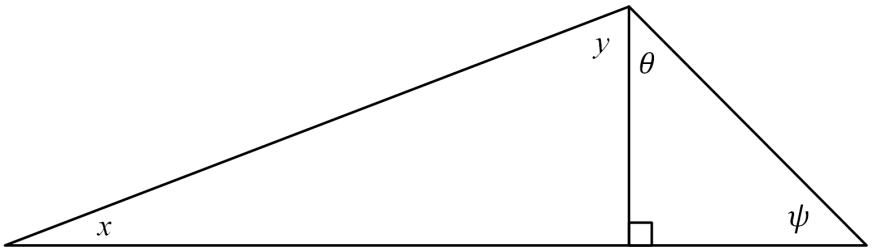
(a) 2 : 7
(b) 1 : 3
(c) 1 : 4
(d) 2 : 5
(e) 3 : 8

28. All of the following figures show three-dimensional shapes that are made up of cylinders and/or cones, all of which have the same radius. Based on the labeled heights, which one of the figures has a volume that is different from the others?



(a) Figure 1
(b) Figure 2
(c) Figure 3
(d) Figure 4
(e) Figure 5

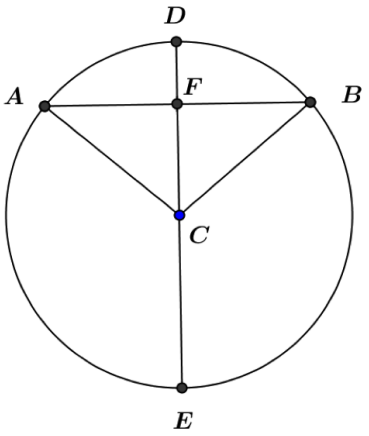
29. In the following image, the angle measures x, y, θ, and ψ are in degrees.



Which of the following sets of conditions would result in infinitely many solutions for x and y?

(a) $\theta = x + 8$ and $\psi = 2y - 66$
(b) $\theta = x + 8$ and $\psi = y - 8$
(c) $\theta = x + 8$ and $\psi = 3y - 124$
(d) $\theta = x + 8$ and $\psi = 2y - 59$
(e) $\theta = x + 8$ and $\psi = 3y - 131$

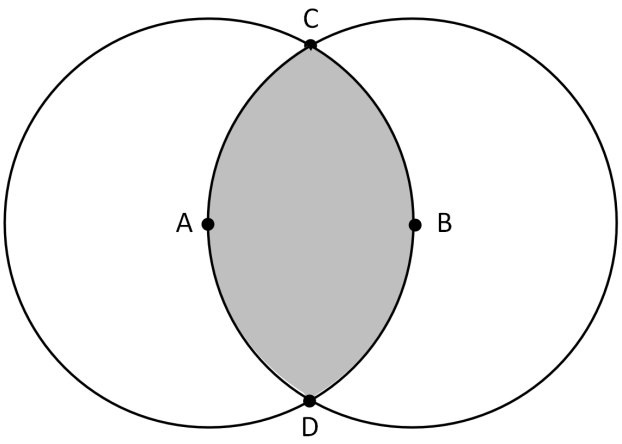
30. Theorem: If a diameter of a circle bisects a chord, then it must be perpendicular to the chord.



One proof strategy is to establish the congruence of $\triangle AFC$ and $\triangle BFC$ and then to leverage that congruence to conclude perpendicularity. Which statement would most likely be found in this proof?

(a) Because of the definition of bisection, we know that $\angle ACF = \angle BCF$.
(b) By the commutative property, $FC = FC$.
(c) By SAS (side-angle-side), $\triangle AFC$ is congruent to $\triangle BFC$.
(d) Because $\overline{AC}$ and $\overline{BC}$ are radii of the same circle, $AC = BC$.
(e) Because $\angle AFC = \angle BFC$ and because these angles are complementary, they must be right angles.

31. Two circles with centers at Points A and B both have a radius of 2 cm. The two circles intersect at exactly two points, C and D. What is the area of the shaded portion?

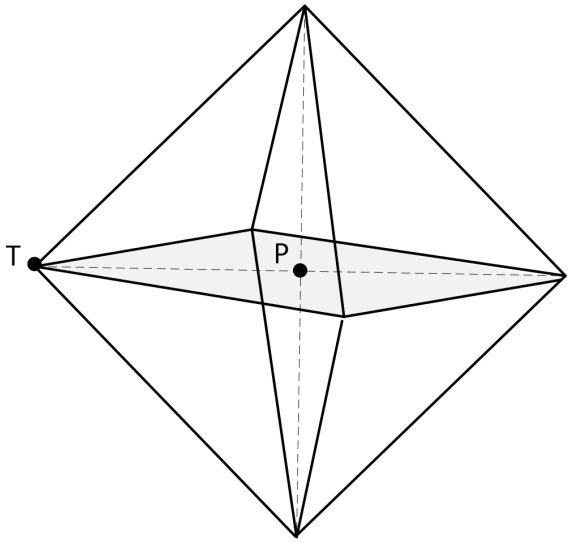


(a) $\left(\frac{4\pi}{3} + 8 - 4\sqrt{3}\right)$ cm^2
(b) $\frac{2}{3}\pi$ cm^2
(c) $\frac{4}{3}\pi$ cm^2
(d) $\left(\frac{8\pi}{3} - 2\sqrt{3}\right)$ cm^2
(e) $2\sqrt{3}$ cm^2

32. A sphere is inscribed in a cylinder whose height equals its diameter. What is the ratio of the sphere's volume to the cylinder's volume?

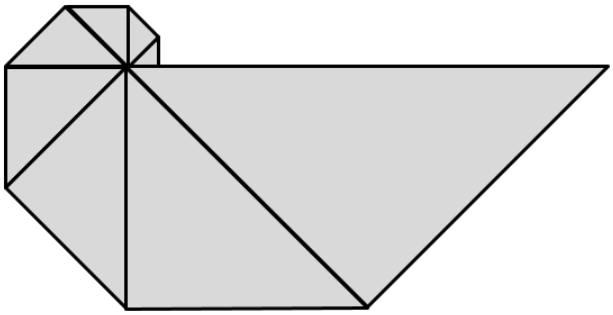
(a) $2 : \pi$
(b) $2 : 3$
(c) $3 : 4$
(d) $\pi : 4$
(e) $3 : 5$

33. A regular octahedron is centered at Point P, as shown. If $TP = 1$, then what is the volume of the octahedron?



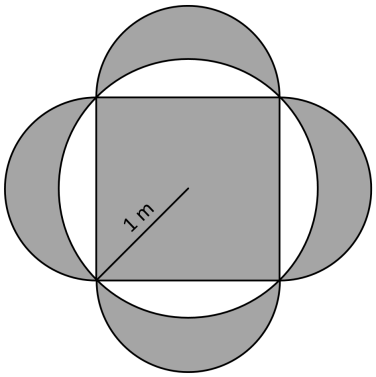
(a) 1 cubic unit
(b) $4\sqrt{3}$ cubic units
(c) $\frac{\sqrt{2}}{3}$ cubic units
(d) $\frac{4\pi}{3}$ cubic units
(e) $\frac{4}{3}$ cubic units

34. The following image was created from 8 isosceles right triangles. A leg of the smallest triangle has a length of a, and the length of the hypotenuse of the largest triangle is b. What is the ratio $a : b$?



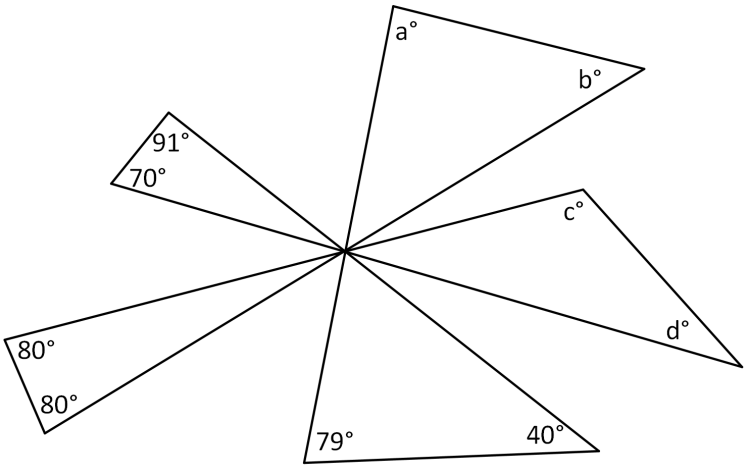
(a) $1 : 10$
(b) $1 : 8\sqrt{2}$
(c) $1 : 13$
(d) $1 : 16$
(e) $1 : 16\sqrt{2}$

35. A square is inscribed inside a circle of radius 1 meter. Each edge of the square is a diameter of a semicircle. What is the total area of the unshaded regions?



(a) $(\pi + 4)$ m^2
(b) $(4 - \pi)$ m^2
(c) $(2 - \pi)$ m^2
(d) 2 m^2
(e) $(\pi - 2)$ m^2

36. The following image was created with exactly ten straight lines, five of which intersect at a single point. What is the value of the sum $a + b + c + d$?

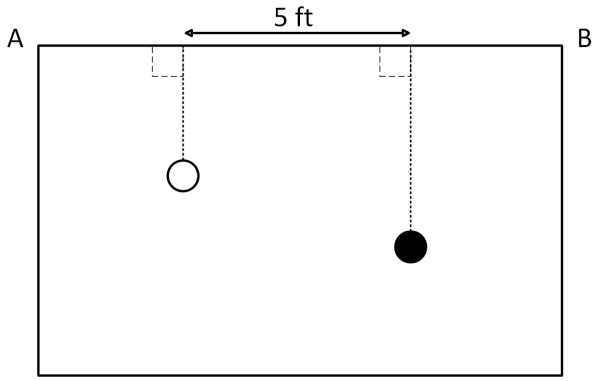


(a) 280
(b) 100
(c) 220
(d) 440
(e) 500

37. A cylindrical tank contains some water. A spherical marble is completely submerged in the water, causing the water level to rise by a distance that is precisely the same as the marble's radius. What is the tank's radius divided by the marble's radius?

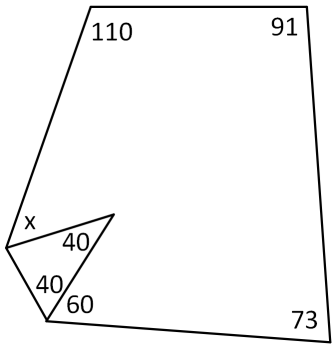
(a) 1
(b) $\frac{2}{3}$
(c) $\sqrt{2}$
(d) $\frac{4}{3}$
(e) $\frac{2\sqrt{3}}{3}$

38. You are playing billiards. The white ball, as shown, is 2 feet away from the wall AB, and the black ball is 3 feet away from the wall AB. If the white ball is struck and hits wall AB before hitting the black ball, how far will the white ball travel before it hits the black ball? Note that the measures of the angles made between the wall AB and the path of the ball are the same as the ball approaches and as it leaves the wall.



(a) 5 units
(b) 10 units
(c) $\sqrt{34}$ units
(d) $5\sqrt{2}$ units
(e) $10\sqrt{2}$ units

39. The values in the following diagram are angle measures given in degrees. What is the value of x?



(a) 66

(b) 80

(c) 70

(d) 76

(e) 67

40. A nearby storage building on level ground is shaped like a rectangular prism with a length of 16 feet and a width of 10 feet. If a dog is on a 20-foot leash tied to an outside ground-level corner of this building, with no other obstructions, what is the total area of the yard that the dog can reach while tied up?

(a) 400π ft^2

(b) 369π ft^2

(c) 300π ft^2

(d) 325π ft^2

(e) 329π ft^2