1. An isosceles triangle has a side of length $\sqrt{2}$ units and another side of length 3 units. Which of the following best completes the statement "The length of the third side of this triangle ..."?
(a) is $\sqrt{2}$ units.
(b) is $\sqrt{7}$ units.
(c) is 3 units.
(d) is $\sqrt{11}$ units.
(e) cannot be determined with the given information.
2. A basketball with a diameter of 12 inches is placed in a box. When the basketball is placed in the box, the sides of the box just touch the ball. Which of the following is the best approximation to the amount of open space in the box when the basketball is in the box?
(a) 1577 cubic inches
(b) 1426 cubic inches
(c) 1276 cubic inches
(d) 823 cubic inches
(e) 654 cubic inches
3. A point is reflected across the $y$-axis and the image point is then rotated $180^{\circ}$ around the origin to reach its final position. Which of the following transformations will always result in the original point ending up in the same final position?
(a) The original point is rotated $90^{\circ}$ around the origin.
(b) The original point is reflected across the $x$-axis.
(c) The original point is rotated $90^{\circ}$ clockwise around the origin and is then rotated $180^{\circ}$ around the origin.
(d) The original point is reflected across the $x$-axis and is then reflected across the $y$-axis.
(e) The original point is rotated $180^{\circ}$ around the origin and is then reflected across the $x$-axis.
4. Ben is flying his kite fully extended on its 140 -foot string. Ben is barely able to hold onto the end of the string as the kite hits the top of a tree 100 feet away. If Ben is 4 feet tall, approximately how tall is the tree?
(a) 19 feet
(b) 41 feet
(c) 44 feet
(d) 102 feet
(e) 164 feet
5. The graph of a function $G$ is the reflection of the graph of a function $F$ over the line $y=x$. If the point $(-2,8)$ is on the graph of the function $F$, which of the following points must be on the graph of the function $G$ ?
(a) $(2,8)$
(b) $(2,-8)$
(c) $(-8,-2)$
(d) $(-8,2)$
(e) $(8,-2)$
6. A polygon satisfying which of the following statements must be a square?
(a) The polygon is a parallelogram with four right angles.
(b) The polygon is a parallelogram with four congruent sides.
(c) The polygon is a rhombus with four congruent sides.
(d) The polygon is a rhombus with four right angles.
(e) The polygon is a rectangle.
7. Segment $\overline{\mathrm{AB}}$ is tangent to a circle centered at point C . Point A is on the circle and the distance between points $A$ and $B$ is 16 units. If the distance between points $B$ and $C$ is 20 units, what is the length of the radius of the circle?
(a) 16 units
(b) 12 units
(c) 9 units
(d) 8 units
(e) 4 units
8. What is the area of a trapezoid if the trapezoid has a base of length 12 cm , a height of 6 cm , and a median of length 18 cm ?
(a) $72 \mathrm{~cm}^{2}$
(b) $90 \mathrm{~cm}^{2}$
(c) $108 \mathrm{~cm}^{2}$
(d) $144 \mathrm{~cm}^{2}$
(e) $180 \mathrm{~cm}^{2}$
9. When two lines are cut by a transversal, what is the maximum number of distinct pairs of congruent angles that can be created?
(a) 8 distinct pairs
(b) 12 distinct pairs
(c) 20 distinct pairs
(d) 28 distinct pairs
(e) 32 distinct pairs
10. The perpendicular bisectors of the sides of a triangle ABC can NEVER meet at which of the following?
(a) The incenter of the triangle
(b) The centroid of the triangle
(c) The circumcenter of the triangle
(d) A point whose distance from A is less than or equal to its distance from B
(e) A point whose distance from A is greater than its distance from B
11. The base of a hemisphere has an area of approximately $452 \mathrm{~mm}^{2}$. What is the approximate height of the hemisphere?
(a) 8 mm
(b) 10 mm
(c) 12 mm
(d) 15 mm
(e) 21 mm
12. Consider the functions: $f(\mathrm{x})=x^{2}, g(x)=\sqrt[3]{x}, h(x)=4 x+3$, and $k(x)=x^{3}-4 x$. The graphs of how many of these functions have a line of symmetry?
(a) none
(b) one
(c) two
(d) three
(e) four
13. In a right triangle with sides of lengths 3 units, 4 units, and 5 units, what is the sum of the cosine, sine, and tangent of the smallest angle in the triangle?
(a) $\frac{43}{20}$
(b) $\frac{41}{15}$
(c) $\frac{11}{3}$
(d) $\frac{7}{4}$
(e) $\frac{17}{4}$
14. A right square prism with dimensions $w \times w \times h$ is cut and separated into 5 smaller right square prisms. Each of the 5 smaller prisms also has a square base with dimensions $w \times w$. If the total surface area of the 5 smaller prisms is exactly twice that of the original prism, what is the volume of the original prism?
(a) $1.5 w^{3}$ cubic inches
(b) $2.5 w^{3}$ cubic inches
(c) $2 w^{3}$ cubic inches
(d) $\sqrt{2} w^{3}$ cubic inches
(e) $\sqrt[3]{5} w^{3}$ cubic inches
15. A square is inscribed in a circle. The diameter of the circle is 12 inches. Which of the following best approximates the area of the region enclosed by the circle that is not inside the square?
(a) $20 \mathrm{in}^{2}$
(b) $41 \mathrm{in}^{2}$
(c) $72 \mathrm{in}^{2}$
(d) $113 \mathrm{in}^{2}$
(e) $380 \mathrm{in}^{2}$
16. Which of the following will be the graph of the function that gives the length of the shadow of a flagpole throughout a sunny day as a function of time?
(a)
(b)

(c)

(d)

(e)

17. Suppose we are trying to draw triangle ABC so that the measure of angle ABC is $30^{\circ}$, the length of segment $\overline{\mathrm{BC}}$ is 20 units, and the length of segment $\overline{\mathrm{AC}}$ is among the lengths 9.5 units, 10 units, 15 units, 20 units, and 25 units. For how many of these choices for the length of $\overline{\mathrm{AC}}$ will we be able to draw two non-congruent triangles satisfying the stated properties?
(a) one
(b) two
(c) three
(d) four
(e) five
18. What is the measure of a central angle of a circle if the central angle intersects the same arc of the circle as an inscribed angle that has a measure of $75^{\circ}$ ?
(a) $37.5^{\circ}$
(b) $60^{\circ}$
(c) $75^{\circ}$
(d) $90^{\circ}$
(e) $150^{\circ}$
19. A goat is tied to the corner of a rectangular shed that is 30 feet by 45 feet. The shed sits in a large grassy area. The rope used to tie the goat will allow the goat to reach a maximum distance of 40 feet. What is the area of the region that the goat has available for grazing?
(a) $1225 p$ square feet
(b) 1050p square feet
(c) 1025 p square feet
(d) 1000 p square feet
(e) 900 p square feet

## A

20. The letters ABC when written vertically are B. Which of the following sets of letters when C written vertically will have symmetry?
(a) CAT
(b) TUB
(c) SAT
(d) CUT
(e) HAT
21. The vertices of a triangle are the points $(0,2),(18,-22)$, and $(-14,-46)$ in the $x y$-plane. What is the area of the circumscribed circle for this triangle?
(a) $225 p$ square units
(b) 400 p square units
(c) 550 p square units
(d) 625 p square units
(e) 900 p square units
22. The area of the shaded region in a regular octagon (shown below) is 12 square units. What is the perimeter of the octagon?
(a) $8 \sqrt{12 \sqrt{2}-12}$ units
(b) $8 \sqrt{12 \sqrt{2}+12}$ units
(c) $8 \sqrt{6 \sqrt{2}+6}$ units

(d) $8 \sqrt{6 \sqrt{2}-6}$ units
(e) $8 \sqrt{12 \sqrt{2}-6}$ units
23. The area of Circle 1 is twice the area of Circle 2. Which of the following statements about these two circles are true?

Statement 1: The circumference of Circle 1 is twice the circumference of Circle 2.
Statement 2: The radius of Circle 1 is twice the radius of Circle 2.
Statement 3: The square of the diameter of Circle 2 is equal to the square of the radius of Circle 1.
(a) Only statement 1 is true.
(b) Only statement 2 is true.
(c) Only statements 1 and 3 are true.
(d) Only statements 2 and 3 are true.
(e) None of the statements are true.
24. In triangle ABC , the length of $\overline{\mathrm{BC}}$ is 100 units. Points D and E are selected on $\overline{\mathrm{AB}}$ and $\overline{\mathrm{AC}}$, respectively, to form trapezoid DECB. If the median of trapezoid DECB has a length of 60 units, what is the length of $\overline{\mathrm{DE}}$ ?
(a) 10 units
(b) 20 units
(c) 30 units
(d) 40 units
(e) 50 units
25. On the coordinate grid below, points $\mathrm{A}, \mathrm{B}$, and C have integer coordinates. Which of the following is closest to the measure (in degrees) of angle ABC ?
(a) 132 degrees
(b) 133 degrees
(c) 134 degrees
(d) 135 degrees
(e) 136 degrees

26. In parallelogram $A B C D$, vertex $A$ is located at (1.1, 1.7), vertex $B$ is located at $(2.1,-1.3)$, and vertex C is located at $(5.1,-0.7)$. Vertex D is located at $(a, b)$. What is the value of $a$ ?
(a) 2.6
(b) 3.1
(c) 3.6
(d) 4.1
(e) 4.6
27. What is the area of a rectangle inscribed in a semicircle if the base of the rectangle has a length of 3 inches and lies along the 5 -inch diameter of the semicircle?
(a) 6 square inches
(b) $3 \sqrt{5}$ square inches
(c) 12 square inches
(d) $\frac{25 \pi}{4}$ square inches
(e) $\frac{25 \pi}{8}$ square inches
28. Three circles, with radii of 1 unit, 2 units, and 3 units, are externally tangent as shown below. Which of the following is closest to the area of the triangle whose vertices are the centers of the three circles?
(a) $\frac{1^{2}+2^{2}+3^{2}}{4}$ square units
(b) $1+2+3$ square units
(c) $\sqrt{1}+\sqrt{2}+\sqrt{3}$ square units

(d) $1^{2}+2^{2}+3^{2}$ square units
(e) $\frac{\sqrt{1}+\sqrt{2}+\sqrt{3}}{4}$ square units
29. Chains and furlongs are old units for measuring length. An acre of land is defined to be the area of a rectangle which is one chain by one furlong. If there are 640 acres in a square mile and 10 chains in a furlong, how many chains are there in a mile?
(a) 10
(b) 16
(c) 32
(d) 64
(e) 80
30. A giant 50-pound block of tofu (bean curd) is $98 \%$ water. If it dehydrates until it is just $96 \%$ water, what is the weight of the dehydrated lump?
(a) 24 pounds
(b) 25 pounds
(c) 47.5 pounds
(d) 48 pounds
(e) 49 pounds
31. Three circles have their centers on the diameter of a fourth circle as shown below. The largest circle has a diameter of 20 units. Which of the following best completes the statement "The sum of the circumferences of all four circles ..."?
(a) is 20 p units.
(b) is 30 p units.
(c) is 40 p units.
(d) is 50 p units.
(e) cannot be determined with the given information.

32. A band leader stands on a platform looking out at the band members that are lined up in rows. The band leader's eyes are 16 feet off the ground and the band leader cannot see any of the ground in the gap between the last two rows of band members. If the last row is 1000 feet from the base of the platform and all of the band members are six feet tall, what is the maximum width of the gap between the last two rows?
(a) 6 feet
(b) 15 feet
(c) 64 feet
(d) 375 feet
(e) 994 feet
33. A two-wheeled wagon has two different sized wheels. The wheel on the left has a radius that is one foot larger than the wheel on the right. When the wagon is pulled a certain distance the left wheel rotates $x$ times and the smaller right wheel rotates $3 x$ times. What is the radius of the right wheel?
(a) 0.5 feet
(b) 1.0 feet
(c) 1.5 feet
(d) 2.0 feet
(e) 2.5 feet
34. The altitude from vertex $A$ in triangle $A B C$ intersects $\overline{\mathrm{BC}}$ at point D . The length of this altitude is $\sqrt{3}$ units and the length of $\overline{\mathrm{BC}}$ is 3 units. If the length of $\overline{\mathrm{DC}}$ is equal to the length of $\overline{\mathrm{AB}}$, what is the length of $\overline{\mathrm{AC}}$ ?
(a) $\sqrt{3}$ units
(b) 2 units
(c) $\sqrt{5}$ units
(d) $\sqrt{6}$ units
(e) $\sqrt{7}$ units
35. The first three numbers in a sequence are $S_{1}=2, S_{2}=3$, and $S_{3}=5$. For $n \geq 4$, the nth term in the sequence is given by $\mathrm{S}_{n}=\mathrm{S}_{n-1}+\mathrm{S}_{n-2}$. Which of the following is closest to the ratio $\frac{\mathrm{S}_{100}}{\mathrm{~S}_{99}}$ ?
(a) The mathematical constant e
(b) The mathematical constant p
(c) The golden ratio
(d) The square root of 2
(e) The cube root of 5
36. Chapter 498 of the TN Public Acts of 1905 which defines the Tennessee state flag starts as follows:

An oblong flag or banner in length one and two thirds times its width, the large or principal field of same to be of color red, but said flag or banner ending at its free or outer end in a perpendicular bar of blue, of uniform width, running from side to side; that is to say, from top to bottom of said flag or banner, and separated from the red field by a narrow margin or stripe of white of uniform width; the width of the white stripe to be one-fifth that of the blue bar; and the total width of the bar and stripe together to be equal to one-eighth of the width of the flag. In the center of the red field shall be a smaller circular field of blue, . . .

If a Tennessee flag is 72 centimeters tall (called the `width' in the description above), then how wide is the thin vertical white stripe?
(a) 0.75 centimeters
(b) 1.5 centimeters
(c) 1.8 centimeters
(d) 2.5 centimeters

(e) 0.36 centimeters
37. Two intersecting lines have slopes $\frac{4}{3}$ and $\frac{-5}{12}$, respectively. A line with which of the following slopes will bisect one of the pairs of vertical angles formed by these intersecting lines?
(a) $\frac{2}{9}$
(b) $\frac{3}{11}$
(c) $\frac{8}{33}$
(d) $\frac{7}{33}$
(e) $\frac{9}{37}$
38. In triangle ABC , point D is located two-fifths of the way from A to B and point E is located two-fifths of the way from C to B . What is the ratio of the area of triangle DBE to the area of triangle ABC ?
(a) $\frac{2}{5}$
(b) $\frac{3}{5}$
(c) $\frac{2}{3}$
(d) $\frac{4}{25}$
(e) $\frac{9}{25}$
39. Which of the following is the best approximation of the area of the shaded region (shown below) between three externally tangent circles, each with a radius of 1 unit?
(a) 0.14159 square units
(b) 1.40954 square units
(c) 0.16125 square units
(d) 0.86603 square units
(e) 0.15227 square units

40. Sally is standing $x$ feet from the base of a building and notices that the angle of elevation to the top of the building is $70^{\circ}$. If she steps back another 20 feet from the base of the building, she notices that the angle of elevation becomes $50^{\circ}$. What is the height of the building?
(a) $\frac{20 \sin \left(70^{\circ}\right) \sin \left(50^{\circ}\right)}{\sin \left(20^{\circ}\right)}$ feet
(b) $\frac{20 \sin \left(50^{\circ}\right) \sin \left(20^{\circ}\right)}{\sin \left(70^{\circ}\right)}$ feet
(c) $\frac{20 \sin \left(50^{\circ}\right) \sin \left(20^{\circ}\right)}{\sin \left(30^{\circ}\right)}$ feet
(d) $\frac{20 \sin \left(70^{\circ}\right) \sin \left(20^{\circ}\right)}{\sin \left(50^{\circ}\right)}$ feet
(e) $\frac{20 \sin \left(70^{\circ}\right) \sin \left(50^{\circ}\right)}{\sin \left(120^{\circ}\right)}$ feet

