

FORTY-NINTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry 2005

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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1. Assuming "For every two distinct points, there exists exactly one line" is true, what is the minimum and maximum number of distinct lines which could exist if you have exactly four distinct points in your system?
 - a) 1 and 2 b) 1 and 4 c) 1 and 6 d) 2 and 4 e) 2 and 6

2. A six foot long ladder is leaning against a vertical wall while the bottom of the ladder rests on horizontal ground. If x is the measure of the angle the ladder makes with the wall, what is the measure of the angle the ladder makes with the ground?
 - a) x° b) $90^\circ - x^\circ$ c) 90° d) $180 - x^\circ$ e) none of these

3. In triangle ABC, $m\angle A = 30^\circ$ and the measure of an exterior angle at B is 120° . Which is the longest side of the triangle?
 - a) \overline{AB} b) \overline{BC} c) \overline{AC} d) cannot be determined e) all are equal in length

4. Two angles whose measures are in the ratio 5:1 form a linear pair. If the larger of these two angles is trisected, what is the measure of each of the three angles thus formed?
 - a) 40° b) 45° c) 50° d) 55° e) 60°

5. The graph of which of the following figures can include the points with coordinates $(-1, -1)$, $(0, 4)$, $(7, 1)$, $(4, 5)$, $(3, 0)$, and $(8,6)$?
 - a) trapezoid only b) rectangle only c) parallelogram only d) b and c only
 - e) a, b and c

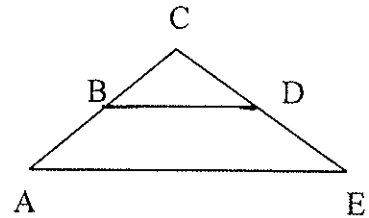
6. The endpoints of a line segment are $(2a, a+3b)$ and $(4a-2b, 5a-b)$. Express the coordinates of the midpoint of the segment in terms of a and b .
 - a) $(3a-b, 3a+b)$ b) $(3a, 2a+2b)$ c) $(2a, 3a)$ d) $(3a-2b, 3a)$ e) $(6a-2b, 6a+2b)$

7. If in manufacturing the wheel of a vehicle, it is required that the wheel's diameter be 30.00 ± 0.01 inches, how precise must the radius of the wheel be manufactured?
 - a) 15.000 ± 0.0025 b) 15.000 ± 0.005 c) 15.000 ± 0.0075 d) 15.00 ± 0.01
 - e) 15.00 ± 0.05

8. The scale on a map indicates that $\frac{1}{2}$ inch on the map is equivalent to 15 miles in the "real world." If it is a straight-line distance of 210 miles from Paduka to where you live, what is the length of this route on the map?
 - a) $5 \frac{1}{2}$ in. b) 6 in. c) $6 \frac{1}{2}$ in. d) 7 in. e) $7 \frac{1}{2}$ in.

9. Which of the following choices is the smallest real number?
 a) $7/15$ b) $\sqrt{50/14}$ c) $5/\sqrt{40}$ d) $\sqrt{2/3}$ e) $\sqrt{100/400}$
10. You have a cube-shaped box, one foot on each side. You decide you want twice the amount of volume in a new cubic box you want to build. The new length of each side should be between...
 a) 1 and $1\frac{1}{4}$ feet b) $1\frac{1}{4}$ and $1\frac{1}{2}$ feet c) $1\frac{1}{2}$ and $1\frac{3}{4}$ feet
 d) $1\frac{3}{4}$ and 2 feet e) none of these, it is more than 2 feet
11. If $\triangle ABC \cong \triangle PQR$ and $\triangle PQR \cong \triangle XYZ$, then which property must be involved in concluding that $\angle A \cong \angle X$?
 a) Symmetric property b) Reflexive property c) Distributive property
 d) Associative property e) Transitive property or Substitution

12. Which of the following segments corresponds to \overline{CD} when we are considering the two similar triangles pictured?

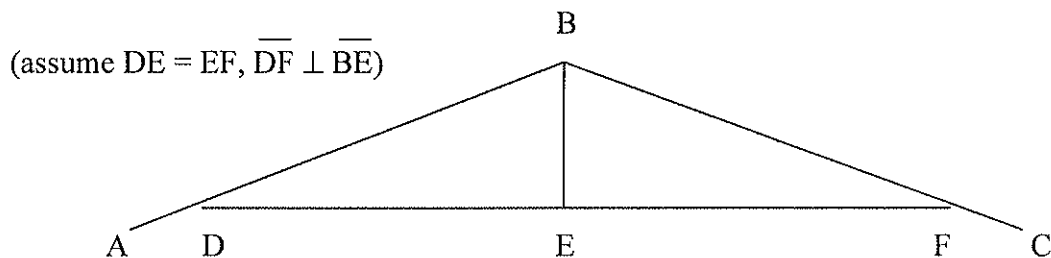


(assume \overline{BD} is parallel to \overline{AE})

- a) \overline{CB} b) \overline{BA} c) \overline{DE} d) \overline{CE} e) \overline{CA}
13. How many points in a plane are 3 units from line m and also 4 units from a point P on line m ?
 a) 0 b) 2 c) 3 d) 4 e) infinite number
14. The area of an equilateral triangle is $\frac{\sqrt{3}}{4}$. What is the length of one of the sides?
 a) 1 b) $\frac{\sqrt{3}}{2}$ c) $\sqrt{2}$ d) $3/2$ e) $3\sqrt{2}$
15. If the sum of the measures of the interior angles of a polygon equals the sum of the measures of all the exterior angles (include two at each vertex), the polygon is
 a) a triangle b) a quadrilateral c) a pentagon d) a hexagon e) an octagon
16. If the perpendicular bisectors of the sides of a triangle intersect at a point outside the triangle, then the triangle must be
 a) equilateral b) acute c) right d) obtuse e) isosceles

17. A triangle is to a parallelogram as a pyramid is to a
- a) cone b) cylinder c) sphere d) trapezoid e) prism
18. The area of a sector of a circle is $9\pi/40$ and its central angle has measure 36° . What is the radius of the circle?
- a) $3/2$ b) $9/4$ c) $40/9$ d) $81/10$ e) 160
19. The length of each side of a building built in the shape of a regular hexagon is 80 feet. How wide is the building at its widest (corner to corner)?
- a) 80 feet b) $80\sqrt{2}$ feet c) $80\sqrt{3}$ feet d) 160 feet e) $160\sqrt{3}$ feet

20. A roof truss is to be constructed, as shown in the diagram below, using just four planks (\overline{AB} , \overline{BC} , \overline{DF} , and \overline{BE}). A 2 foot "overhang" is necessary on each side (\overline{AD} and \overline{FC}). If $DF = 30$ feet and $BE = 8$ feet, what is the total length of all four planks?



- a) 72 feet b) 74 feet c) 76 feet d) 82 feet e) 84 feet
21. For a fundraiser at a fair, a 5 foot by 10 foot piece of plywood is used as a backboard. Then 24 spherical balloons, each of diameter 10 inches, are taped to the board as targets in the contest. No balloon sticks out past the edge of the board. Contestants are given a dart and allowed to try to hit the balloon (if they miss the board they may try again until they hit it). Assuming they are random shots, what is the probability that a contestant will hit a balloon?
- a) $2\pi/5$ b) $\pi/10$ c) $4\pi/25$ d) $\pi/12$ e) $\pi/50$
22. A certain distance can be "measured" or divided by lengths of 6 meters, 7 meters, 8 meters, and 9 meters (and in each case leave no remainder). What is the minimum this distance can be in meters?
- a) 126 b) 168 c) 504 d) 1008 e) 3024

23. A stack of rectangular shoeboxes is to be equally tall as it is wide but twice as long as it is high. If the stack is in the shape of a rectangular box itself and each individual shoebox has dimensions $\frac{1}{2}$ ft. x $\frac{3}{4}$ ft. x 1 ft., express the approximate number of shoeboxes as a function of the height (in feet) of the stack h .

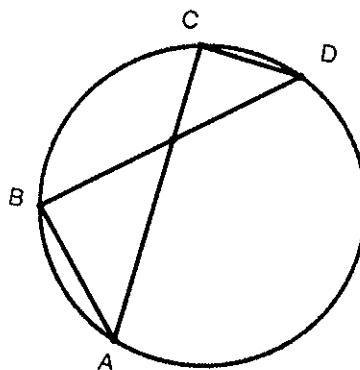
- a) $3h^3/8$ b) $11h^2/2$ c) $\frac{3}{4}h^3$ d) $16h^3/3$ e) $4h^3$

24. How many diagonals does a convex n -sided polygon have?

- a) $(n-2) + (n-3) + (n-4) + \dots + 2 + 1$ b) $(n-3) + (n-4) + \dots + 2 + 1$
 c) $(n-3) + (n-3) + (n-4) + \dots + 2 + 1$ d) $2[(n-3) + (n-4) + \dots + 2 + 1]$
 e) $2(n-3) + (n-2) + (n-3) + \dots + 2 + 1$

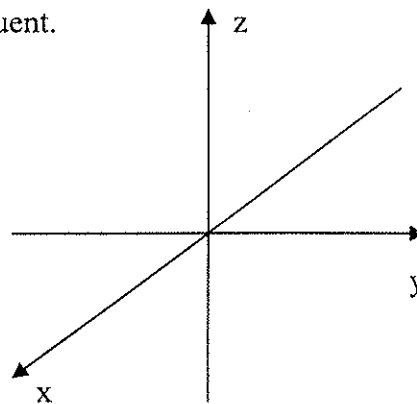
25. Which of the following facts is necessary to prove that $\angle B \cong \angle C$ in the figure?

(Suppose you only know that A, B, C, and D are points on the circle.)



- a) Arc \widehat{AD} is inscribed by both angles.
 b) Vertical angles are congruent.
 c) Alternate interior angles are congruent.
 d) If two angles of a triangle are congruent to two angles in another triangle, then the third angles in each triangle are congruent.
 e) Corresponding parts of congruent triangles are congruent.

26. An x - y - z coordinate system is established as pictured with the arrow denoting the positive "half" of each axis. If we call the x - y plane the "floor," the x - z plane the "left wall" and the y - z plane the "right wall," which of the following describes the location of the point $(x,y,z) = (-2, 3, -1)$?

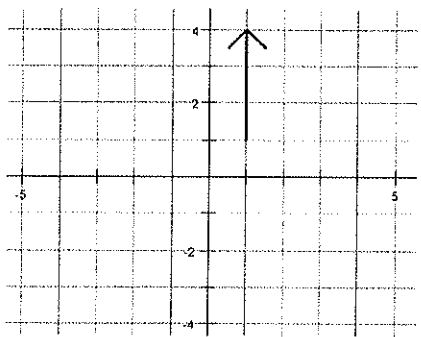


- a) in front of the two walls but below the floor
 b) behind the left wall and below the floor
 c) above the floor but behind both walls
 d) behind both walls and below the floor
 e) in front of the left wall, behind the right wall, and below the floor

27. If line m is perpendicular to line n , line p is perpendicular to line q , and line m and line p are parallel, then what must be the relationship between lines n and q ?

- a) parallel b) perpendicular c) skew d) on the same plane e) none of these

28. If a quadrilateral has two pairs of congruent opposite sides, then which classification must the quadrilateral fall into?
- a) parallelogram b) square c) rectangle d) rhombus e) trapezoid
29. One line separates a plane into 2 regions. A second line drawn separates it into at most 4 regions. At most, into how many regions would four lines separate a plane?
- a) 10 b) 11 c) 12 d) 13 e) 14
30. The area of a trapezoid is 36, one base is 6, and the height is 8. Find the length of the other base.
- a) 3 b) 4 c) 4.5 d) 6 e) 9
31. M is the midpoint of \overline{AB} and N is the midpoint of \overline{AM} . If the coordinates of A are (2, -4) and the coordinates of N are (1, -1), find the coordinates of B.
- a) (-2, 8) b) (-2, 6) c) (-1, 6) d) (0, 2) e) (1, -1)
32. Circles O and P are externally tangent at point T. \overline{QR} , \overline{TR} , and \overline{RS} are tangent segments to external point R. Given that for any particular circle, tangent segments to an external point are equal, which property would be needed to conclude that $QR = RS$?
- a) Associative property b) Commutative property c) Reflexive property
d) Symmetric property e) Transitive property
33. Suppose the arrow " \uparrow " is drawn on the Cartesian plane as in the figure below so that the top point is at the point (1,4) and the base is at the point (1,1). If the arrow is transformed first by reflection in the y-axis and then by a counterclockwise rotation of 270° about its base point, in which orientation is the arrow?



- a) \uparrow b) \rightarrow c) \downarrow d) \leftarrow e) none of these

34. If the images of points (4,8) and (2,6) under the transformation $f(x,y) = (kx, ky)$ are the points (10, 20) and (5, 15), then what is the image of the point (10, 16)?
- a) (20, 32) b) (24, 36) c) (25, 40) d) (30, 48) e) (32, 54)
35. If the Euclidean parallel postulate were not assumed to be true, then which of the following can still be concluded?
- a) If parallel lines are cut by a transversal, then alternate interior angles are congruent.
b) If two sides of a triangle are congruent, then the angles opposite those sides must be congruent.
c) The sum of the measures of the angles of any triangle is 180° .
d) For any line l and a point P not on l , there exists exactly one line m through P such that l is parallel to m .
e) Two lines both perpendicular to a third line are parallel to each other.
36. What is the area of rectangle ABCD if its perimeter is 43 and \overline{CD} has length 13.5 ?
- a) 108 b) 172 c) 175.5 d) 216 e) 870.25
37. If the n sides of a regular polygon each measure 6 inches, then which of the following gives the distance from the center of the regular polygon to any side of the polygon?
- a) $6 \sin (360^\circ/n)$ b) $3\sqrt{3} \cos (360^\circ/n)$ c) $3 \cot (180^\circ/n)$
d) $6\sqrt{2} \sin (180^\circ/n)$ e) none of these
38. If the distance from the center of a regular hexagon to one of the sides is $7\sqrt{3}$, what is the perimeter of the hexagon?
- a) 42 b) 63 c) $42\sqrt{2}$ d) $42\sqrt{3}$ e) 84
39. In triangle XYZ, $\angle X$ measures 2 more than twice the measure of $\angle Y$ and $\angle Y$ measures 4 less than half of $\angle Z$. What is the measure of $\angle X$?
- a) 66° b) 70° c) 72.5° d) 76° e) 78°
40. Triangle PQR has a right angle at P. If $QR = 16$, what is the length of median \overline{PS} ?
- a) 4 b) $4\sqrt{3}$ c) 8 d) $8\sqrt{3}$ e) 16

