FORTY-FIRST ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry 1997

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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Forty-first Annual Mathematics Contest
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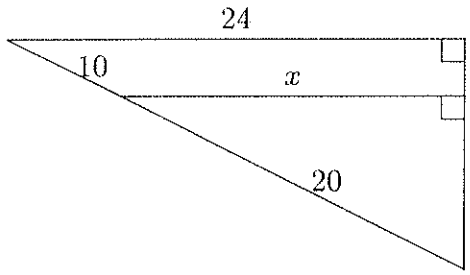
Notation:
$\overline{AB}$ denotes the straight line segment joining points A and B,
$\angle AOB$ denotes the angle with vertex O determined by points A, O, and B, and
a right angle (90° angle) is denoted by $\llcorner$.

Caution:
Many problems on this test are accompanied by diagrams. Though none are meant to be intentionally misleading, these diagrams may not be exactly to scale.

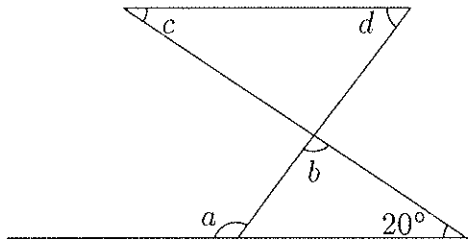
1. What is the value of length x?

(a) 12
(b) 14
(c) 16
(d) 18
(e) 20



2. Which of the following gives the measure of angle a in terms of angles b, c, and d?

(a) $200° - c - d$
(b) $160° - c - d$
(c) $160° - 2b + c + d$
(d) $c + d - 20°$
(e) $200° - b + c + d$



3. What is the length of an arc intercepted by one side of a regular hexagon inscribed in a circle of radius 18?

(a) 60 (b) 2π (c) 3π (d) 6π (e) 60π

4. The radius of a circle is increased by 2 units. How is the new circumference obtained from the old circumference?

(a) add 2 units
(b) add 2π units
(c) add 4π units
(d) multiply by 2
(e) There is not enough information given.

5. The lengths of two sides of a triangle are 10 and 14. Which of the following *could be* the length of the third side?

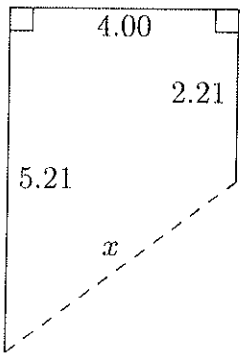
(a) 2 (b) 4 (c) 22 (d) 24 (e) 26

6. A circle passes through the vertices of a right triangle with sides 6, 8, and 10. What is the radius of the circle?

(a) 4 (b) 4.6 (c) 5 (d) 6 (e) 6.4

7. What is the value of length x?

(a) 1.79
(b) 3.00
(c) 3.21
(d) 5.00
(e) 5.21



8. What is the length of the longest thin rigid straight rod that can fit entirely within a rectangular box having side lengths 1, 2, and 3?

(a) $\sqrt{5}$ (b) $\sqrt{6}$ (c) $\sqrt{8}$ (d) $\sqrt{10}$ (e) $\sqrt{14}$

9. What is the sum of the degree measures of the interior angles of a polygon with 10 sides?

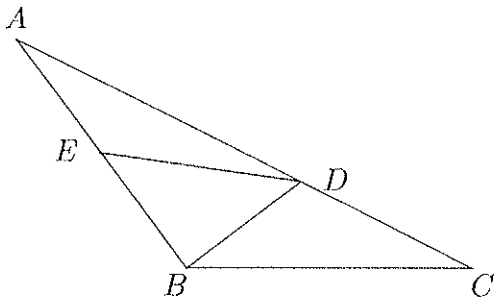
(a) 1440° (b) 1620° (c) 1800° (d) 1980° (e) 2160°

10. All of the exterior angles of a polygon measure 20°. How many sides does the polygon have?

(a) 8 (b) 9 (c) 12 (d) 18 (e) 27

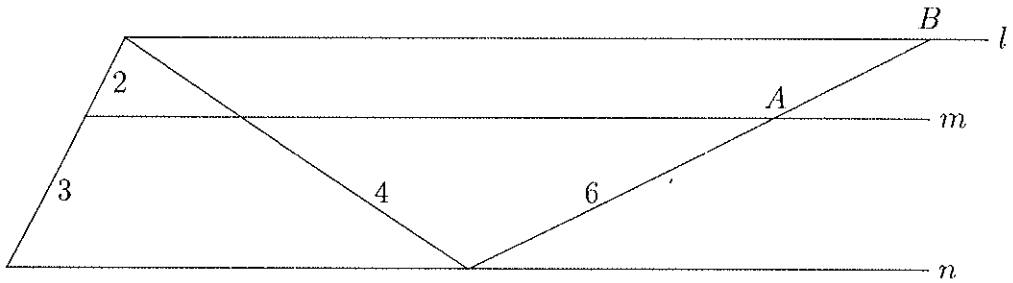
11. Suppose that the length of $\overline{BA}$ equals the length of $\overline{BC}$, the length of $\overline{BE}$ equals the length of $\overline{BD}$, and the measure of $\angle DBC$ is 40°. What is the degree measure of $\angle ADE$?

(a) 15°
(b) 20°
(c) 30°
(d) 40°
(e) 45°



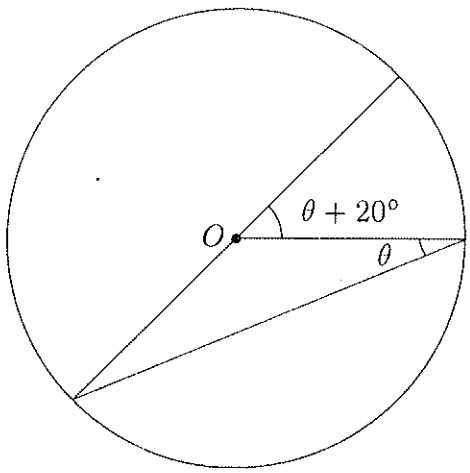
12. Lines l, m, and n are parallel. What is the length of $\overline{AB}$?

(a) 1
(b) 8/3
(c) 3
(d) 4
(e) 5



13. Suppose that O is the center of the circle shown. What is the degree measure of θ?

(a) $5°$
(b) $10°$
(c) $15°$
(d) $20°$
(e) $30°$



14. A circle of radius 4 and a circle of radius 14 are tangent to each other in the plane. The larger circle remains fixed, while the smaller circle rolls without slipping around the outside of the larger. How many complete revolutions will the smaller circle make before the original two tangent points are tangent again?

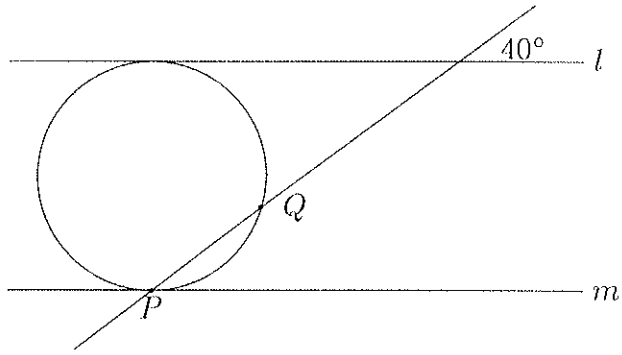
(a) 3 (b) 4 (c) 5 (d) 7 (e) 14

15. An annulus is a region in the plane bounded by two circles having a common center. Suppose that the area of an annulus is equal to the area of its smaller circle. What is the ratio of the radius of its larger circle to the radius of its smaller circle?

(a) 2π (b) $\sqrt{2\pi}$ (c) $\sqrt{2\pi}$ (d) $\sqrt{2}$ (e) 2

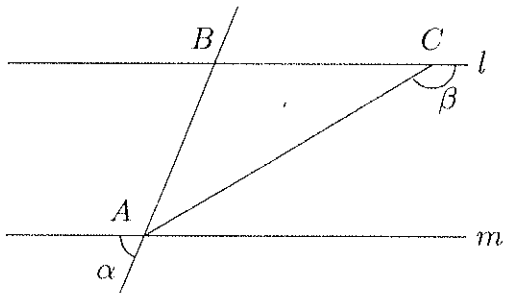
16. A circle of radius 9 is tangent to the two parallel lines l and m. What is the length of the arc $\overset{\frown}{PQ}$?

(a) 2π
(b) 4π
(c) 8π
(d) 4
(e) 36π



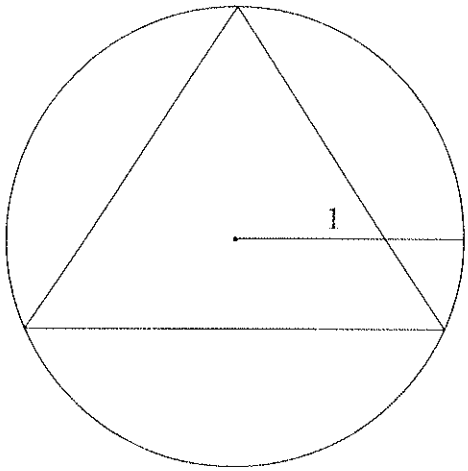
17. The lines l and m are parallel, and the length of $\overline{BA}$ equals the length of $\overline{BC}$. What is the degree measure of β in terms of α?

(a) $180° - \alpha/2$
(b) 2α
(c) $90° - \alpha/2$
(d) $2(180° - \alpha)$
(e) $90° - 2\alpha$



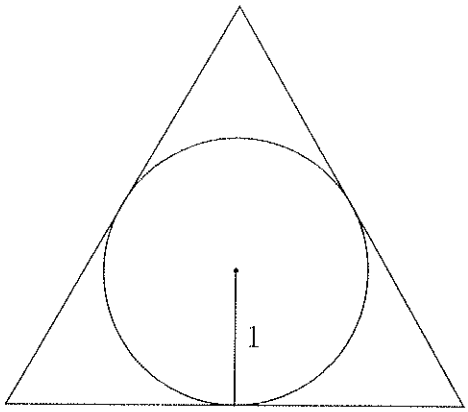
18. What is the area between a circle of radius 1 and its inscribed equilateral triangle?

(a) $1/(2\sqrt{3}) - \pi$
(b) $\pi - 3\sqrt{3}/4$
(c) $\pi - 9/(2\sqrt{3})$
(d) $9/(4\sqrt{3})$
(e) $\pi - 3\sqrt{3}/2$



19. What is the area between a circle of radius 1 and its circumscribed equilateral triangle?

(a) $\sqrt{3}$
(b) $\pi - 6\sqrt{3}$
(c) $2 + 2\sqrt{5} - \pi$
(d) $3/2$
(e) $3\sqrt{3} - \pi$

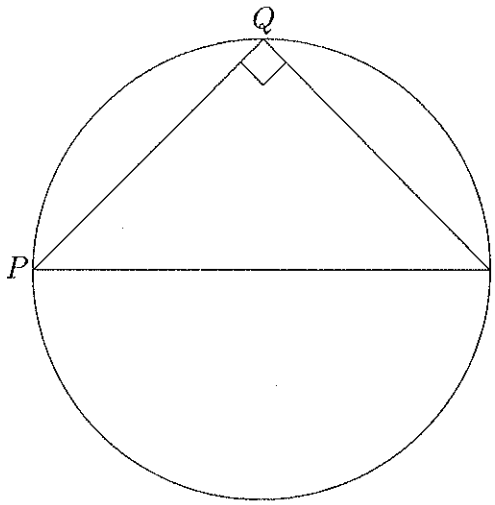


20. A circle is inscribed in a square which in turn is inscribed in a circle. The area of the outer circle is 16π. What is the area of the inner circle?

(a) 4π (b) 8π (c) 12π (d) $16\pi - 8$ (e) $16\pi - 16$

21. An isoceles right triangle is inscribed in a circle of radius 4. What is the length of the arc $\widehat{PQ}$?

(a) $\pi/2$
(b) $3\pi/4$
(c) 4π
(d) 2π
(e) π



22. What is the degree measure of the angle formed by the two diagonals drawn from one vertex of a regular pentagon?

(a) 30° (b) 36° (c) 54° (d) 60° (e) 72°

23. What is the distance between parallel sides of a regular hexagon of side length 1?

(a) 2 (b) $\sqrt{2}$ (c) $\sqrt{3}/2$ (d) $2/\sqrt{3}$ (e) $\sqrt{3}$

24. Suppose that the diameter of a hemisphere is tripled. By what factor does the volume increase?

(a) 3 (b) 9/2 (c) 9 (d) 27/2 (e) 27

25. Suppose that the volume of a sphere is doubled. By what factor does the surface area change?

(a) 2 (b) $3^{3/2}$ (c) $2^{2/3}$ (d) 4 (e) 8

26. A right circular cone is changing size, with the diameter of its base always equal to its height. Suppose that its height is doubled. How does its volume change?

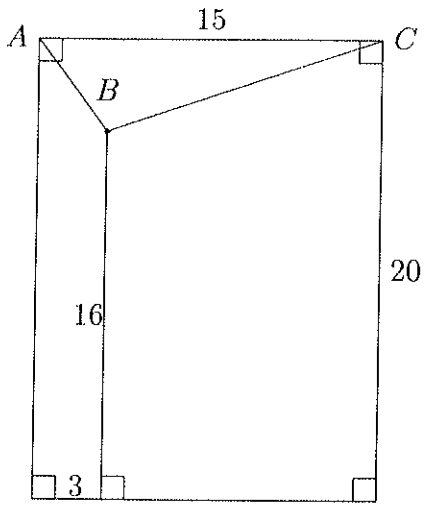
(a) stays the same
(b) is doubled
(c) is multiplied by 4π
(d) is multiplied by 8
(e) is multiplied by $8\pi^2$

27. What is the area of a triangle having side lengths 2, 3, and 4?

(a) $\sqrt{15}/4$ (b) $\sqrt{135}/4$ (c) $9/2$ (d) $\sqrt{345}/4$ (e) 4

28. What is the sum of the length of $\overline{BA}$ and the length of $\overline{BC}$?

(a) $\sqrt{15}$
(b) 5
(c) 165
(d) $5 + 4\sqrt{10}$
(e) $4\sqrt{10}$

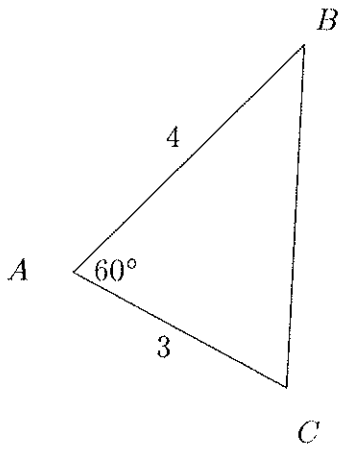


29. Three circles have centers A, B, and C and respective radii a, b, and c, and are tangent to each other in the plane. The length of $\overline{AB}$ is 14, the length of $\overline{AC}$ is 10, and the length of $\overline{BC}$ is 18. What is (a, b, c)?

(a) $(2, 10, 6)$ (b) $(3, 11, 7)$ (c) $(4, 10, 6)$ (d) $(3, 10, 7)$ (e) $(3, 11, 6)$

30. What is the length of $\overline{BC}$?

(a) 5
(b) $\sqrt{13}$
(c) $\sqrt{7}$
(d) 13
(e) $25 - 6\sqrt{12}$



31. An equilateral triangle has area 1. What is the length of its altitude?

(a) $\sqrt[4]{3}$ (b) $\sqrt{3}$ (c) $\sqrt{3}/2$ (d) $2/\sqrt{3}$ (e) $\sqrt{3}/4$

32. A point is 5 units from the center of a circle of radius 3. The two tangent lines to the circle through the point are drawn. What is the distance between the two points of tangency?

(a) 4.0 (b) 4.2 (c) 4.4 (d) 4.6 (e) 4.8

33. The angle formed by two tangents drawn to a circle from the same external point measures 70°. What is the degree measure of the minor intercepted arc?

(a) 70° (b) 90° (c) 100° (d) 110° (e) 180°

34. What is the radius of the circle inscribed in a right triangle, in terms of the triangle's leg lengths a and b?

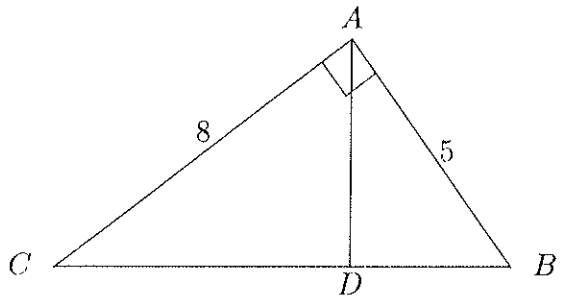
(a) $(a + b - \sqrt{a^2 + b^2})/2$
(b) $(a^2 + b^2)/2$
(c) $(ab - a - b)/ab$
(d) $(a + b + \sqrt{a^2 + b^2})/ab$
(e) $a + b + \sqrt{a^2 + b^2}$

35. A trapezoid has area 39 square inches, height 6 inches, and one base of length of 6 inches. What is the length of the other base?

(a) 39/18 (b) 33/12 (c) 7 (d) 9 (e) 18

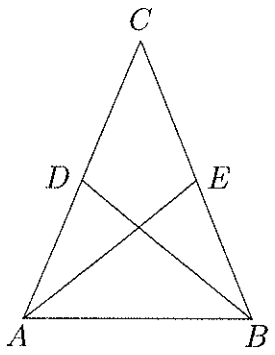
36. What is the ratio of the length of $\overline{AD}$ to the length of $\overline{CD}$?

(a) 5/8
(b) 8/5
(c) $\sqrt{89}/8$
(d) $\sqrt{89}/5$
(e) $5/\sqrt{89}$



37. Suppose that the length of $\overline{CA}$ equals the length of $\overline{CB}$, the length of $\overline{CD}$ equals the length of $\overline{CE}$, the measure of $\angle CDB$ is 100°, and the measure of $\angle DCE$ is 45°. What is the degree measure of $\angle AEC$?

(a) 80°
(b) 100°
(c) 115°
(d) 135°
(e) 145°



38. The side of a square is also the diameter of a circle. What is the ratio of the area of the square to the area of the circle?

(a) 4 (b) $4/\pi$ (c) $2/\pi$ (d) $1/\pi$ (e) π

39. In parallelogram $ABCD$, the measure of $\angle ABC$ exceeds the measure of $\angle DAB$ by $10°$. What is the degree measure of $\angle ABC$?

(a) $75°$ (b) $85°$ (c) $95°$ (d) $105°$ (e) $115°$

40. A 25 foot ladder rests on a horizontal floor and leans against a vertical wall, with the base of the ladder 20 feet from the wall. The ladder slides until the base is 24 feet from the wall. What is the location of the point that is common to both ladder positions, relative to where the wall and floor meet?

(a) 3 feet up and 7 feet out
(b) 21/11 feet up and 192/11 feet out
(c) 3 feet up and 16 feet out
(d) 74/13 feet up and 144/11 feet out
(e) 4 feet up and 144/11 feet out