

THIRTY-SIXTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry 1992

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Rhodes College, Memphis
Coordinated by Steve Gadbois

Scoring formula: $4R - W + 40$

Edited by: Larry Bouldin, Roane State
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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

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NOTE: 1993 CONTEST DATE--APRIL 6

Thirty-sixth Annual Mathematics Contest
Tennessee Mathematics Teachers Association
GEOMETRY 1992

Notation:

\overline{AB} denotes the straight line segment joining points A and B ,

$\angle AOB$ denotes the angle with vertex O determined by points A , O , and B , and

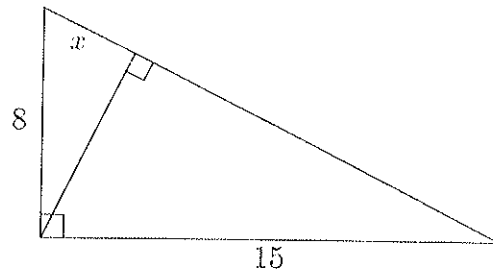
a right angle (90° angle) is denoted by \square .

Caution:

Many problems on this test are accompanied by diagrams. Though none are meant to be intentionally misleading, these diagrams may not be exactly to scale.

1. What is the value of length x ?

- (a) $64/23$
- (b) $64/17$
- (c) $64/15$
- (d) $120/17$
- (e) $136/15$



2. What is the length of the diagonal of a cube with side length 1? (The diagonal passes through the center of the cube.)

- (a) $\sqrt{2}$
- (b) $\sqrt{3}$
- (c) $\sqrt{5}$
- (d) $\sqrt[3]{2}$
- (e) $\sqrt[3]{3}$

3. The ratio of the area of one equilateral triangle to another is nine. What is the ratio of the lengths of their altitudes?

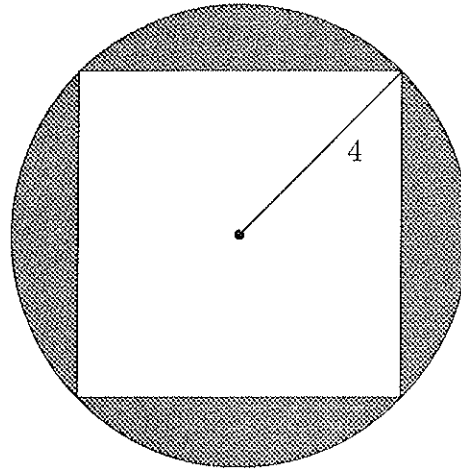
- (a) 3
- (b) 9
- (c) $\sqrt{3}/2$
- (d) $3\sqrt{3}/4$
- (e) $9\sqrt{3}/4$

4. What is the sum of the measures of the angles of a polygon of 20 sides?

- (a) 360°
- (b) 1620°
- (c) 2460°
- (d) 3240°
- (e) 3600°

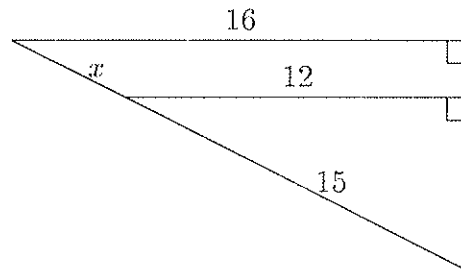
5. A square is inscribed in a circle of radius 4. What is the area of the region which is inside the circle and outside the square?

- (a) $8\pi - 2\sqrt{2}$
- (b) $8\pi - 16$
- (c) $8\pi - 32$
- (d) $16\pi - 16$
- (e) $16\pi - 32$



6. What is the value of length x in the figure?

- (a) 5
- (b) 6
- (c) 8
- (d) 10
- (e) 12



7. What is the maximum number of regions that the plane can be divided into by three straight lines?

- (a) 6
- (b) 7
- (c) 8
- (d) 9
- (e) 10

8. Suppose that one triples the diameter of a sphere. By what factor does the volume increase?

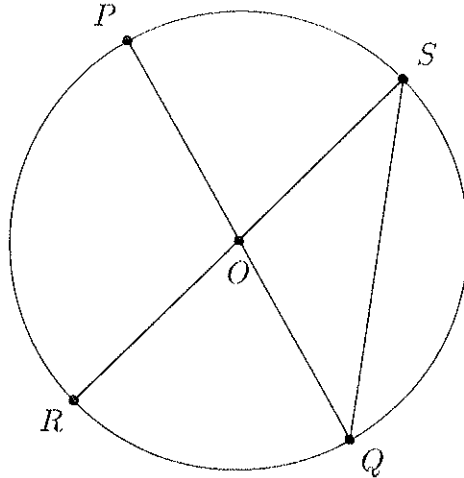
- (a) 3
- (b) 6
- (c) 8
- (d) 9
- (e) 27

9. Suppose that one triples the diameter of a sphere. By what factor does the surface area increase?

- (a) 3
- (b) 6
- (c) 8
- (d) 9
- (e) 27

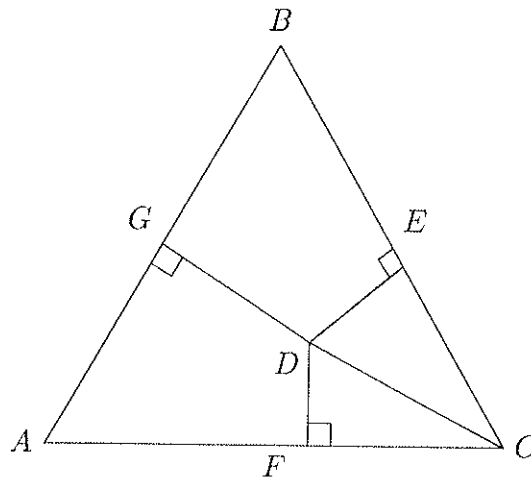
10. If \overline{PQ} and \overline{RS} are diameters of a circle and the measure of $\angle PQS$ is 20° , what is the measure of $\angle POR$?

- (a) 60°
- (b) 70°
- (c) 90°
- (d) 120°
- (e) 140°



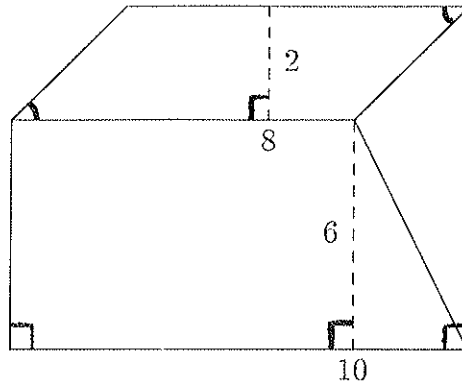
11. If the length of \overline{DC} is 6, the length of \overline{EC} is 4, the length of \overline{FC} is 5, and the length of \overline{DG} is 8, what is the length of the altitude of the equilateral triangle ABC ?

- (a) 12
- (b) 14
- (c) $\sqrt{55}$
- (d) $6 + \sqrt{20}$
- (e) $8 + \sqrt{20} + \sqrt{11}$



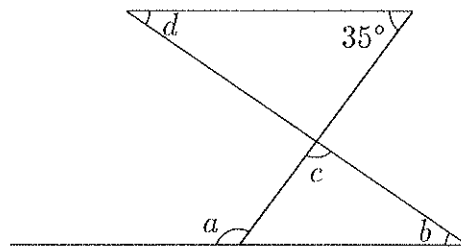
12. If the two marked angles are congruent, what is the area of the entire region in the figure?

- (a) 37
- (b) 74
- (c) 78
- (d) 82
- (e) 83



13. Which of the following gives the measure of angle a in terms of angles b , c , and d ?

- (a) $b + d$
- (b) $35^\circ + d + b$
- (c) $145^\circ + b - d$
- (d) $145^\circ + d - b$
- (e) $180^\circ + d - b$

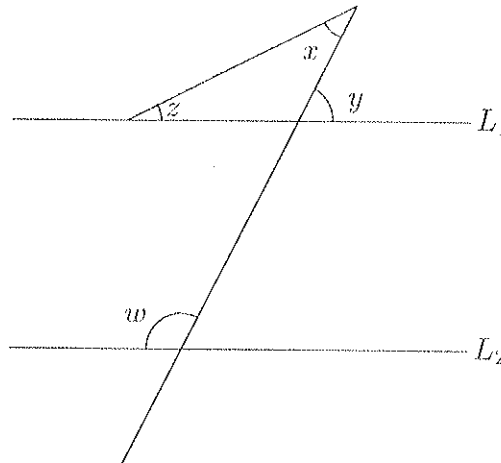


14. Suppose that ABC is a triangle and \overline{DE} is a line segment. If D lies on \overline{AC} and B is the midpoint of \overline{DE} , which of the following statements is *always* true?

- (a) $\angle BAD$ is congruent to $\angle BDC$
- (b) $\angle BAD$ is congruent to $\angle CBD$
- (c) $\angle BAD$ is congruent to $\angle CBE$
- (d) $\angle BAD$ is less than $\angle CBE$
- (e) $\angle BAD$ is greater than $\angle CBE$

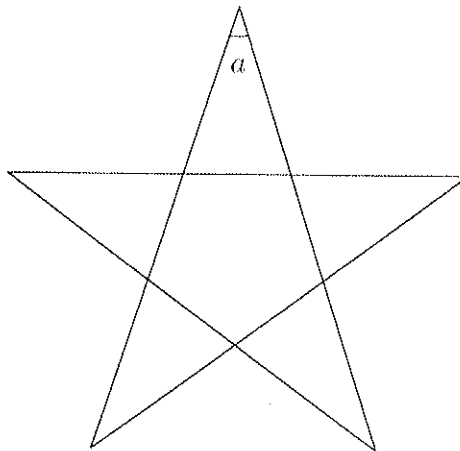
15. Suppose that lines L_1 and L_2 are parallel. Which of the following gives the measure of the angle w in terms of the angles x , y , and z ?

- (a) $180^\circ - x$
- (b) $180^\circ - z$
- (c) $180^\circ + y - z$
- (d) $180^\circ + x - z$
- (e) $180^\circ - x - z$



16. What is the measure of the angle a in the regular star shown?

- (a) 36°
- (b) 40°
- (c) 54°
- (d) 60°
- (e) 72°

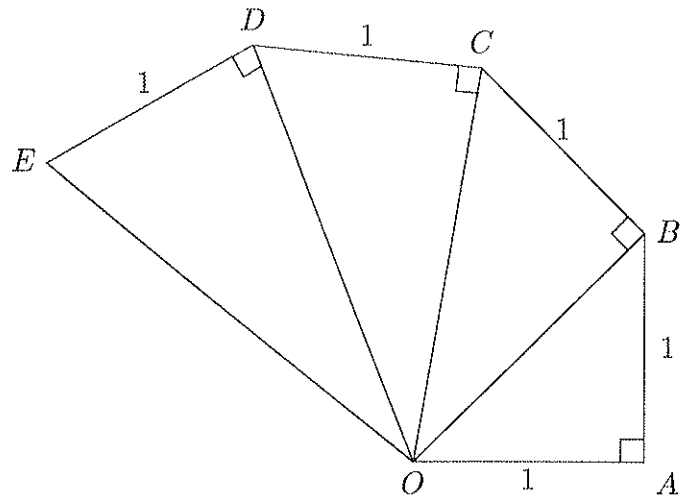


17. An annulus is a region in the plane bounded by two circles with a common center. If an annulus has an area of 4π and if the radius of one of its boundary circles is 3, what could be the radius of the other boundary circle?

- (a) $\sqrt{5}$
- (b) $\sqrt{13}$
- (c) 5 or 13
- (d) 5 or $\sqrt{13}$
- (e) $\sqrt{5}$ or $\sqrt{13}$

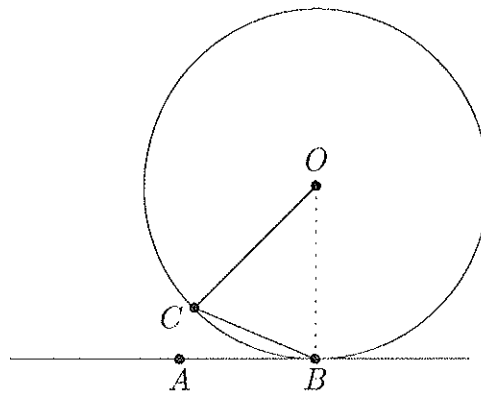
18. What is the length of \overline{OE} ?

- (a) 1
- (b) 2
- (c) $\sqrt{5}$
- (d) $\sqrt{6}$
- (e) 5



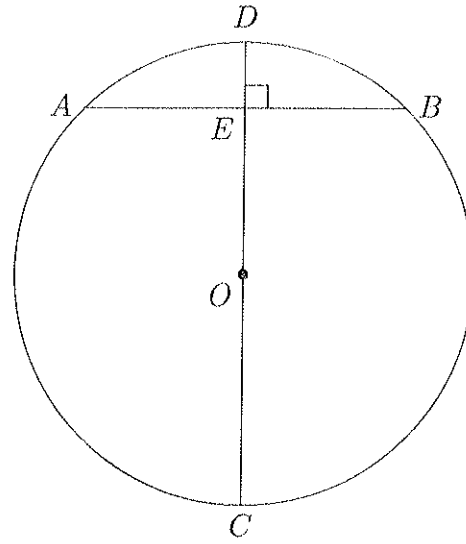
19. Suppose that a circle of radius 6 and center O rolls without slipping along the line from where C coincides with A to the position in the diagram. If the length of \overline{AB} is π , what is the radian measure of $\angle OCB$?

- (a) $\pi/6$
- (b) $\pi/2$
- (c) $2\pi/3$
- (d) $5\pi/12$
- (e) $5\pi/6$



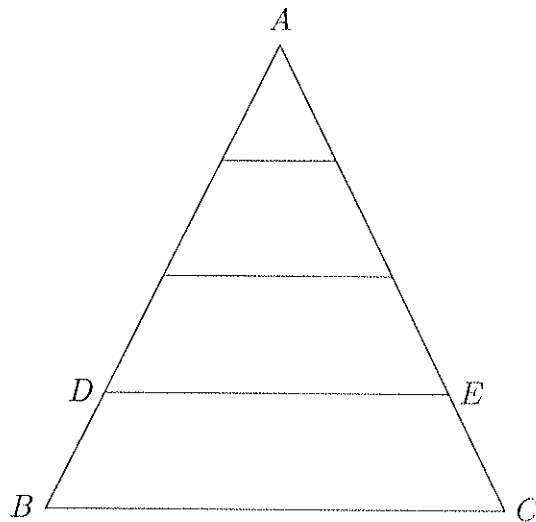
20. Suppose that O is the center of the circle, the length of \overline{AB} is 16, the length of \overline{ED} is 4, and \overline{OD} is perpendicular to \overline{AB} . What is the length of \overline{CD} ?

- (a) 4
- (b) 5
- (c) 8
- (d) 10
- (e) 20



21. Two sides of triangle ABC are each divided into four segments of equal length and the corresponding points are connected as shown in the diagram. If the area of the region $BCED$ is 10, what is the area of the triangle ABC ?

- (a) $100/7$
- (b) $160/7$
- (c) 20
- (d) 40
- (e) 160

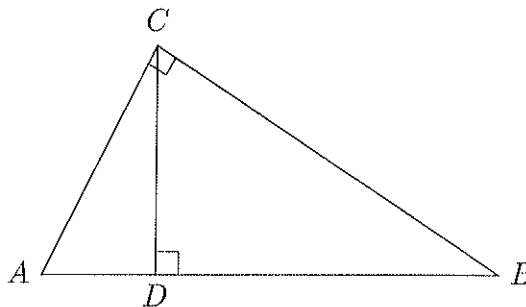


22. A regular n -gon is a polygon with n equal sides and equal interior angles. Assuming that $n > 3$, what is the degree measure of each interior angle?

- (a) $180/n$
- (b) $360/n$
- (c) $540/n$
- (d) $(n - 2)180$
- (e) $(n - 2)180/n$

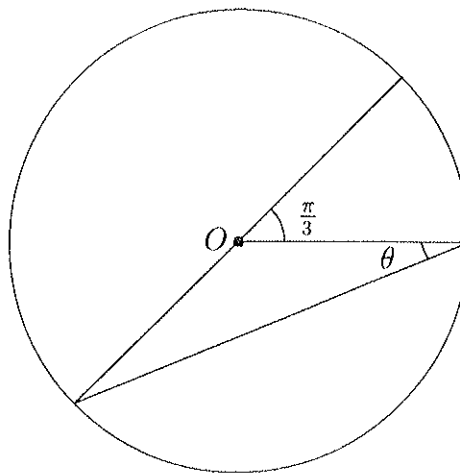
23. Suppose that \overline{AC} is perpendicular to \overline{BC} , \overline{CD} is perpendicular to \overline{AB} , the length of \overline{AD} is 2, and the length of \overline{BD} is 8. What is the length of \overline{CD} ?

- (a) 4
- (b) 5
- (c) 16
- (d) $\sqrt{20}$
- (e) $\sqrt{80}$



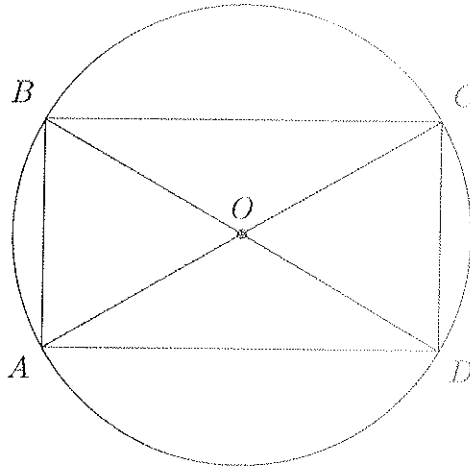
24. If O is the center of the circle shown, what is the radian measure of θ ?

- (a) $\pi/12$
- (b) $\pi/6$
- (c) $\pi/4$
- (d) $\pi/3$
- (e) $2\pi/3$



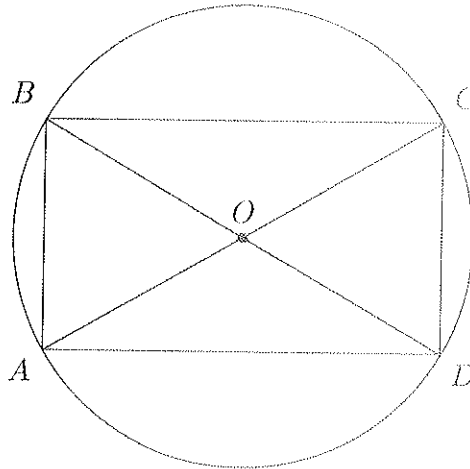
25. Suppose that \overline{AC} and \overline{BD} are diameters of a circle with center O . If the length of \overline{AB} is 4 and the length of \overline{DC} is 2, what is the length of \overline{BC} ?

- (a) 4
- (b) $\sqrt{5}$
- (c) $2\sqrt{2}$
- (d) $2\sqrt{3}$
- (e) $2\sqrt{5}$



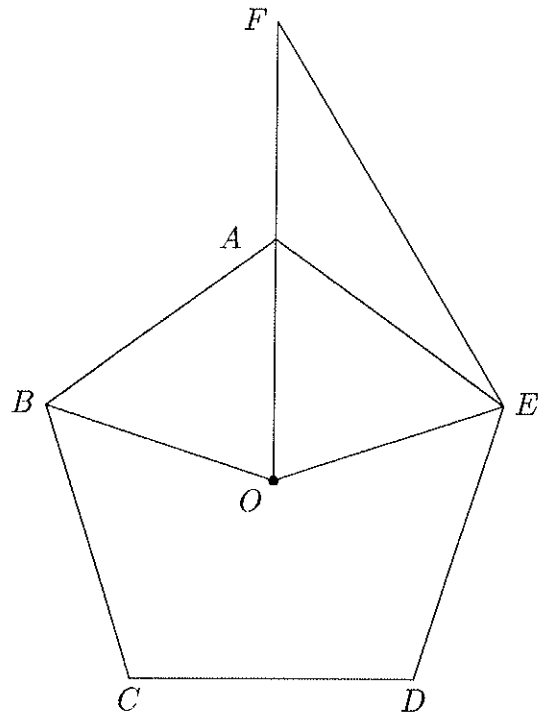
26. Suppose that \overline{AC} and \overline{BD} are diameters of a circle with center O . If the length of \overline{AB} is 4 and the length of \overline{DC} is 2, what is the measure of $\angle BCO$?

- (a) 30°
- (b) 35°
- (c) 40°
- (d) 45°
- (e) 50°



27. Suppose that $ABCDE$ is a regular pentagon with center O . If the measure of $\angle AEF$ is 25° , what is the measure of $\angle EFA$?

- (a) 21°
- (b) 29°
- (c) 31°
- (d) 54°
- (e) 126°

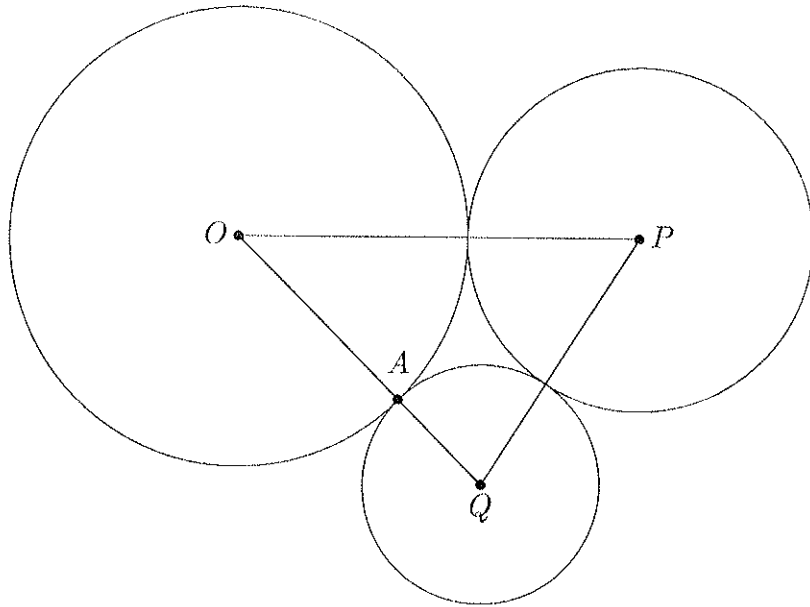


28. An icosahedron is a solid polyhedron with twenty faces, each an equilateral triangle. How many edges does an icosahedron have?

- (a) 10
- (b) 12
- (c) 20
- (d) 24
- (e) 30

29. Suppose that O , P , and Q are the centers of three mutually tangent circles as shown. If the length of \overline{OP} is 7, the length of \overline{OQ} is 5, and the length of \overline{PQ} is 5, what is the length of \overline{OA} ?

- (a) $5/2$
- (b) 3
- (c) $7/2$
- (d) 4
- (e) 8

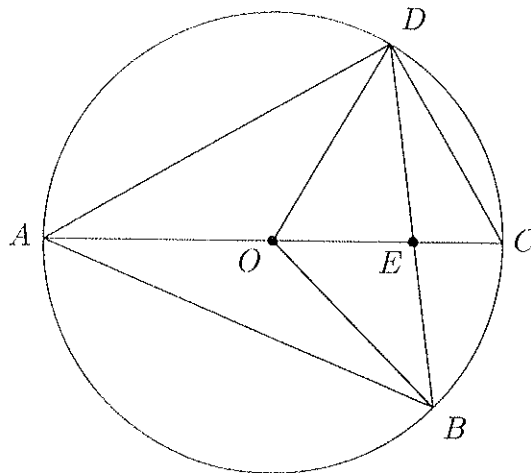


30. Suppose that one draws three circles of radius r , each centered at a different corner of an equilateral triangle ABC of side length r . What is the area of the region enclosed by the circular arcs AB , BC , and CA (including the triangle)?

- (a) r^2
- (b) $\pi r^2/2$
- (c) $(\pi - \sqrt{3})r^2/2$
- (d) $(2\pi - \sqrt{3})r^2/2$
- (e) $(2\pi + \sqrt{3})r^2/4$

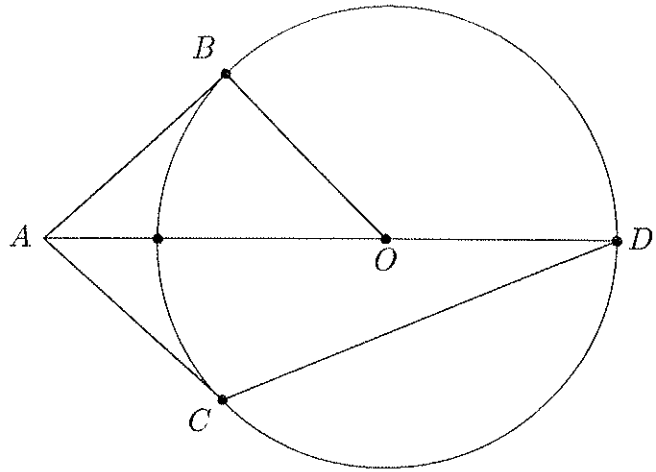
31. Suppose that \overline{AC} is a diameter of a circle centered at O , the measure of $\angle BOC$ is 50° , and the measure of $\angle AOD$ is 110° . What is the measure of $\angle BEC$?

- (a) 75°
- (b) 80°
- (c) 85°
- (d) 90°
- (e) 100°



32. Suppose that \overline{AB} and \overline{AC} are both tangent to a circle centered at O and that the measure of $\angle OAB$ is 30° . What is the measure of $\angle ADC$?

- (a) 20°
- (b) 25°
- (c) 30°
- (d) 35°
- (e) 40°



33. The circumference of a circle is 25,000 feet. If the circumference is increased by 3 feet, which of the following most closely approximates the increase in the radius of the circle?

- (a) 0.5 feet
- (b) 1.0 feet
- (c) 1.5 feet
- (d) 1.8 feet
- (e) 2.0 feet

34. What is the area of an equilateral triangle inscribed in a circle of radius 4?

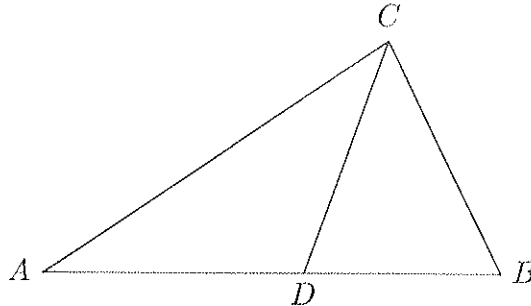
- (a) 12
- (b) 16
- (c) $12\sqrt{2}$
- (d) $12\sqrt{3}$
- (e) $16\sqrt{3}$

35. An equilateral triangle is inscribed in a circle of radius 12. Which of the following most closely approximates the area of one of the three sectors inside the circle and outside the triangle?

- (a) 74.2
- (b) 78.5
- (c) 83.6
- (d) 88.4
- (e) 92.5

36. Suppose that $\angle ACD$ and $\angle DCB$ have the same measure, the length of \overline{AC} is 18, the length of \overline{BC} is 12, the length of \overline{AD} is 9, and the length of \overline{BD} is 6. What is the length of \overline{CD} ?

- (a) 10
- (b) 12
- (c) 14
- (d) $9\sqrt{2}$
- (e) $10\sqrt{3}$



37. Two vertical poles with heights 2 feet and 8 feet are located a distance 5 feet apart. A wire stretches from the top of each of them to a point on the level ground between them. How far should this point be from the base of the 2 foot pole in order to minimize the total length of the wire?

- (a) 1 foot
- (b) 2 feet
- (c) $5/2$ feet
- (d) 3 feet
- (e) 4 feet

38. From any point inside an equilateral triangle with side length s , construct three segments perpendicular to the three sides. What is the sum of the lengths of these segments?

- (a) s
- (b) $2s/3$
- (c) $3s/2$
- (d) $\sqrt{2}s$
- (e) $\sqrt{3}s/2$

39. A person who is 1.5 meters tall stands 12 meters from a light pole that is 6 meters high. What is the length of the shadow she casts due to the light on the top of the pole?

- (a) 4 meters
- (b) 6 meters
- (c) 10 meters
- (d) 12 meters
- (e) 16 meters

40. A circular arch having width 24 feet and height 9 feet is to be constructed. What is the radius of the circle of which the arch is an arc?

- (a) 10
- (b) 12.5
- (c) 13.5
- (d) 14
- (e) 16

