TWENTY-NINTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY 1985

Scoring formula: 4R - W + 40

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on the the next. The working time for the entire test is 80 minutes.

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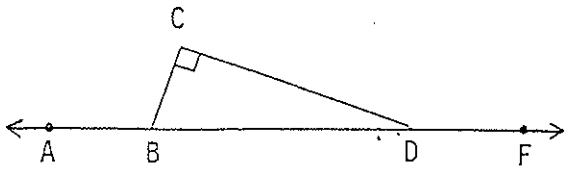
1. If in the figure below, m $\angle ABC = 110^\circ$, and $\angle C$ is a right angle, then m $\angle CDF$ =



a. 110°

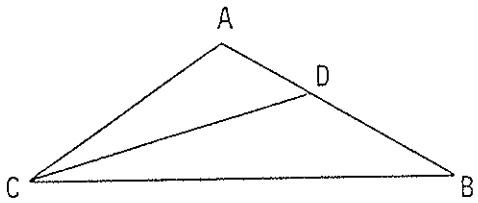
b. 120°

c. 140°

d. 160°

e. 170°

2. In $\triangle ABC$, AC = AB, m $\angle B = 50^\circ$, and $\overline{CD}$ bisects $\angle ACB$. Find m $\angle ACD$.



a. 20°

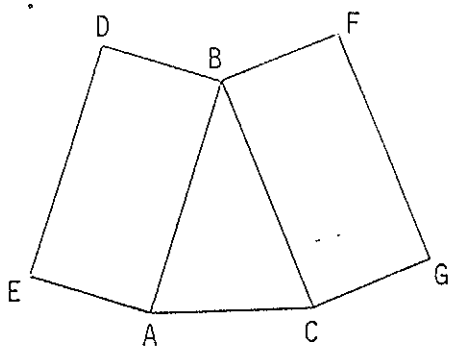
b. 25°

c. 35°

d. 40°

e. 50°

3. In the figure below, AB = CB, quadrilaterals ABDE and CBFG are both rectangles, and m $\angle BAC = 70^\circ$. Find m $\angle DBF$.



a. 105°

b. 110°

c. 120°

d. 130°

e. 140°

4. The area enclosed by the curve $(x - 2)^2 + (y + 3)^2 = 16$ is

a. 16π

b. 32π

c. 256π

d. 8π

e. 96π

5. If the areas of the three noncongruent faces of a rectangular box are x, y and z, then xyz is equal to

   a. the square of the volume of the box.

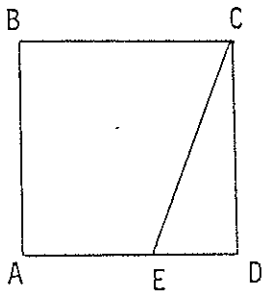
   b. the cube of the volume of the box.

   c. twice the volume of the box.

   d. three times the volume of the box.

   e. four times the volume of the box.

6. In the figure below, quadrilateral ABCD is a square, CE = 7, and ED = 3. The area of square ABCD is



   a. 38

   b. 39

   c. 40

   d. 41

   e. 58

7. A rhombus is formed by two radii and two chords of a circle of radius 2. The area of the rhombus is

   a. $\sqrt{3}$

   b. $2\sqrt{3}$

   c. $3\sqrt{3}$

   d. $4\sqrt{3}$

   e. 3

8. Which of the following angles cannot be constructed with just a compass and straight edge?

   a. an angle of measure 15°

   b. an angle of measure $22\frac{1}{2}^\circ$

   c. an angle of measure 40°

   d. an angle of measure 60°

   e. an angle of measure 105°

9. Two points on a circle determine a minor arc and a major arc. If the measure of the major arc is 40° more than 3 times the measure of the minor arc, find the measure of the major arc.

   a. 120°

   b. 240°

   c. 260°

   d. 280°

   e. 320°

10. In the figure below, the smaller circle is tangent to the larger circle, A is the center of the larger circle, and the area of the smaller circle is 3. Find the area of the larger circle.

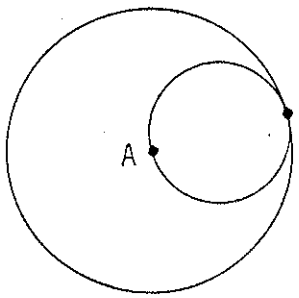
   a. 8

   b. 12

   c. 15

   d. 6π

   e. 9π



11. A rectangular piece of paper when folded in half forms a square with perimeter n. What is the area of the rectangle?

   a. $2n$

   b. n

   c. n^2

   d. $\frac{n^2}{2}$

   e. $\frac{n^2}{8}$

12. In the figure below O is the center of the circle, $m \angle BOC = 60^\circ$, and $AB = 6$. Find AC.

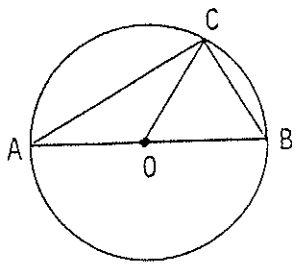
   a. $3\sqrt{3}$

   b. $4\sqrt{2}$

   c. 5

   d. 6

   e. $6\sqrt{3}$



13. In the rectangle ABCD below, if AD = 4, AB = 5, and AE = EF = FC, then the area of $\triangle BEF$ is

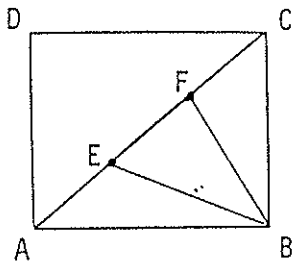
a. $\frac{5}{2}$

b. 3.

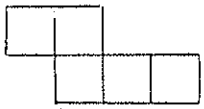
c. $\frac{10}{3}$

d. $\frac{15}{4}$

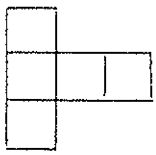
e. 4

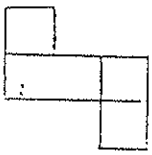


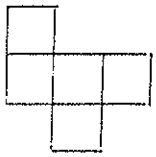
14. Which of the following patterns of squares can not be folded into a box with an open top?

a. 

b.

c. 

d. 

e. 

15. Find the area of the trapezoid ABCD given AB = 14, DC = 6, $\angle EDC$ is a right angle and the area of $\triangle EDC$ is 30.

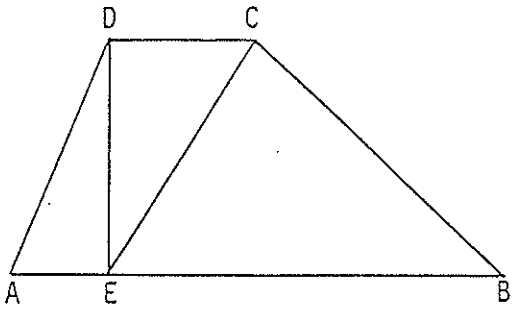
a. 75

b. 80

c. 90

d. 100

e. 120



16. In the figure, $\triangle ABC$ is equilateral, m $\angle DAB = 30^\circ$, and AE = AD. Find m $\angle EDC$.

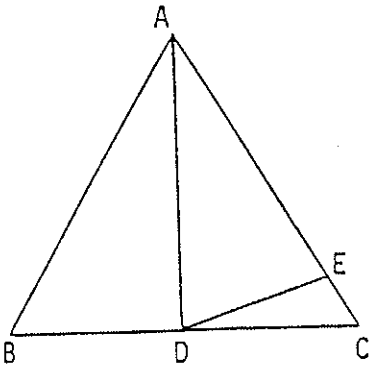
a. 15°

b. 20°

c. $12\frac{1}{2}^\circ$

d. 30°

e. $7\frac{1}{2}^\circ$



17. In the figure below, AB = 8, m $\angle$ A = 30°, and m $\angle$ C = 45°. Find BC.

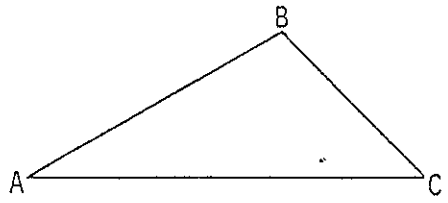
a. 4

b. $4\sqrt{2}$

c. $4\sqrt{3}$

d. 5

e. $3\sqrt{5}$



18. The area of square ABCD is 121, E is the center of the square, $\angle$GEF is a right angle and FC = 3. Find the area of quadrilateral GEFC.

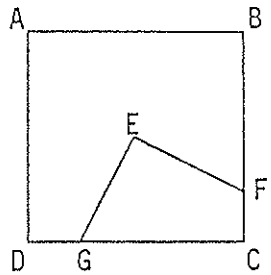
a. 29

b. $29\frac{5}{8}$

c. 30

d. $30\frac{1}{4}$

e. 32



19. Two hundred sixteen unit cubes are fastened together to form a cube with each edge of length 6. This cube is painted and then separated into the original cubes. How many of these cubes have at least one painted face?

a. 140

b. 144

c. 150

d. 152

e. 160

20. In the figure below, $\overline{DF} \parallel \overline{AC}$, $\overline{DC} \parallel \overline{EF}$, BE = 4 and DE = 6. Find DA.

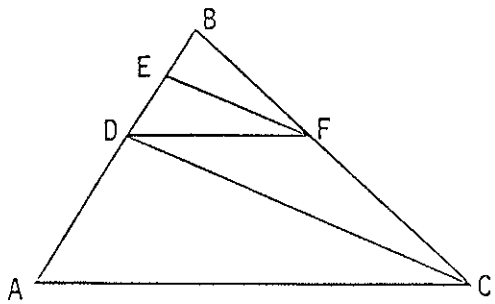
a. 6

b. 8

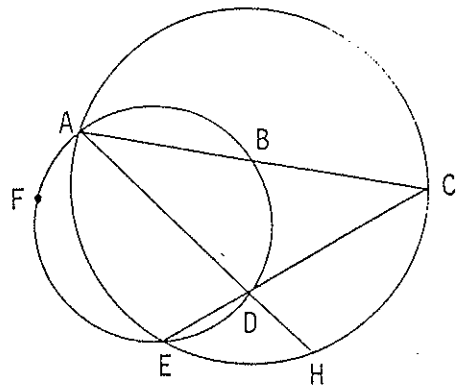
c. 10

d. 12

e. 15



21. Two noncongruent circles overlap as shown. If $m\overset{\frown}{AFE} = 170^\circ$ and $m\angle C = 40^\circ$, then $m\overset{\frown}{CH} =$



a. 80°

b. 85°

c. 90°

d. 130°

e. 140°

22. If each angle of a regular polygon has measure 140°, how many sides does the polygon have?

a. 6

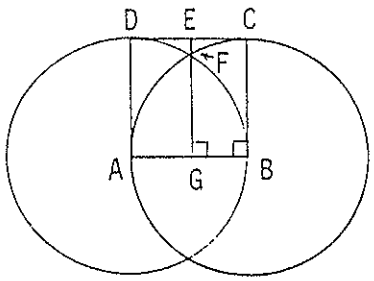
b. 7

c. 8

d. 9

e. 10

23. In the figure below, A and B are centers of circles, quadrilateral ABCD is a square, and $\overline{EG} \perp \overline{AB}$. Find FE.



a. $\frac{1}{2}(\sqrt{5} - 2)AB$

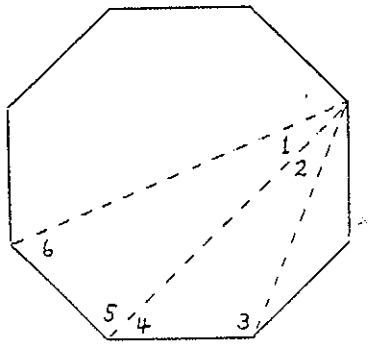
b. $\frac{1}{2}\sqrt{3}\,AB$

c. $\frac{1}{2}(3 - \sqrt{3})AB$

d. $\frac{1}{2}(\sqrt{3} - 1)AB$

e. $\frac{1}{2}(2 - \sqrt{3})AB$

24. Consider the six angles as indicated in the regular octagon below. Which one of the following statements is incorrect?



a. $m\angle 3 = m\angle 4 + m\angle 6$

b. $m\angle 3 = m\angle 1 + m\angle 5$

c. $m\angle 4 = m\angle 1 + m\angle 2$

d. $m\angle 5 = m\angle 1 + m\angle 4$

e. $m\angle 3 + m\angle 4 = m\angle 5 + m\angle 6$

25. If two poles 80 feet and 20 feet tall are 100 feet apart, then the height of the intersection of the line joining the top of each pole to the foot of the opposite pole is

a. 12 feet

b. 16 feet

c. 24 feet

d. 32 feet

e. 40 feet

26. If an arc of 60° on a first circle has the same length as an arc of 90° on a second circle, then the ratio of the area of the first circle to the area of the second circle is

a. $\frac{2}{3}$

b. $\frac{4}{9}$

c. $\frac{3}{2}$

d. 2

e. $\frac{9}{4}$

27. A sphere is cut in half as shown. If the area of the cross-section is 36π, what is the volume of the hemisphere?

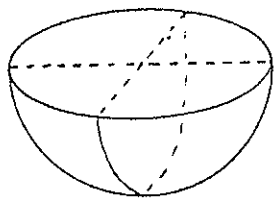
a. 144π

b. 288π

c. 36π

d. 72π

e. 108π



28. A circle with center O has perpendicular diameters, $\overline{AB}$ and $\overline{CD}$. If $\overline{AM}$ is a chord intersecting $\overline{CD}$ at E, then AE • AM is

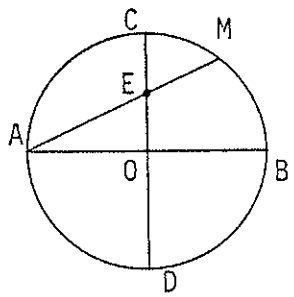
a. AO • OB

b. AO • AB

c. CE • CD

d. CE • ED

e. CO • OE



29. In $\triangle ABC$, $\angle B$ is an obtuse angle, AB = 6, and CB = 8. Which of the following is a true statement?

a. $8 < AC < 10$

b. $8 < AC < 12$

c. $10 < AC < 12$

d. $10 < AC < 14$

e. $12 < AC < 14$

30. In the figure $\overline{AB}$ is a diameter of circle O and $\overleftrightarrow{CB}$ is a tangent at B. If the radius of the circle is 6 and AG = 8, then CB is

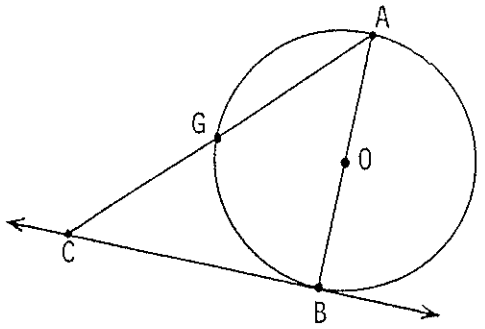
a. $4\sqrt{5}$

b. $6\sqrt{2}$

c. $6\sqrt{5}$

d. $2\sqrt{21}$

e. $8\sqrt{2}$



31. How many squares are on the surface of the cube?

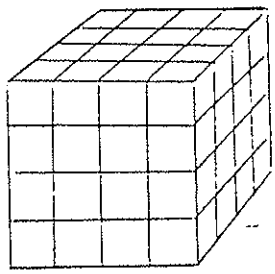
a. 192

b. 48

c. 64

d. 90

e. 180



32. If $\triangle ABC$ is formed by three tangents to circle O as drawn below and $m\angle BCA = 40°$, then $m\angle AOB$ is

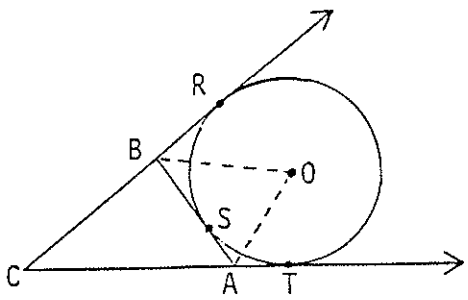
a. 45°

b. 50°

c. 55°

d. 60°

e. 70°



33. A line intersects two sides of an equilateral triangle and is parallel to the third side. If this line divides the triangular region into a trapezoid and a smaller triangle having equal perimeters, then the ratio of the area of the smaller triangle to the area of the trapezoid is

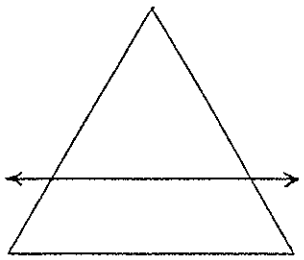
a. $\frac{9}{7}$

b. $\frac{\sqrt{3}}{2}$

c. $\frac{7}{4}$

d. $\frac{3}{2}$

e. $\frac{16}{9}$



34. The semicircle below is folded up to form a cone as shown. What is the volume of the cone?

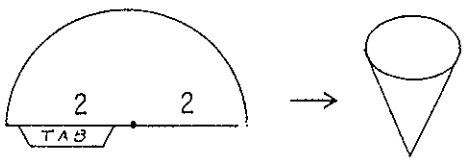
a. $\frac{\pi\sqrt{3}}{3}$

b. $\frac{\pi}{3}$

c. $\frac{\pi}{4}$

d. $\pi\sqrt{3}$

e. Cannot be determined



35. If $\triangle ABC$ is equilateral, point P is interior to $\angle BAC$ but exterior to $\triangle ABC$, X = distance from P to $\overrightarrow{AB}$, Y = distance from P to $\overleftrightarrow{BC}$, and Z = distance from P to $\overrightarrow{AC}$, then the altitude of $\triangle ABC$ is

a. X + Y + Z

b. -X + Y + Z

c. X - Y + Z

d. X + Y - Z

e. -X + Y - Z

36. An equilateral triangle is inside a circle. The sides of the triangle are extended to the circle and labeled as shown. Which one of the following is correct?

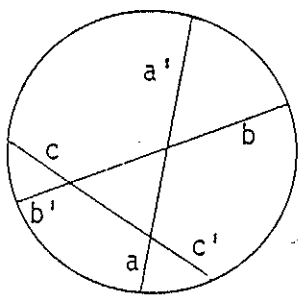
a. a + b + c = a' + b' + c'

b. -a + b + c = -a' + b' + c'

c. -a + b - c = a' - b' + c'

d. -a + b + c -a' + b' + c'

e. a + b + c a' + b' + c'



37. The base of isosceles $\triangle ABC$ is $4\sqrt{2}$. The medians to the legs intersect each other at right angles. The area of $\triangle ABC$ is

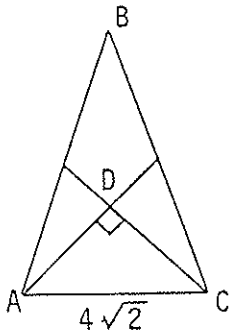
a. 8

b. 24

c. 32

d. 40

e. 48



38. A cylindrical gasoline tank is resting on its side. What portion of the tank is filled when the gasoline level is one-fourth of the way up from the bottom?

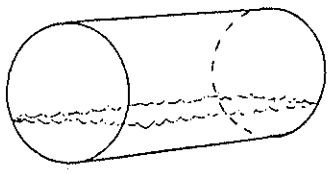
a. $\frac{1}{4}$

b. $\frac{1}{16}$

c. $\frac{\pi - 2}{6}$

d. $\frac{3\pi - 2\sqrt{5}}{12\pi}$

e. $\frac{4\pi - 3\sqrt{3}}{12\pi}$



39. If in the circle $\overset{\frown}{AC} \cong \overset{\frown}{CD}$, AB = 3, BE = 2, ED = x, and $\overline{CE} \perp \overline{BD}$, then

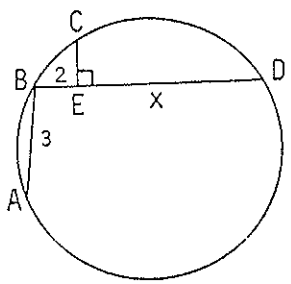
a. x = 4

b. x = 5

c. x = 6

d. $3 < x < 4$

e. $4 < x < 5$

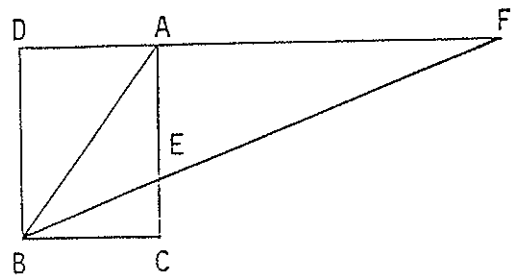


40. Quadrilateral ACBD is a rectangle. If from B a line is drawn intersecting $\overline{AC}$ at E and $\overline{DA}$ is extended to F such that m $\angle EBC$ = 1/3 m $\angle ABC$, then

a. BF = 2 BD

b. BF = 2 BA

c. EF = 2 BA

d. EF = 2 BD

e. AF = 2 BD