

TWENTY-SIXTH ANNUAL MATHEMATICS CONTEST  
Sponsored by  
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY TEST 1982

Edited by: The University of  
Tennessee at Martin

Scoring Formula:  $4R - W + 40$

This test was prepared from a list of Geometry questions submitted by Memphis State University.

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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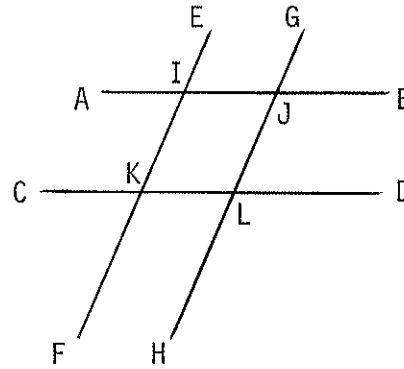
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1. An indirect proof is basically carried out by
  - a) assuming the hypothesis to be true and then by logical reasoning led to a known accepted conclusion.
  - b) assuming the hypothesis to be true and thereby by faulty reasoning led to a known faulty conclusion.
  - c) assuming the suspected conclusion to be false and then by logical reasoning led to a contradiction of some accepted fact.
  - d) assuming the suspected conclusion to be true and then by logical reasoning being led to a part or parts of the conclusion.
  - e) assuming the hypothesis to be false and then by logical reasoning being led to the suspected conclusion.
  
2. If  $a$  and  $b$  are the coordinates of a point  $P(a, b)$ , which of the following statements is/are true?
  - a) If the product of  $a$  and  $b$  is negative,  $P$  lies in quadrant II or quadrant IV.
  - b) If the product of  $a$  and  $b$  is positive,  $P$  lies in quadrant I or quadrant III.
  - c) If the product of  $a$  and  $b$  is zero,  $a$  and/or  $b$  is zero.
  - d)  $a$  is the abscissa and  $b$  is the ordinate.
  - e) All of the above.
  
3. The sum of the interior angles of a decagon is
  - a)  $1440^\circ$
  - b)  $1080^\circ$
  - c)  $360^\circ$
  - d)  $540^\circ$
  - e)  $720^\circ$

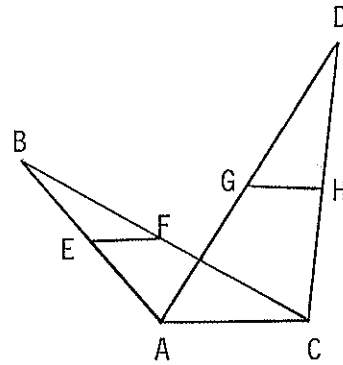
4. In the figure  $\overline{AB}$  is parallel to  $\overline{CD}$  and  $\overline{EF}$  is parallel to  $\overline{GH}$ . Angles  $\angle AJG$  and  $\angle DKF$  are

- a) complementary
- b) supplementary
- c) alternate interior angles
- d) equal in measure
- e) none of these



5. In the figure the non-collinear points E, F, G and H are midpoints of  $\overline{AB}$ ,  $\overline{CB}$ ,  $\overline{AD}$ , and  $\overline{DC}$ , respectively. Which of the following is true?

- a)  $\overline{EF}$  and  $\overline{GH}$  are neither parallel nor congruent.
- b)  $\overline{EF}$  and  $\overline{GH}$  are both parallel and congruent.
- c)  $\overline{EF}$  and  $\overline{GH}$  are parallel but not congruent.
- d)  $\overline{EF}$  and  $\overline{GH}$  are congruent but not parallel.
- e) All of the above are false.



6. If the vertices of a quadrilateral lie on a circle, the perpendicular bisectors of all the sides

- a) intersect at a point
- b) are equal
- c) are perpendicular to each other
- d) bisect the opposite sides also
- e) none of these

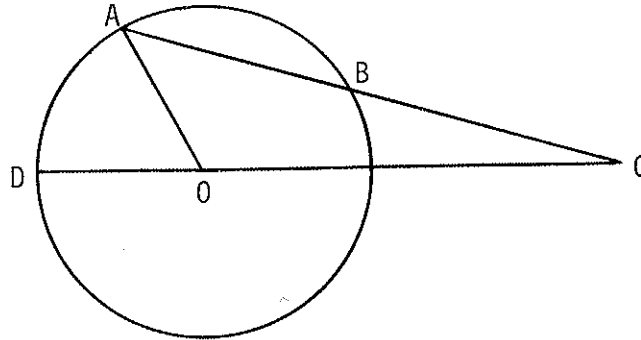
7. A circle can always be circumscribed about any

- a) parallelogram
- b) rhombus
- c) quadrilateral
- d) isosceles trapezoid
- e) polygon

8. Given the statement: "If Mary can vote, then Mary is over 18." Which of the following are then necessarily true?
- If Mary cannot vote, she is not over 18.
  - If Mary is over 18, she can vote.
  - Either Mary can vote, or she is not over 18.
  - Either Mary can vote, or she is over 18.
  - If Mary is not over 18, then she cannot vote.
9. If two triangles of equal area  $A$  have the same base  $a$ , then the locus of the third vertex of the two triangles is
- a line perpendicular to  $a$  at its extremity.
  - a circle with  $a$  as diameter
  - a line parallel to  $a$  at a distance  $1/(2a)$  from it
  - two lines parallel to  $a$  at a distance of  $2A/a$  from it, one on either side
  - indeterminant
10. Given: "If  $p$ , then  $q$ ," which of the following is not represented by "If not  $q$ , then not  $p$ ?"
- The inverse
  - The contrapositive
  - The inverse of the converse
  - The converse of the inverse
  - none of these
11. Two arcs can be made to coincide if they have
- equal central angles
  - the same number of arc degrees
  - the same number of inches in length
  - equal central angles in the same circle
  - equal chords

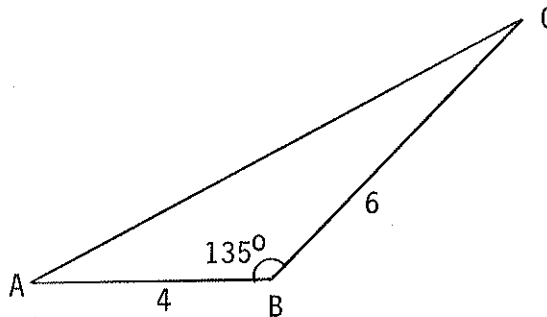
12. In the circle below,  $\overline{OA}$  is a radius,  $\overline{BC} = \overline{OA}$ ,  $m(\angle AOD)$  equals  $x$  and  $m(\angle ACO) = y$ . Which of the following is true?

- a)  $x = 2y$
- b)  $x = 5y/2$
- c)  $x = 3y$
- d)  $x = y$
- e)  $x = 4y$



13. In triangle ABC,  $AB = 4$ ,  $BC = 6$ , and  $\angle ABC = 135^\circ$ . The area of the triangle is

- a) 12
- b)  $6\sqrt{3}$
- c)  $6\sqrt{2}$
- d)  $6\sqrt{2} + 9$
- e) none of these



14. In any triangle ABC, if  $\angle A + \angle B = \angle C$ , which one of the following statements is not necessarily true?

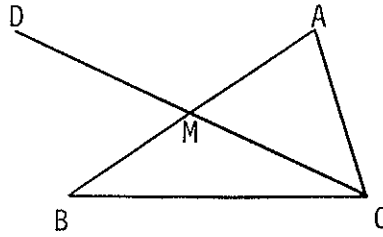
- a) ABC is a right triangle.
- b) ABC is an isosceles triangle.
- c) ABC could be inscribed in a semicircle.
- d) The center of the inscribed circle is on the bisector of angle C.
- e) none of the above

15. The sides of a triangle are 5, 6, and 9 units, respectively. The area of the triangle in square units is

- a) 15
- b)  $10\sqrt{2}$
- c) 27
- d)  $10\sqrt{462}$
- e) impossible to determine

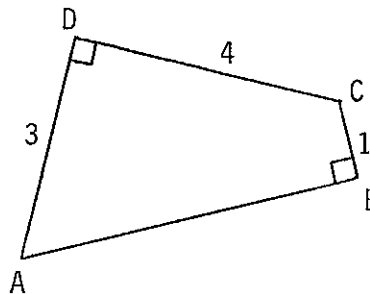
16. In triangle ABC, M is the midpoint of  $\overline{AB}$ . Extend  $\overline{CM}$  to D so that  $\overline{CM} = \overline{MD}$ . Then ACBD is always

- a) a rhombus
- b) a rectangle
- c) a pyramid
- d) a parallelogram
- e) a quadrilateral that can be inscribed in a circle



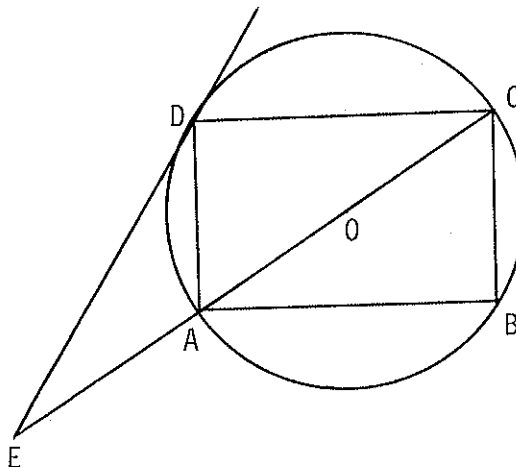
17. What is the area of ABCD if angles B and D are right angles?

- a) 8
- b)  $6 + \sqrt{6}$
- c) 8.5
- d) 17
- e)  $12 + 2\sqrt{6}$



18. ABCD is a rectangle inscribed in circle O.  $AC = 2 \cdot BC$  and  $\overline{ED}$  is tangent to circle O at D. The measure of angle DEA is

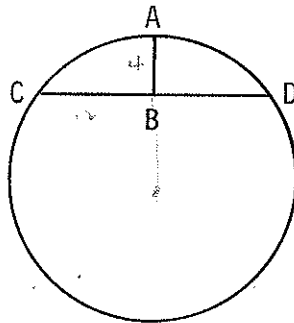
- a)  $30^\circ$
- b)  $20^\circ$
- c)  $60^\circ$
- d)  $45^\circ$
- e) none of these



19. To find the center of the circle inscribed in a triangle, you would
- construct the medians of the triangle
  - construct the altitudes of the triangle
  - construct the perpendicular bisectors of the sides
  - construct the exterior angle bisectors
  - construct the angle bisectors

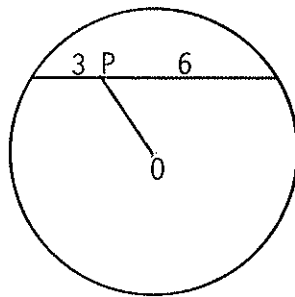
20. In the figure,  $\overline{AB}$  is 4 and  $\overline{AB}$  is perpendicular to  $\overline{CD}$  at the midpoint of  $\overline{CD}$ . If  $\overline{CD}$  is 24, find the radius of the circle.

- $4\sqrt{10}$
- 20
- 16
- 24
- $4\sqrt{3}$



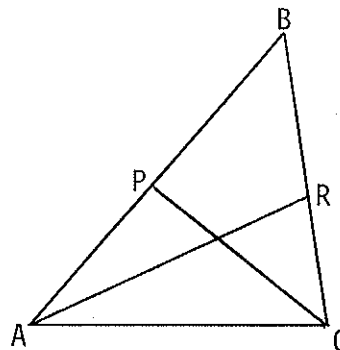
21. In circle O, a point P divides a chord into segments 3 inches and 6 inches long. If the diameter of the circle is 11 inches, what is the distance from P to the center O?

- 3.5 inches
- 3.2 inches
- 3.8 inches
- 3.0 inches
- 2.8 inches



22. In triangle ABC,  $\angle A = 50^\circ$ ,  $\angle C = 80^\circ$ .  $\overline{CP}$  bisects  $\angle C$  and  $\overline{AR}$  bisects  $\angle A$ . What is the measure of  $\angle ARC$ ?

- $105^\circ$
- $75^\circ$
- $65^\circ$
- $40^\circ$
- $25^\circ$



23. A rectangular room is 12 ft. by 16 ft. with height 8 ft. Find the distance from one upper corner to the diagonally opposite lower corner. The distance in feet is

- a) 20
- b)  $4\sqrt{13}$
- c)  $4\sqrt{29}$
- d)  $8\sqrt{5}$
- e) none of these

24. The difference between the base and altitude of a rectangle is 91 and a diagonal is 221. The dimensions of the rectangle are

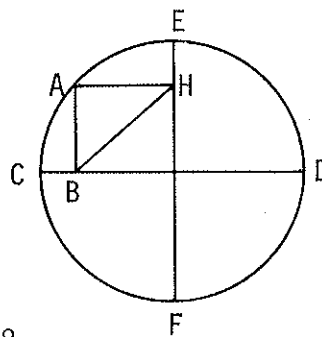
- a) 52 x 195
- b) 200 x 109
- c) 195 x 104
- d) 60 x 151
- e) cannot be found

25. Given a square whose side has length  $a$ , find the volume of a right circular cylinder whose lateral area is equal to the area of the square and whose altitude is equal to the length of a side of the square. The volume in cubic units is

- a)  $\frac{a^2}{2\pi}$
- b)  $4\pi a^3$
- c)  $\frac{a^3}{4\pi}$
- d)  $\frac{a^3}{12\pi}$
- e) none of these

26. A circle of radius 8 inches has diameters,  $\overline{CD}$  and  $\overline{EF}$  that are perpendicular to each other.  $\overline{AH}$  is parallel to  $\overline{CD}$  and  $\overline{AB}$  is perpendicular to  $\overline{CD}$ . The length of  $\overline{BH}$  is

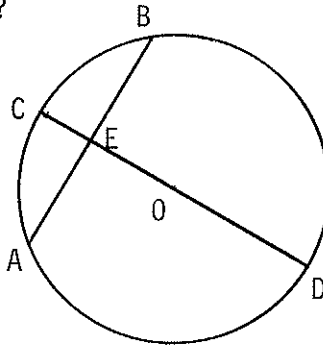
- a)  $\sqrt{89}$
- b) 8
- c) 7
- d)  $\sqrt{39}$
- e) 6





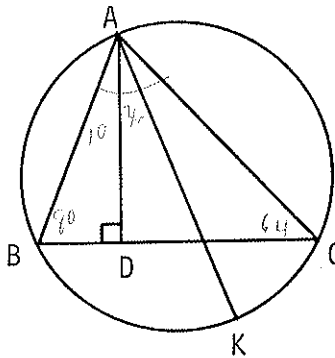
27. In circle  $O$  chord  $\overline{AB}$  is perpendicular to diameter  $\overline{CD}$  at  $E$ . Which of the following is not necessarily true?

- a)  $OA = OB$
- b)  $AE = EB$
- c)  $AD = DB$
- d)  $CE = EO$
- e)  $\widehat{AC} = \widehat{CB}$



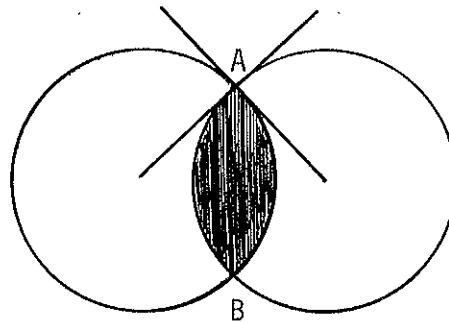
28. Let  $\triangle ABC$  have vertices on a circle. Let  $\overline{AD}$  be an altitude of the triangle and let  $\overline{AK}$  be a diameter of the circle. If  $m(\angle ABC) = 80^\circ$  and  $m(\angle BCA) = 64^\circ$ , then the  $m(\angle DAK) =$

- a)  $10^\circ$
- b)  $12^\circ$
- c)  $16^\circ$
- d)  $18^\circ$
- e)  $26^\circ$



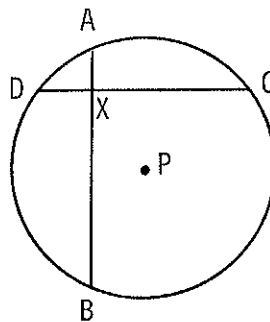
29. Two circles, each of which has a radius of 2, intersect as shown. If the tangents drawn to the two circles at point  $A$  are perpendicular, the area of the shaded region in square units is

- a)  $4 - \pi$
- b)  $2\pi - 4$
- c)  $2\pi$
- d)  $4\pi - 2$
- e) none of these



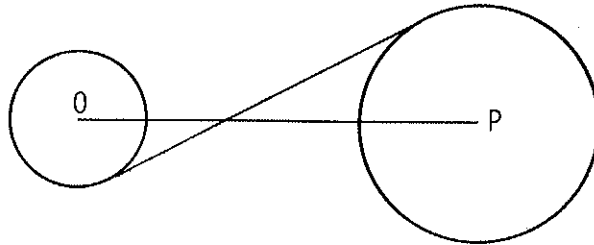
30. In circle  $P$ , suppose  $\overline{AB}$  is perpendicular to  $\overline{CD}$  at  $X$ .  $AX = 3$  cm,  $BX = 8$  cm,  $CX = 6$  cm,  $DX = 4$  cm.

- a)  $PC = \frac{5\sqrt{2}}{2}$  cm
- b)  $PC = \frac{3\sqrt{3}}{2}$  cm
- c)  $PC = 5\sqrt{2}$  cm
- d)  $PC = 3\sqrt{3}$  cm
- e) none of the above



31. The centers of two circles, O and P, are 16 inches apart. The larger circle has a radius of 5 inches, and the smaller circle a radius of 3 inches. The length of the common internal tangent is

- a)  $10\sqrt{2}$  inches
- b)  $6\sqrt{2}$  inches
- c)  $8\sqrt{5}$  inches
- d)  $8\sqrt{3}$  inches
- e) none of the above

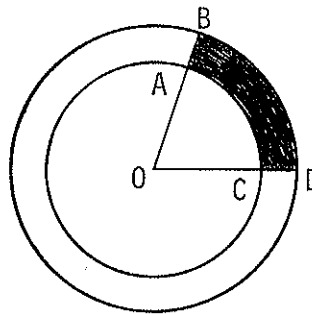


32. Which of the following is always a convex plane figure?

- a) an angle
- b) the interior of a polygon
- c) the union of a polygon with its interior
- d) the union of a polygon with its exterior
- e) a line segment.

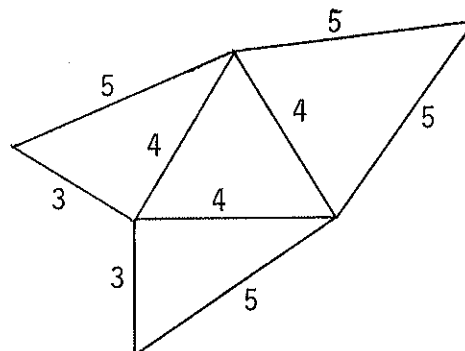
33. Two circles have centers at O. One has a radius  $OC = 4$  feet. The other has a radius  $OB = 6.5$  feet.  $\angle BOD = 72^\circ$ . What will it cost to paint the area  $ABDC$  if the quoted price is \$3.60 per square yard? Use  $\pi = 22/7$ .

- a) \$6.70
- b) \$6.80
- c) \$6.60
- d) \$6.20
- e) \$7.00



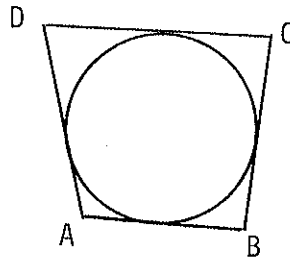
34. This figure gives the faces of a tetrahedron. Its volume in cubic units is

- a)  $4\sqrt{3}$
- b)  $8\sqrt{3}$
- c)  $12\sqrt{3}$
- d)  $4\sqrt{7}$
- e) none of the above



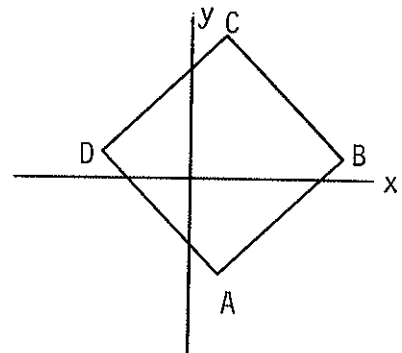
35. In the figure, ABCD is a circumscribed quadrilateral with  $CD = 6$  cm,  $BC = 3.5$  cm and  $AB = 3.4$  cm. Then AD is equal to

- a) 5.9 cm
- b) 6.0 cm
- c) 6.55 cm
- d) 6.9 cm
- e) 7.0 cm



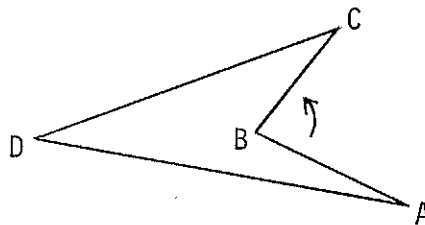
36. Given the coordinates of A, B, C, and D, ABCD is a square, you could determine the coordinates of D by

- a) using the distance formula
- b) solving simultaneously the equations of lines AD and CD
- c) using the midpoint formula for a segment
- d) a or b
- e) a, b, or c



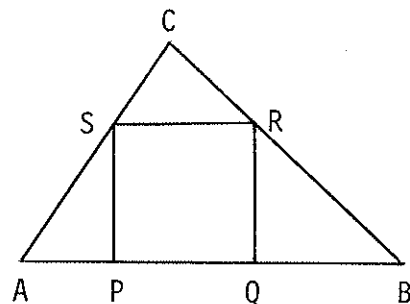
37. The tip on an arrow has the shape as shown. If the  $\angle ABC$  marked by the curved arrow is an acute angle, then the sum of the interior angles of the quadrilateral ABCD

- a) is less than  $180^\circ$
- b) is less than  $360^\circ$
- c) is exactly  $360^\circ$
- d) is more than  $360^\circ$
- e) none of these



38. Square PQRS is inscribed in the acute triangle ABC. The area of the square is

- a)  $1/2$  the area of  $\triangle ABC$
- b) greater than  $1/2$  the area of  $\triangle ABC$
- c) less than  $1/2$  the area of  $\triangle ABC$
- d) a or b
- e) none of the above



39. Three circles of equal radii are inscribed in a larger circle. The area of the shaded region in the center is 1 square unit. The area of one of the smaller circles is

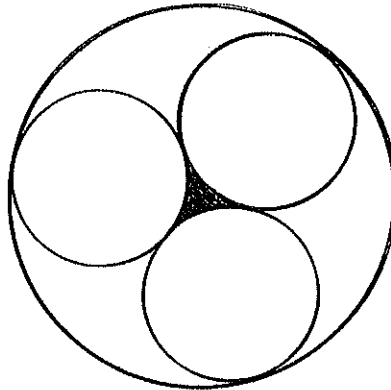
a)  $\frac{2\pi}{2\sqrt{3} - \pi}$

b)  $\frac{2\pi}{4\sqrt{3} - \pi}$

c)  $\frac{4\pi}{2\pi - \sqrt{3}}$

d)  $\frac{4\pi}{2\sqrt{3} - \pi}$

e) none of these



40. If in the figure, C is a point on the semicircle with  $\overline{AB}$  as its diameter, if  $\overline{AM}$  is perpendicular to  $\overline{MN}$ ,  $\overline{BN}$  is perpendicular to  $\overline{MN}$ , and  $\overline{CD}$  is perpendicular to  $\overline{AB}$ , then the FALSE statement is

a)  $CD = CM = CN$

b)  $(CD)^2 = (AM)(BN)$

c)  $(CD)^2 = (AD)(DB)$

d)  $\angle BCN = 1/2 \text{ arc } AC$

e) none of these

