

THIRTIETH ANNUAL MATHEMATICS CONTEST  
Sponsored by  
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

COMPREHENSIVE 1986

Prepared by: Mathematics Department  
University of the South,  
Sewanee  
W. M. Priestley, Coordinator

Scoring formula:  $4R - W + 40$

Edited by: Larry Bouldin, Roane State  
Community College

---

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

---

Contributors to TMTA for Annual Mathematics Contest:

Dr. Hal Ramer, President, Volunteer State Community College, Gallatin,  
Tennessee  
Donnelley Printing Company, Gallatin, Tennessee  
Chattanooga Coca-Cola Bottling Company, Chattanooga, Tennessee  
Sears, Madison, Tennessee  
Shoney's Incorporated, Nashville, Tennessee  
Beasley Distributing Company, Chattanooga, Tennessee  
IBM Corporation, Nashville, Tennessee  
Provident Life and Accident Insurance Company, Chattanooga, Tennessee  
TRW, Ross Gear Division, Lebanon, Tennessee

1. Evaluate  $(x^x)(x^x)$  when  $x = 2$ .
- A. 16      B. 64      C. 256      D. 1024      E. 65,536
2. If  $a = 2$ ,  $b = 20$ ,  $c = 200$ , and  $d = 2000$ , then  $(a+b+c-d) + (a+b-c+d) + (a-b+c+d) + (-a+b+c+d)$  is equal to
- A. 4444      B. 2222      C. 1111      D. 8484      E. 2424
3. The sum of 100 numbers is what percentage of the average of the 100 numbers?
- A. 0.1%      B. 10%      C. 200%      D. 1000%      E. 10,000%
4. The sum of all integers between 40 and 340 which end in 1, is
- A. 5580      B. 5339      C. 4908      D. 4577      E. 4366
5. The sum of the first fifty positive odd integers subtracted from the sum of the first fifty positive even integers is
- A. 25      B. 30      C. 40      D. 50      E. 100
6. Suppose that I am going to shoe your horse and that you have agreed to pay me 1 cent for the first nail, 2 cents for the second nail, 4 cents for the third nail, . . . , and more generally,  $2^{n-1}$  cents for the  $n$ -th nail. Each shoe requires 8 nails, so that a total of 32 nails are necessary to shoe the horse. Your total cost falls in which of the following ranges?
- A. Less than \$10      D. Between \$1000 and \$ 25,000  
 B. Between \$10 and \$100      E. More than \$25,000  
 C. Between \$100 and \$1000
7. If  $\exp$  denotes the inverse of the natural log function, which of the following rules defines an odd function?
- $\exp(x)$ ,  $\exp(x^2)$ ,  $\exp(x^3)$ ,  $\exp|x|$
- A. None      D. Only the third  
 B. The first and third only      E. All  
 C. The second and fourth only

Comprehensive

8. Exactly one of the following statements is true. Which is it?
- A. The product of two odd functions is odd.
  - B. The sum of two odd functions is even.
  - C. If  $(a,b)$  is on the graph of an even function, then so is  $(-a,b)$ .
  - D. The graph of an odd function is symmetric with respect to the vertical axis.
  - E. If  $(a,b)$  is on the graph of an odd function, then so is  $(-a,b)$ .
9. The solution set of the inequality  $x^2 - 5x + 6 > 0$  is described by which of the following conditions?
- A.  $x > 3$  and  $x < 2$
  - B.  $x > 3$  or  $x < 2$
  - C.  $2 < x < 3$
  - D.  $2 < x < 3$
  - E.  $x < 0$  or  $x > 5$
10. If  $1 - 6/x + 9/x^2 = 0$ , then  $3/x$  equals
- A. 1
  - B. -1
  - C. 3
  - D. -1 or 3
  - E. -1 or -3
11. What is the least positive value of  $b$  for which the equation
- $$7x^2 - bx + 48 = 0$$
- has a real root?
- A. 37
  - B. 0
  - C.  $\sqrt{1344}$
  - D.  $\sqrt{336}$
  - E. 55
12. If the expression  $(3x-y)^{12}$  is expanded according to the binomial theorem, then the coefficient of the term  $x^3y^9$  in the resulting expression will be
- A. 5940
  - B. -660
  - C. -5940
  - D. -17,820
  - E. 17,820
13. In how many ways can 4 boys and 4 girls be arranged in a row so that boys and girls alternate?
- A. 1296
  - B. 48
  - C. 256
  - D. 1152
  - E. 576
14. In how many ways can 12 different books be divided equally among 3 different children?
- A. 34,650
  - B. 1485
  - C. 4096
  - D. 3456
  - E. 924
15. A certain organization has exactly 4 committees. Each member of the organization serves on exactly 2 committees, yet every pair of committees has exactly one member in common. How many persons belong to this organization?
- A. 4
  - B. 6
  - C. 8
  - D. 10
  - E. 16

16. The Coca-Cola Company has authorized a survey to determine the reaction of consumers to several of its products. Each person in the survey will be asked to taste-test and to express an opinion (like or dislike) about 1) Classic Coke, 2) New Coke, and 3) Cherry Coke. 475 liked Classic Coke, 425 liked New Coke, 400 liked Cherry Coke, 225 liked both New Coke and Classic Coke, 250 liked both New Coke and Cherry Coke, 250 liked both Classic Coke and Cherry Coke, 150 liked all three, and 175 disliked all three Coca-Cola products. How many people were surveyed?

A. 1300    B. 725    C. 1475    D. 900    E. 2350

17. Suppose that each of  $a$ ,  $b$ ,  $c$ , and  $d$  is a real number and that  $ad - bc = 2$ . The inverse of the matrix,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ is which of the following?}$$

A.  $\begin{pmatrix} 2a & 2b \\ 2c & 2d \end{pmatrix}$                       B.  $\begin{pmatrix} 1/a & 1/b \\ 1/c & 1/d \end{pmatrix}$                       C.  $\begin{pmatrix} 2/a & 2/b \\ 2/c & 2/d \end{pmatrix}$

D.  $\begin{pmatrix} d/2 & -b/2 \\ -c/2 & a/2 \end{pmatrix}$                       E.  $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$

18. Which of the following is a description of the solution set of this system of simultaneous equations?

$$\begin{aligned} 3/X + 5/Y &= 21 \\ 7/X - 4/Y &= 2 \end{aligned}$$

A.  $\{(2,3)\}$     B.  $\{\}$     C.  $\{2,3\}$     D.  $\{(1/2, -1/3)\}$     E.  $\{1/2, 1/3\}$

19. Let  $p$  and  $q$  denote propositions. Consider the following propositions:

- I. If  $q$ , then not  $p$ .
- II. If not  $q$ , then not  $p$ .
- III. (Not  $p$ ) or  $q$ .
- IV. (Not  $p$ ) and  $q$ .
- V. ( $p$  and not  $q$ ) or (not  $q$  and  $p$ ).

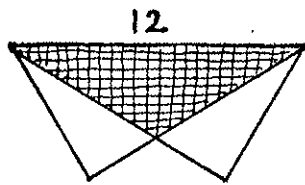
Which pair of these propositions are each logically equivalent to the proposition "If  $p$ , then  $q$ ."

A. (II, III)    B. (I, IV)    C. (I, III)    D. (III, IV)  
E. (II, V)

20. Express the function  $\cos(3t)$  as a polynomial involving  $\cos(t)$ .
- A.  $2\cos^2(t)-1$     B.  $3\cos(t)$     C.  $3\cos^2(t)-2\cos(t)$   
 D.  $4\cos^2(t)-4\cos(t)+1$     E.  $4\cos^3(t)-3\cos(t)$
21. Definition: The period of a periodic function  $f$  is the smallest positive number  $p$  such that  $f(t+p)=f(t)$  for all  $t$ . The amplitude of a periodic function  $f$  is the number  $A$  such that  $2A$  is the difference between the maximum value and the minimum value of  $f$ . Determine the period  $p$  and the amplitude  $A$  for the function  $f(t) = 5\sin(t/3)$ .
- A.  $A = 10; p = 2\pi/3$     D.  $A = 10; p = 6\pi$   
 B.  $A = 5; p = 2\pi/3$     E.  $A = 5; p = 2\pi$   
 C.  $A = 5; p = 6\pi$
22. What is the largest set of real numbers that can serve in the domain of the function,  $f(x) = \ln(\cos^{-1}x)$ ? (Here  $\ln$  denotes the natural logarithm function, and  $\cos^{-1}$  denotes the principal value of the arccosine function.)
- A.  $-\infty < x < \infty$     B.  $-1 < x < 1$     C.  $-1 \leq x < 1$   
 D.  $0 < x$     E.  $-1 \leq x \leq 1$
23.  $\log_{10}(1/1000^a)$  simplifies to which of the following:
- A.  $-3a$     B.  $1$     C.  $10$     D.  $3a$     E.  $10a$
24. Some functions (for example, the logarithmic function given by  $f(x) = \log(x)$ ) satisfy the identity  $f(ab) = f(a) + f(b)$ .
- If such a function  $f$  has an inverse  $g$ , then which of the following identities must hold?
- A.  $g(a)g(b) = g(ab)$     D.  $g(a+b) = g(a)g(b)$   
 B.  $g(a+b) = g(ab)$     E.  $g(a) + g(b) = g(a)g(b)$   
 C.  $g(a) + g(b) = g(ab)$
25. A bear walks 3 km due east, then 4 km due south, then 4 km due east, and then 3 km due south. Ignoring the curvature of the earth, the bear's distance from his starting position in km is
- A.  $7\sqrt{2}$     B.  $10$     C.  $14$     D.  $14\sqrt{2}$     E.  $2\sqrt{7}$

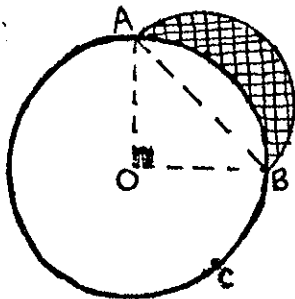
26. A regular polygon of 12 sides is inscribed in a circle whose radius is 1 inch. The area within the polygon is
- A.  $2 \text{ in}^2$     B.  $3 \text{ in}^2$     C.  $10 \text{ in}^2$     D.  $12\sqrt{3}/7 \text{ in}^2$   
 E.  $\sqrt{2} + \sqrt{3} \text{ in}^2$

27. Two congruent  $30^\circ-60^\circ-90^\circ$  triangles are placed so that they overlap partly and their hypotenuses coincide. If the hypotenuse of each triangle is 12, the area common to both triangles is



- A.  $6\sqrt{3}$   
 B.  $8\sqrt{3}$   
 C.  $9\sqrt{3}$   
 D.  $12\sqrt{3}$   
 E. 24

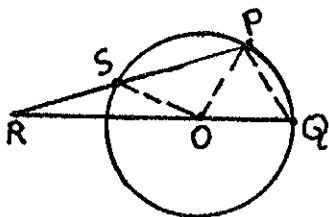
28. Find the area of the shaded moon-shaped figure bounded by two arcs of circles, one being the unit circle centered at O and the other having as diameter the hypotenuse of the right triangle AOB, where  $AO = BO = 1$ .



- A.  $1/2 \text{ sq. unit}$   
 B.  $\sqrt{2}/2 \text{ sq. unit}$   
 C.  $\sqrt{3}/3 \text{ sq. unit}$   
 D.  $\pi/6 \text{ sq. unit}$   
 E.  $\sqrt{2}/3 \text{ sq. unit}$

29. See the figure described for the preceding problem. If C is on the longest of the arcs joining A and B, then what is the measure of angle ACB?
- A. The answer varies depending on where C falls on the arc.  
 B. 45 degrees    C. 30 degrees    D. 22.5 degrees    E. 60 degrees

30. In the figure below,  $O$  lies on the segment  $RQ$  and is the center of a circle on which lie the points  $S$ ,  $P$ , and  $Q$ . Furthermore,  $R$  is the intersection of  $PS$  and  $OQ$  extended. If  $RS = OQ$ , then which of the following relations must hold?



$m\angle$  represents "the measure of the angle".

- A.  $m\angle POQ = m\angle PQO$   
 B.  $m\angle POR = 2m\angle POQ$   
 C.  $m\angle RSO = 2m\angle SOP$   
 D.  $m\angle POQ = 3m\angle PRQ$   
 E.  $\angle RPQ$  is a right angle
31. The point  $(5,7)$  is on the circle  $x^2 + y^2 - 4x - 6y = 12$ . Find the slope of the line which is tangent to this circle at  $(5,7)$ .

- A.  $-10/7$     B.  $4/3$     C.  $1$     D.  $-3/4$     E.  $-7/10$

32. Definition: The point of concurrency of the medians of a triangle is called its centroid. Find the first coordinate of the centroid of the triangle whose vertices are  $A=(0,0)$ ,  $B=(10,0)$ , and  $C=(3,4)$ .

- A.  $22/5$     B.  $11/3$     C.  $13/3$     D.  $4$     E.  $4/3$

33. Let  $A=(-a,0)$  and  $B=(a,0)$  where  $a>0$ . Let  $S$  be the set of all points  $P$  in the plane such that  
 $\text{distance}(A,P) + \text{distance}(B,P) = 2a$ .

Then  $S$  is

- A. a parabola whose axis is the horizontal axis.  
 B. a parabola whose axis is the vertical axis.  
 C. a hyperbola.  
 D. an ellipse whose major axis is vertical.  
 E. an ellipse whose major axis is horizontal.

34. The intersection of the cylinder  $x^2 + y^2 = 16$  with the plane  $y = -3$  is

- A. a point.    D. a line.  
 B. a segment.    E. two parallel lines.  
 C. a circle.

35. In a board game, moves are determined by throwing a pair of dice. A player can escape from JAIL if she throws a seven or an eleven or doubles. What is the probability that after she has landed in JAIL, she escapes from JAIL on her second try?

- A.  $7/18$     B.  $77/324$     C.  $7/9$     D.  $49/324$     E.  $25/325$

36. Ten fair coins are tossed and the number of heads recorded. This procedure is done 1000 times, generating 1000 numbers. The average of this set of numbers is most likely to be near which of the following?

A. 500    B. 5    C. 100    D. 250    E. 1000

37. Marcia and John play a game of chess. If the odds in favor of Marcia's winning are 2:5 and the odds in favor of John's winning are 3:10, what is the probability of a draw? (A draw is the outcome in which neither player wins.)

A. 44/91    B. 47/91    C. 3/10    D. 44/47    E. 47/44

38. A hundred people each toss a coin one time. Let  $p$  denote the probability that exactly 50 heads come up, assuming that the coin is fair and that each toss is independent of the others. Which of the following numbers is closest to  $p$ ?

A. 0.000    B. 0.100    C. 0.500    D. 0.999    E. 1.000

39. In doing arithmetic to a certain base, a student finds that the square of the number represented by 16 is represented by 230. She was working in one of the following bases. Which one?

A. eight    B. nine    C. twelve    D. fourteen    E. sixteen

40. If  $k$  is a positive number and  $f$  is a function such that, for every positive number  $X$ ,

$$(f(2X + 1))^{\sqrt{X}} = k;$$

then, for every positive number  $Y$ ,

$$(f((8+Y)/Y))^{\sqrt{16/Y}} \text{ is equal to}$$

A.  $Y\sqrt{k}$     B.  $2k$     C.  $k^2$     D.  $\sqrt{k}$     E.  $4k$