

TWENTY-THIRD ANNUAL MATHEMATICS CONTEST
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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

COMPREHENSIVE TEST

1979

Scoring Formula: $4R - W + 40$

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This test was prepared from a list of Comprehensive questions submitted by the University of Tennessee at Nashville.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

- (a) $2/3$ (b) 3 (c) 6
(d) $3/2$ (e) none of the above

A B C D E
1.

The correct answer for the sample problem is $3/2$, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

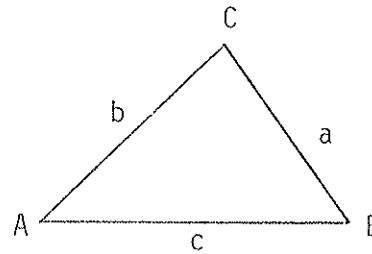
If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

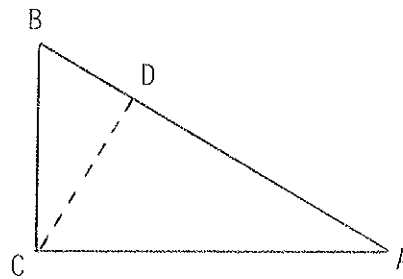
1. In triangle ABC, the length of side a is 20, the measure of angle A is 45° , the measure of angle C is 75° . The length of side b is

- (a) 20
 (b) $10\sqrt{6}$
 (c) $10\sqrt{2}$
 (d) $20\sqrt{2}$
 (e) $10\sqrt{3}$



2. The area of the following right triangle is 12 square units. Angle C is the right angle. The leg AC has length 6. The length of the altitude CD is

- (a) $\frac{12}{\sqrt{13}}$
 (b) 5
 (c) $\frac{6}{\sqrt{13}}$
 (d) 4
 (e) none of the above



3. The value of the determinant $\begin{vmatrix} 1 & 2 & 0 & 1 \\ -1 & -1 & 2 & 1 \\ 2 & 1 & 0 & 1 \\ 1 & 3 & 0 & 1 \end{vmatrix}$ is

- (a) -2
 (b) 2
 (c) 5
 (d) -10
 (e) -5

4. If the line whose equation is $y = mx + 3m + 4$ is perpendicular to the line whose equation is $3x + 2y - 5 = 0$, then m is

- (a) $-\frac{2}{3}$ (d) $\frac{2}{3}$
 (b) $-\frac{3}{2}$ (e) 3
 (c) $\frac{3}{2}$

5. Let $f(x) = x^2 - 3x - 4$ for all real numbers x . For what values of x is the function f decreasing?

- (a) $x > \frac{3}{2}$
- (b) $x < 1$ or $x > 4$
- (c) $-1 < x < 4$
- (d) $x < \frac{3}{2}$
- (e) none of the above

6. Which of the following is the equation of an asymptote of the graph of the equation

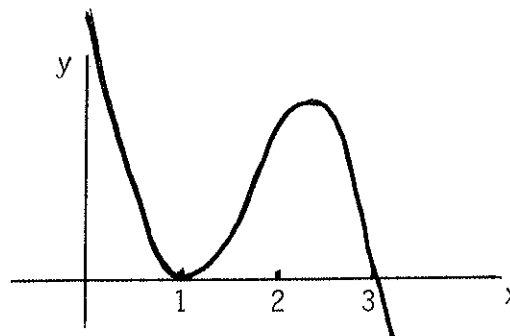
$$y = \frac{x - 2}{(x + 3)(x - 1)} ?$$

- I. $x = -3$
- II. $x = 1$
- III. $x = 2$

- (a) I only
- (b) II only
- (c) III only
- (d) I and II only
- (e) I, II, and III

7. Which of the following can be the equation of the following graph?

- (a) $y = (x - 1)(x - 3)$
- (b) $y = (1 - x)(3 - x)$
- (c) $y = (1 - x)(3 - x)^2$
- (d) $y = (x - 1)^2(x - 3)$
- (e) $y = (1 - x)^2(3 - x)$



8. A polar-coordinate equation equivalent to $x^2 + y^2 = 4y$ is

- (a) $r = 2$
- (b) $r = 4$
- (c) $r = 4 \cos \theta$
- (d) $r = 4 \sin \theta$
- (e) $r^2 = 4 \sin \theta$

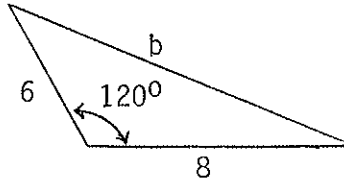
9. A car traveling on a circular track at a rate of 90 miles per hour is turning about the center at an angular rate of 60° per minute. The radius of the track in miles is
- (a) 9π
 - (b) $\frac{9}{2}$
 - (c) $\frac{9}{2\pi}$
 - (d) $\frac{9}{\pi}$
 - (e) none of the above.
10. $\sin 3x \cos x - \cos 3x \sin x$ may be simplified to yield
- (a) $\sin 4x$
 - (b) $\sin 2x$
 - (c) $\cos 4x$
 - (d) $\cos 2x$
 - (e) $\sin x \cos x$
11. In a bridge game the dealer finds himself one card short. He notices his 12-card hand contains only black cards. A bridge deck contains 52 cards, 26 of which are black. What is the probability that the lost card is black?
- (a) $\frac{1}{4}$
 - (b) $\frac{1}{3}$
 - (c) $\frac{7}{20}$
 - (d) $\frac{1}{2}$
 - (e) $\frac{13}{20}$
12. The graph of $x^2 + 4y^2 - 4xy = 0$ is
- (a) an ellipse
 - (b) a hyperbola
 - (c) a circle
 - (d) a straight line
 - (e) a point

13. What is the solution for x in the following system of equations?
- $$\begin{aligned}3x + 2y - z &= 3 \\x + y + z &= 2 \\2x + 3y + 2z &= 3\end{aligned}$$
- (a) -2
(b) 2
(c) 1
(d) 0
(e) -1
14. The square of the length of the diagonal of a rectangle is 121. If two such rectangles are placed end to end, the square of the length of the diagonal of the rectangle formed is 229. The length of one side of the original rectangle is
- (a) 36
(b) $\sqrt{85}$
(c) 5
(d) $\sqrt{108}$
(e) none of the above
15. At the closest point, what is the distance between the y -axis and the curve $x = 6y - y^2 - 25$?
- (a) 4
(b) 5
(c) 0
(d) 25
(e) 16
16. Two men, A and B, set out at the same time from two points 87 miles apart to walk directly toward each other. A walks at a uniform rate of 4 miles per hour, while B walks 6 miles the first hour, 5 and one-half miles the second hour, 5 miles the third hour and so on. In how many hours will the two meet?
- (a) 12
(b) 11
(c) 10
(d) 13
(e) none of the above

17. In a geometric progression the first term is 4, the last term is 108, and the sum is 160. The common ratio r is
- (a) 2
 - (b) 3
 - (c) 4
 - (d) 5
 - (e) none of the above
18. $\sec^2 x + \csc^2 x$ may be rewritten as
- (a) $2 \sec^2 x + 1$
 - (b) $\frac{1}{\sin^2 x + \cos^2 x}$
 - (c) $\tan^2 x + \cot^2 x$
 - (d) $\sec^2 x \csc^2 x$
 - (e) none of the above
19. What is the probability of obtaining 2 heads and 2 tails if 4 unbiased coins are tossed simultaneously?
- (a) $\frac{1}{2}$
 - (b) $\frac{3}{4}$
 - (c) $\frac{3}{8}$
 - (d) $\frac{47}{128}$
 - (e) $\frac{1}{4}$
20. A student takes an examination in a subject about which he knows absolutely nothing. There are 5 questions to be answered "yes" or "no." What is the probability that he will answer exactly 60% of the questions correctly?
- (a) $\frac{1}{4}$
 - (b) $\frac{5}{16}$
 - (c) $\frac{2}{5}$
 - (d) $\frac{1}{2}$
 - (e) $\frac{3}{5}$

21. In the triangle shown below, the length of side b is

- (a) 12
- (b) $2\sqrt{37}$
- (c) 10
- (d) $2\sqrt{25 + 12\sqrt{3}}$
- (e) none of the above



22. The solution set for the inequality $\frac{3x^2 - x - 2}{3x + 2} \leq 0$ is

- (a) $\{x | x \leq 1\}$
- (b) $\{x | x < \frac{-2}{3}\}$
- (c) $\{x | x \leq 0\}$
- (d) $\{x | x < \frac{-2}{3} \text{ or } \frac{-2}{3} < x \leq 1\}$
- (e) $\{x | x \neq \frac{-2}{3}\}$

23. The solution set for the equation $\log_2 |x| + \log_2 |x - 1| = \frac{1}{2} \log_2 4$ is

- (a) $\{1\}$
- (b) $\{0\}$
- (c) $\{-1, 2\}$
- (d) $\{-1\}$
- (e) $\{2\}$

24. Bill can complete a certain task in 2 days. Jim can complete the same work in 3 days. Tom can complete the same task in 4 days. In how many days can they complete the task if they all work together?

- (a) $\frac{11}{12}$
- (b) $\frac{4}{3}$
- (c) $\frac{12}{13}$
- (d) $\frac{3}{4}$
- (e) none of the above

25. Twenty-five slips numbered from 1 to 25 are placed in a hat. If two slips are drawn from the hat without replacement, what is the probability that the sum of the two numbers on the slips is odd?
- (a) $\frac{11}{13}$
 - (b) $\frac{13}{25}$
 - (c) $\frac{13}{50}$
 - (d) $\frac{12}{25}$
 - (e) $\frac{2}{5}$
26. $\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x}$ may be simplified to yield
- (a) $2 \cot x$
 - (b) 2
 - (c) $\frac{2}{\sin x (1 + \cos x)}$
 - (d) $2 \sec x$
 - (e) $2 \csc x$
27. The solution set for the equation $\log(\log(\log x)) = 0$, using base ten logarithms, is
- (a) $\{10\}$
 - (b) $\{1000\}$
 - (c) $\{10000\}$
 - (d) $\{10^{10}\}$
 - (e) $\{1\}$
28. The base sixteen numeral 74.2 , written as a base two numeral, is
- (a) 1110100.001
 - (b) 111100.1
 - (c) 111100.01
 - (d) 11110.01
 - (e) none of the above

29. How many possible 5-card hands, chosen from a deck of 52 cards, contain the ace of hearts?

(a) $\frac{52!}{5! 47!}$

(b) 13

(c) $\frac{51!}{4! 47!}$

(d) 51

(e) 52

30. What is the probability that the first head is obtained in the sixth toss of a fair coin?

(a) $\frac{1}{64}$

(b) $\frac{1}{32}$

(c) $\frac{5}{64}$

(d) $\frac{1}{2}$

(e) $\frac{5}{32}$

31. The set of all real numbers x such that $0 \leq x < 2\pi$ which are solutions of the equation $\cos 2x - \sin x = 0$ is

(a) $\left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\}$

(b) $\left\{ \frac{\pi}{6}, \frac{3\pi}{2} \right\}$

(c) $\left\{ \frac{3\pi}{2} \right\}$

(d) $\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \right\}$

(e) $\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{3\pi}{2} \right\}$

32. If $0 < \theta < 90^\circ$ and $\cot(2\theta) = \frac{-7}{24}$, then $\cos \theta$ is equal to
- (a) $\frac{3}{5}$
 - (b) $\frac{4}{5}$
 - (c) $\frac{-7}{25}$
 - (d) $\frac{7}{25}$
 - (e) $\frac{-24}{25}$
33. The solution set for the equation $\log_{10}100 = (\log_x 4)(\log_3 81)$ is
- (a) $\{16\}$
 - (b) $\{2\}$
 - (c) $\{4\}$
 - (d) $\{\frac{1}{2}\}$
 - (e) no solution
34. $\sum_{i=1}^n (4i + 3)$ equals
- (a) $2n^2 + 2n + 3$
 - (b) $4n + 3$
 - (c) $2n^2 + n$
 - (d) $2n^2 + 5n$
 - (e) none of the above
35. The sum of the infinite series $\frac{-1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \dots$ is
- (a) $\frac{1}{3}$
 - (b) $\frac{1}{4}$
 - (c) $\frac{-1}{2}$
 - (d) $\frac{-1}{4}$
 - (e) $\frac{-1}{3}$

36. $(\sqrt{2} + i\sqrt{2})^8$ equals

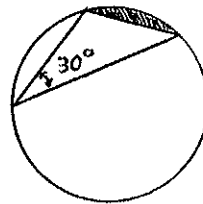
- (a) 16
- (b) $16i$
- (c) 256
- (d) -128
- (e) none of the above

37. The area inside an ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab . The area inside the ellipse whose equation is $4x^2 + y^2 - 8x + 6y + 5 = 0$ is

- (a) 4π
- (b) 16π
- (c) 6π
- (d) $\frac{5\pi}{2}$
- (e) none of the above

38. Given the following circle where the chords form an angle of 30° and the area of the shaded region is $2\pi - 3\sqrt{3}$ square units. The length of the radius of the circle is

- (a) 3
- (b) $2\sqrt{3}$
- (c) $3\sqrt{2}$
- (d) $\frac{10}{\pi}$
- (e) none of the above



39. From a stake midway between two trees 60 feet apart, guy wires are stretched which meet the trees 40 feet and 20 feet above the ground, respectively. Assuming the ground is level, the tangent of the angle between the wires is

- (a) $\frac{2}{3}$
- (b) $\frac{6}{17}$
- (c) $\frac{4}{3}$
- (d) $\frac{-17}{6}$
- (e) -18

40. $\frac{(2 - i)^2}{2\sqrt{6} - i}$ is equal to

(a) $\frac{2\sqrt{6} + i}{5 + 4i}$

(b) $\frac{2\sqrt{6} + i}{3 + 4i}$

(c) $\frac{2\sqrt{6} - i}{3 - 4i}$

(d) $\frac{2\sqrt{6} + i}{25}$

(e) none of the above