

NINTH ANNUAL MATHEMATICS CONTEST

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COMPREHENSIVE TEST

1965

Scoring Formula: $4R - W$.

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DIRECTIONS:

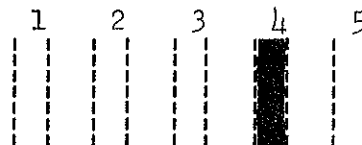
Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work the problems, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. If $2x = 3$, then x equals:

(1) $2/3$; (2) 3 ; (3) 6 ;

(4) $3/2$; (5) none of these.



The correct answer for the sample problem is " $3/2$ ", which is answer (4); so you would answer this problem by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. The set of values of x such that $|x - 3|$ is less than 2 is:
 - (1) all $x < 2$
 - (2) all x such that $1 < x < 5$
 - (3) all $x < 5$
 - (4) all $x > 1$
 - (5) all x such that $0 < x < 5$.

2. The area of an equilateral triangle is $4\sqrt{3}$. The perimeter is:
 - (1) 13
 - (2) $2\sqrt{5}$
 - (3) $6\sqrt{3}$
 - (4) 9
 - (5) none of the above

3. The logarithm of 5 to the base 10 is approximately .69897. Which of the following numbers is the best approximation to $\log_{10}(1/2)$?
 - (1) .34948
 - (2) - .23299
 - (3) .30103
 - (4) - .30103
 - (5) - .34948

4. Let a and b be two positive numbers. Which of the following statements is true?
 - (1) The geometric mean of a and b is always greater than the arithmetic mean of a and b .
 - (2) The geometric mean of a and b is always greater than or equal to their arithmetic mean and is sometimes actually equal.
 - (3) The geometric mean of a and b is sometimes greater and sometimes less than the arithmetic mean of a and b , depending on a and b .
 - (4) The geometric and arithmetic means of a and b are never equal, no matter what values a and b take.
 - (5) The geometric mean of a and b is always less than or equal to the arithmetic mean of a and b .

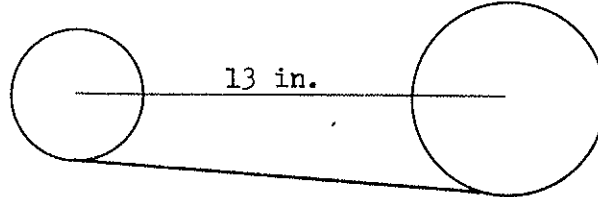
5. The sum of the numbers from 1 to 100 inclusive is:
 - (1) 5050
 - (2) 4938
 - (3) 6092
 - (4) 10,502
 - (5) none of the above

6. Which of the following is a square root of $i = \sqrt{-1}$?

- (1) i (2) $\frac{1+i}{\sqrt{2}}$ (3) $\frac{1}{\sqrt{2i}}$ (4) $\sqrt{-i}$ (5) $3 - 2i$

7. Two circles of radii 5 inches and 10 inches respectively have their centers 13 inches apart. The length of their common external tangent is:

- (1) 12 in.
 (2) 13 in.
 (3) 15 in.
 (4) $\sqrt{175}$ in.
 (5) none of the above



8. A motor boat which travels 12 mph in still water can go 12 miles downstream on a river and 12 miles back in 2 hours and 8 minutes. What is the speed of the stream?

- (1) 12 mph
 (2) 2 mph
 (3) 6 mph
 (4) 3 mph
 (5) 4 mph

9. $\frac{7x^{-1} - 6 - 2x^{-2}}{x^{-2} - 2x^{-1}}$ is equal to:

- (1) $-2x + 1$
 (2) $-2 + 3x$
 (3) $-3x$
 (4) $2x - 1$
 (5) $2 - 3x$

10. The sum of the roots of $18x^2 - 144x + 55 = 0$ is:

- (1) 55
 (2) 8
 (3) 5
 (4) 73
 (5) $55/18$

11. Which of the following has a graph with period π and amplitude 3?

- (1) $y = 3 \tan(x)$
 (2) $y - 2 = \frac{1}{3} \sec(2x + 3\pi)$
 (3) $3y = \csc(2x + 3)$
 (4) $y = -3 \cos(2x - \pi)$
 (5) $y = 2 \sin(3x)$

12. $\tan 105^\circ$ is equal to:

- (1) $-\tan 15^\circ$
- (2) $-2 - \sqrt{3}$
- (3) $\tan 75^\circ$
- (4) $\sqrt{3}/2$
- (5) $\frac{1 - \sqrt{3}}{1 + \sqrt{3}}$

13. Which one of the following statements is not true?

- (1) The sum of two irrational numbers may be a rational number.
- (2) The sum of an irrational number and a rational number is an irrational number.
- (3) The product of two rational numbers is a rational number.
- (4) The product of two irrational numbers is always an irrational number.
- (5) The product of two complex numbers is a complex number.

14. A man is driving directly toward a mountain. At one point the angle of elevation to the top of the mountain is 30° . Four miles further down the road the angle of elevation to the top of the mountain is 45° . How high is the mountain above the road?

- (1) $\frac{4}{\sqrt{3} - 1}$ miles
- (2) $\frac{3}{1 + \sqrt{2}}$ miles
- (3) $\frac{4}{1 - \sqrt{2}}$ miles
- (4) $\frac{3}{\sqrt{2} + 3}$ miles
- (5) $\frac{4}{1 - \sqrt{3}}$ miles

15. If $\log_x 16 = 4$, then x is equal to:

- (1) 10
- (2) 3
- (3) e
- (4) 2
- (5) 4

16. What values of k will make the zeros of $f(x) = 3x^2 + 12x - 2k$ equal?

- (1) 8
- (2) 3
- (3) -4
- (4) 1
- (5) -6

17. $12.3565656\dots$ is equal to:

- (1) $\frac{12257}{990}$
- (2) $\frac{12278}{990}$
- (3) $\frac{12241}{990}$
- (4) $\frac{12237}{990}$
- (5) $\frac{12233}{990}$

18. $\text{Arcsin}(-\sqrt{3}/2)$ is equal to:

- (1) 60°
- (2) 120°
- (3) $4\pi/3$ radians
- (4) $-\pi/3$ radians
- (5) $\pi/6$ radians

19. If the first term of an arithmetic progression is -7 and the sum of the first eight terms is 84 , then what is the eighth term?

- (1) 19
- (2) 12
- (3) 28
- (4) -12
- (5) 77

20. Consider the system $\begin{cases} y^2 - 3 \log(x) - 16 = 0 \\ y - \log(x) = 2 \end{cases}$

The solutions to the system are:

- (1) $(3, 5)$ and $(-4, -2)$
- (2) $(.0001, -2)$ and $(1000, 5)$
- (3) $(0, 4)$ and $(1, 2)$
- (4) $(.00001, 3)$ and $(100, -2)$
- (5) $(.001, -5)$ and $(1000, -2)$

21. $\frac{8! + 10!}{7! + 11!}$ is equal to:

- (1) $\frac{256}{117}$
- (2) $\frac{311}{589}$
- (3) $\frac{728}{7921}$
- (4) $\frac{719}{7831}$
- (5) $\frac{583}{19}$

22. Two circles of radius r intersect in such a way that one circle passes through the center of the other. The length of the common chord joining the two points where the circles intersect is:

- (1) $2r$
- (2) $\frac{3}{4}r$
- (3) $\sqrt{3}r$
- (4) $\frac{4}{\sqrt{5}}r$
- (5) r

23. Suppose the following probabilities have been assigned to the possible result of putting a penny in a certain defective peanut-vending machine: The probability that nothing comes out is $1/2$; the probability that either you get your money back or you get peanuts (but not both) is $1/3$. What is the probability that you get back your money and also get peanuts?

- (1) $1/3$
- (2) $5/6$
- (3) $1/6$
- (4) $1/2$
- (5) not enough information to determine the probabilities

24. Let a, b, c, d, e, f be six given objects. Form sets using these objects as members as follows:

$$A = \{b, c, a\}, \quad B = \{d, a, e, c\}, \quad C = \{f, a, c\}.$$

Then $(A \cup B) \cap C$ contains how many members?

- (1) 3 (2) 4 (3) 6 (4) 2 (5) 1
25. The expression for a certain integer in the binary system is 101110. In the decimal system, the expression for this number is:
- (1) 101110 (2) 20220 (3) 10^{101110} (4) 46 (5) 138
26. The greatest common divisor of 4725 and 3234 is:
- (1) 1 (2) 3 (3) 15 (4) 35 (5) none of the above
27. Suppose that $|x - 2| < 1$. Then $|x^2 - 4|$ will be less than a positive number e whenever $|x - 2|$ is less than
- (1) $1/5$ (2) e (3) $e/5$ (4) $e/2$ (5) $5e$
28. An airplane A is flying level at an altitude of 4 miles with a speed of 500 mph relative to the ground. An airplane B directly above A is flying at an altitude of 7 miles and is in a shallow dive. The courses of the two airplanes intersect at a point C which is 4 miles from airplane A. Suppose airplane B maintains a constant speed of 700 mph in its dive. When airplane B reaches the point C, airplane A will be
- (1) at point C
 (2) $3/7$ miles short of point C
 (3) $3/7$ miles past point C
 (4) .4137 miles short of point C
 (5) .4137 miles past point C
29. Let A be a set containing 3 members. Then the number of subsets of A (counting the empty set and A itself, of course!) is:
- (1) 6 (2) 8 (3) 7 (4) 4 (5) 9
30. Let \overline{AB} be a given line segment. Consider the set of all triangles having \overline{AB} as base and a given fixed area c . In this set, a triangle of minimum perimeter is:
- (1) a triangle with one side equal to $\frac{1}{2}c$
 (2) an isosceles triangle on \overline{AB}
 (3) a right triangle on \overline{AB}
 (4) a triangle other than any of the above
 (5) this is a trick problem since there is no triangle of minimum perimeter in the given set

31. Let S be the set of people taking this examination. If x and y are members of S , we say x is "related" to y if the height of x differs from the height of y by less than $1/2$ inch. This relation is:
- (1) reflexive and symmetric but not transitive
 - (2) an equivalence relation
 - (3) reflexive and transitive but not symmetric
 - (4) symmetric and transitive but not reflexive
 - (5) none of the above
32. Suppose $a = 1011$ and $b = 111001$, the numbers being expressed in the binary system. The expression for the product $a \cdot b$ in the binary system is:
- (1) 1001110011
 - (2) 627
 - (3) 111001111
 - (4) 10000001
 - (5) 1110011011
33. Let the operation $*$ be defined for the set of odd integers by
- $$a * b = (a + b) - 1.$$
- Which of the following is false?
- (1) $5 * 3 = 7$
 - (2) The set of odd integers is closed with respect to $*$.
 - (3) There is no identity element for this system.
 - (4) The operation $*$ is associative in the set of odd integers.
 - (5) The operation $*$ is commutative in the set of odd integers.
34. The solution to the inequality $2x^2 - x - 6 \geq 0$ is:
- (1) $\{x : x < -3/2 \text{ or } x > 2\}$
 - (2) $\{x : -3/2 < x < 2\}$
 - (3) $\{x : x \leq -3/2 \text{ and } x \geq 2\}$
 - (4) $\{x : x \leq -3/2 \text{ or } x > 2\}$
 - (5) $\{x : x \leq -3/2 \text{ or } x \geq 2\}$
35. Which of the following is not equivalent to the statement: "If two triangles are congruent, then they are similar."
- (1) "Only if two triangles are similar, are they congruent."
 - (2) "A sufficient condition that two triangles be congruent is that they be similar."
 - (3) "Only if two triangles are not congruent, are they not similar."
 - (4) "If two triangles are not similar, then they are not congruent."
 - (5) "A necessary condition that two triangles be congruent is that they be similar."

36. Consider the matrix $\begin{bmatrix} \sqrt{-6} & 11 & \sqrt{27} \\ 5 & 9 & \sqrt{-8} \\ 5i & 7 & \sqrt[3]{2} \end{bmatrix}$. What is the cofactor of 7?

- (1) $18\sqrt{3}$ (2) $-19\sqrt{3}$ (3) $-11\sqrt{3}$ (4) $19\sqrt{3}$ (5) $11\sqrt{3}$

37. Which of the following is true about the set

$$A = \{x : x \text{ is a rational number.}\} \cap \{x : x \geq 0\} ?$$

- (1) A is the set of positive real numbers.
 (2) A is the set of positive fractions.
 (3) $A = \{x : x \text{ is a rational number or } x \text{ is a positive number.}\}$
 (4) $A = \{x : x \text{ is a positive number.}\}$
 (5) A is the set of non-negative rational numbers.

38. The domain of the function $f(x) = \sqrt{\frac{x-5}{x-3}}$ is given by:

- (1) $\{x \mid x \geq 5 \text{ or } x \leq 3\}$
 (2) $\{x \mid x \neq 3 \text{ or } x \neq 5\}$
 (3) $\{x \mid x \geq 5 \text{ and } x > 3\}$
 (4) $\{x \mid x \geq 5 \text{ or } x < 3\}$
 (5) $\{x \mid x > 5 \text{ or } x \leq 3\}$

39. If p and q are integers greater than or equal to 2 with $p \neq q$, then which of the following lists give all the solutions to the equation

$$q(x-1)^p(x+1)^{q-1} + p(x-1)^{p-1}(x+1)^q = 0 ?$$

- (1) 1, -1
 (2) 1, $\frac{p+q}{2}$
 (3) -1, $\frac{p-q}{p+q}$, 1
 (4) -1, $\frac{p+q}{p-q}$, 1
 (5) -1, $\frac{p-q}{2}$, 1

40. If $c < 0$, then the number of real roots of the equation $x^3 + bx^2 + cx = 0$, where b, c represent real numbers, is:

- (1) not any
 (2) 1
 (3) 2
 (4) 3
 (5) not enough information given to determine the number of roots.