

THIRTY-FIRST ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

CHALLENGE 1987

Compiled from 1987 Algebra I, II,
Geometry, Comprehensive and Advanced
Topics Exams

Scoring formula: $4R - W + 40$

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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1. Which property below is not valid for the set of all integers?
- Associative law for addition
 - Commutative law for multiplication
 - Identity law for multiplication
 - Inverse law for multiplication
 - Distributive law of multiplication over addition.
2. If the digits of a two-digit integer are reversed the new integer exceeds the original by 20 less than the old integer. The sum of the digits is 11. The original integer is:
- 29
 - 38
 - 47
 - 56
 - 65
3. Ed can do a job in 8 hours. Bob can do the same job in 12 hours. Ed and Bob worked together for 3 hours and Bob completes the job alone. How long does Bob work alone?
- 1.8 hours
 - 7.2 hours
 - 1.0 hours
 - 4.5 hours
 - 6 hours
4. A vessel contains V cc of acid and water with $A\%$ acid. Add M cc of water and N cc of acid (100%). What percentage of acid is now in the vessel?
- $A + N$
 - $A + \frac{N}{100}$
 - $\frac{VA + N}{V}$
 - $\frac{VA + N}{V + N + M}$
 - $\frac{VA + 100N}{V + M + N}$
5. If the smaller of two numbers is increased by $\frac{2}{3}$ the larger, the resulting number is $\frac{4}{3}$ the larger. s and L denotes smaller and larger. Therefore
- $3s = L$
 - $3s = 2L$
 - $\frac{2}{3}L = 3s$
 - $\frac{4}{3}s = \frac{2}{3}L$
 - $L = \frac{4}{3}s$

Challenge

6. Two trains leave point A at 7 a.m. and 8:30 a.m. The late train overtakes the second train at 12:00 noon. The speed of the faster train is 70 m.p.h. What is the speed of the other train?

- (a) 47 (b) 48
 (c) 49 (d) 50
 (e) 51

7. The cost, C dollars, of K apples is 12¢ less than the cost of K-5 pears. The cost of three pears is S dollars. S =

- (a) $\frac{3(C-12)}{K}$ (b) $\frac{3(C-12)}{K-5}$
 (c) $\frac{3(C+12)}{K}$ (d) $\frac{3(C+12)}{K-5}$ (e) $\frac{K(C-12)}{3}$

8. The solution to $9^{(1-2X)} = 81^{X-1}$ is X =

- (a) 3/4 (b) $\frac{10}{11}$
 (c) 3/5 (d) 1/3
 (e) $\frac{10}{19}$

9. If matrix A = $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & -1 \end{bmatrix}$ and matrix B = $\begin{bmatrix} 1 & 3 \\ 2 & -4 \\ -1 & 1 \end{bmatrix}$

then A x B =

- a) B x A

b) $\begin{bmatrix} -3 & 10 \\ 13 & -14 \end{bmatrix}$

c) $\begin{bmatrix} 7 \\ -1 \end{bmatrix}$

d) $\begin{bmatrix} -3 & 13 \\ 10 & -14 \end{bmatrix}$

- e) does not exist

Challenge

10. A water tank can be emptied by using one pump for 5 hours. A second, smaller pump can empty the tank in 8 hours. If the larger pump is started at 1:00 p.m., at what time should the smaller pump be started so that the tank will be emptied at 5:00 p.m.?
- a) 1:36 p.m. d) 4:00 p.m.
 b) 3:30 p.m. e) 4:10 p.m.
 c) 3:24 p.m.
11. An automobile radiator contains 16 quarts of a 20% solution of antifreeze. How much of the original must be drawn off and replaced by pure antifreeze to make a 25% antifreeze?
- a) 0.2 quarts d) 8 quarts
 b) 0.8 quarts e) 1.5 quarts
 c) 1 quart
12. If a right circular cylinder is circumscribed about a sphere, the surface area of the sphere, compared with the total surface area of the cylinder (including top and bottom) is
- a) one-third as great d) the same
 b) one-half as great e) twice as great.
 c) two-thirds as great
13. $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$ is the power series representation for
- a) $\ln x$ d) e^x
 b) $\sin x$ e) $\frac{1}{1+x}$
 c) $\cos x$

Challenge

14. If the fourth term of an arithmetic sequence is 5 and the ninth term is 20, the sixth term is

- a) 9
- b) 11
- c) 13
- d) 15
- e) 17

15. The graph of $P(x) = x^3 + ax^2 + bx + c$ intersects the x-axis at $x = -2$ and $x = 1$ and the y-axis at $y = 0$. Find $P(12)$.

- a) 1848
- b) 1428
- c) 2346
- d) 1232
- e) 1056

16. If $3^{x+4} = 2^{1-3x}$ then $x =$

- a) $\frac{1 - \ln 81}{1 + \ln 8}$
- b) $\frac{\ln 2 - \ln 81}{\ln 24}$
- c) $\frac{\log_2 2 - \log_{24} 81}{2}$
- d) no real solution
- e) $\frac{\ln 2 + \ln 81}{\ln 24}$

17. A piece of wire 100 inches long is cut into two pieces, and then each piece is bent into the shape of a square. If the sum of the enclosed areas is 397 sq. in., find the length of the shorter piece of wire.

- a) 16
- b) 24
- c) 32
- d) 40
- e) 48

18. $\frac{\cos(x)}{\sec(x) + \tan(x)}$ is equal to
- (a) $1 - \sin(x)$
 - (b) $1 + \sin(x)$
 - (c) $\frac{\cos(x)}{1 + \sin(x)}$
 - (d) $\frac{\cos(x)}{1 - \sin(x)}$
 - (e) $\frac{1 + \sin^2(x)}{1 + \sin(x)}$
19. Jim and Jan each toss three coins. What is the probability that they obtain the same number of heads?
- (a) $5/16$
 - (b) $1/2$
 - (c) $19/20$
 - (d) $1/4$
 - (e) $1/8$
20. If $f(x) = 4x - 2$, then $f^{-1}(x)$ is
- (a) $\frac{1}{4x - 2}$
 - (b) $\frac{4}{x} - 2$
 - (c) $\frac{4}{x} + 2$
 - (d) $1/4(x + 2)$
 - (e) Not defined
21. The number of diagonals that can be drawn in a polygon of 100 sides is
- (a) 4850
 - (b) 4950
 - (c) 9900
 - (d) 98
 - (e) 8800

22. Three people work independently at deciphering a message in code. The respective probabilities that they will decipher it are $1/7$, $1/6$, and $1/5$. What is the probability that the message will be deciphered?

- (a) $90/210$
- (b) $1/210$
- (c) $107/210$
- (d) $3/210$
- (e) $120/210$

23. For what value or values of a is the matrix $\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 2 & 3 & a \end{bmatrix}$ singular?

- (a) 0
- (b) 1 and 3
- (c) 1
- (d) 3
- (e) -1

24. For $0 \leq \theta < 2\pi$, the solution set for $2\sin^2\theta - \cos\theta - 1 = 0$ is

- (a) \emptyset
- (b) $\{\pi/3, \pi\}$
- (c) $\{\pi/3, 5\pi/3, \pi\}$
- (d) $\{\pi/4, 3\pi/4\}$
- (e) $\{\pi/6, 5\pi/6, 11\pi/6\}$

25. Find all x which satisfy $\log_7 (x^2) = 3 \log_7 (2x)$.

- (a) 0, 6
- (b) 1
- (c) $1/8$
- (d) $1/8, 0$
- (e) 0

26. The function f is continuous for all x and has a local (relative) maximum at 0 and a local (relative) minimum at 1 . Label statements I, II, and III as always true, sometimes true, or never true.

I) $f'(0) = 0$

II) f has an inflection point between 0 and 1

III) $f'(1) > 0$.

- | | | |
|----------------------|--------------------|---------------------|
| a) I. Always true | II. Always true | III. Never true |
| b) I. Sometimes true | II. Always true | III. Never true |
| c) I. Sometimes true | II. Always true | III. Sometimes true |
| d) I. Sometimes true | II. Sometimes true | III. Never true |
| e) I. Sometimes true | II. Sometimes true | III. Sometimes true |

27. A formula which describes the growth of a bacterial colony is given by $N = N_0 a^{rt}$, where N is the number of bacteria at the end of time t , N_0 is the original number of bacteria, r is the rate of growth, and a is constant. If $\log N$ is the logarithm of N to the base 10, solving for t yields

a) $\frac{\log(N-N_0)}{r \log a}$

b) $\frac{\log(N-N_0)}{(\log a)^r}$

c) $\frac{\log N + \log N_0}{(\log a)^r}$

d) $\frac{\log(N-N_0)}{\log(a^r)}$

e) $\frac{\log N - \log N_0}{r \log a}$

Challenge

28. The region bounded by the graph of $y=2x - x^2$ and the x-axis is revolved about the y-axis. The volume in cubic units of the resulting solid is:
- a) $5\pi/3$
 - b) $7\pi/16$
 - c) $8\pi/3$
 - d) $3\pi/4$
 - e) $\pi/9$
29. How many of the following curves are tangent to $x^2 - y^2 = 16$?
- 1) $x=4$
 - 2) $y=x$
 - 3) $x=y^2+4$
 - 4) $x^2+y^2=16$
 - 5) $(x-5)^2 + y^2=1$
- a) 1
 - b) 2
 - c) 3
 - d) 4
 - e) 5
30. Which of the following cannot be a zero of any polynomial $p(x)$ with integer coefficients for which $p(1) = 12$?
- a) 2
 - b) 3
 - c) 4
 - d) 5
 - e) 6
31. Determine k so that $f(x) = x^3 + kx^2 - kx + 10$ is divisible by $x + 3$
- a) $3/10$
 - b) $9/16$
 - c) $11/8$
 - d) $17/12$
 - e) $15/4$

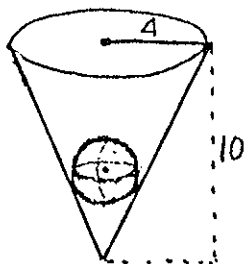
32. The volume of the sphere that can be circumscribed about a regular tetrahedron 2 in. on a side is
- (a) $\pi\sqrt{6}$ cu. in.
 - (b) $\frac{2\pi}{3}$ cu. in.
 - (c) 8π cu. in.
 - (d) $\frac{4\pi\sqrt{3}}{3}$ cu. in.
 - (e) $2\pi\sqrt{3}$ cu. in.
33. A point is 4 in. from each face of a dihedral angle of 60° . How far is the point from the edge of the angle?
- (a) 8 in.
 - (b) $4\sqrt{2}$ in.
 - (c) $4\sqrt{3}$ in.
 - (d) $\frac{4}{\sqrt{3}}$ in.
 - (e) 2 in.
34. The sides of a regular polygon of X sides, $X > 4$, are extended to form a star. The measure in degrees of the angle at a point of the star is:



- (a) $\frac{360}{X}$
- (b) $\frac{180(X-4)}{X}$
- (c) $\frac{180(X-2)}{X}$
- (d) $180 - \frac{90}{X}$
- (e) $\frac{180}{X}$

Challenge

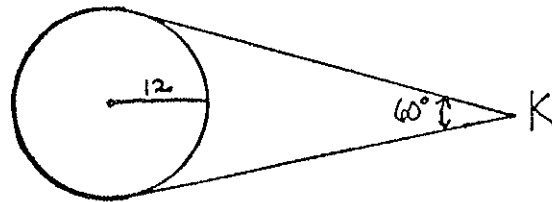
35. A sphere of radius 2 is placed inside a right circular cone of base radius 4 and height 10. The distance from the vertex of the cone to the center of the sphere is



- (a) 5
(b) $5\sqrt{2}$
(c) $\sqrt{29}$
(d) $\sqrt{29} - 2$
(e) $\sqrt{29} + 2$
36. The development of the non-Euclidean geometry which postulates that through a given point not on a given line can be drawn more than one line parallel to the given line is generally credited to which mathematician?
- (a) Riemann
(b) Playfair
(c) Lobachevsky
(d) Bourbaki
(e) Newton
37. Which statement provides a valid conclusion to the following discussion?
- All the Eton men in this college play cricket. None of the cricketers row. All my friends in this college come from Eton. All the scholars are rowing men. All who dine with the Dean are scholars. Therefore,
- (a) All my friends dine with the Dean.
(b) None of my friends play cricket.
(c) All the cricket players are from Eton.
(d) None of my friends dine with the Dean.
(e) All the rowing men dine with the Dean.

38. A wire is stretched from point K around a circle of radius 12 in. and back to K, forming an angle of 60° at K. The length of the wire is

- (a) $48 + 8\pi$
- (b) $12\sqrt{3} + 16\pi$
- (c) $24\sqrt{3} + 16\pi$
- (d) $48\sqrt{3} + 8\pi$
- (e) $48 + 16\pi$



39. Which one of these statements is not one of Euclid's postulates?

- (a) Two points determine a unique line.
- (b) If two lines cut by a transversal have equal alternate interior angles, the lines are parallel.
- (c) All right angles are equal.
- (d) A straight line extends infinitely far in either direction.
- (e) It is possible to describe a circle with any point as center and with a radius equal to any finite straight line drawn from the center.

40. $\lim_{x \rightarrow \infty} \sqrt{x^2 + 5x} - x =$

- a) 0
- b) $5/2$
- c) ∞
- d) 1
- e) 5



