TMTA Calculus and Advanced Topics Test, 2007

1. $\lim_{t\to 3} \frac{2t-1}{3t+2} =$

a) 5/2 b) 2/3 c) 5/11 d) 3/5 e) 7/11

2. $\lim_{x\to 5} \frac{3x-15}{x^2-25} =$

a) 0 b) 3/10 c) 3/5 d) 3 e) undefined

3. The equation for the line tangent to the curve $y = x^2$ at the point (3,9) is

a) 3x + 9y = 0 b) y = 2x(x-3) + 9 c) y = 6x – 18

d) y – 9 = 6x – 3 e) 6x – y = 9

4. The graph of the function $f(x) = \frac{x-3}{x^2-3x+2}$ has a vertical asymptote at which of the following points?

a) x = 3 b) x = 2 c) x = – 3 d) x = – 2 e) x = 0

5. Calculator problem. If y = sin(x), then the derivative y ’(2) =

a) – 0.416147 b) 0.416147 c) – 0.909297 d) 0.909297 e) 1

6. The derivative of $f(x) = x^6 - 3x^5$ is

a) $6x^5 - 3x^6$ b) $6x^5 - 8x^6$ c) $6x^7 - 15x^6$ d) $6x^5 - 15x^4$ e) $6x^6 - 15x^5$

7. The derivative of $y = \frac{x^2 + 1}{x + 1}$ is

a) $\frac{x^2 + 2x + 1}{(x + 1)}$ b) $\frac{x^2 + 2x + 1}{(x + 1)^2}$ c) $\frac{3x^2 + 2x + 1}{(x + 1)^2}$

d) $\frac{x^2 + 2x - 1}{(x + 1)^2}$ e) $\frac{1 - 2x - x^2}{(x + 1)^2}$

8. The derivative of y = $\sin(x^2)$ is

a) 2x cos(x^2) b) 2x sin(x^2) c) 2x sin(x) + x^2 cos(x)

d) 2x cos(x) e) cos(2x)

9. Given that $x^2 + y^2 = xy$ find $\frac{dy}{dx}$

a) $2x + 2y$ b) $x + y$ c) $\frac{y - 2x}{x - 2y}$ d) $\frac{y - 2x}{2y - x}$ e) – 1

10. Given $y = x^2$, use the secant line to estimate the slope when $x = 7$ and $\Delta x = 0.001$. The percent error of this estimate is approximately

a) 0.7% b) 0.07% c) 0.007% d) 0.0007% e) 0.00007%

11. A speedboat is traveling 12 knots as it passes the end of a pier extending 8 feet at right angles from the shore. A man standing 6 feet down the shore from the foot of the pier, at the edge of the water, uses radar to clock the speed of the boat at the instant it passes the end of the pier, traveling parallel to the straight shoreline. What speed does his radar register?

a) 7 knots b) 7.1 knots c) 7.2 knots d) 7.3 knots e) 7.4 knots

12. The cost of making glass statues of Elvis has a one time startup cost of $1000, after which the cost is $4 per statue. A market survey shows that you can expect to sell $\frac{100}{p-4} + 20(50-p)$ statues if you charge p dollars each. At what price should you sell the statues to maximize your expected profit?

a) between $20 and $40 b) between $40.01 and $50

c) between $50.01 and $60 d) between $60.01 and $70

e) between $70.01 and $200

13. $\int \sec(x)\tan(x)dx =$

a) $\sec^2(x) + C$ b) $\tan(x) + C$ c) $\sec(x)$

d) $\tan^2(x) + C$ e) $\sec(x) + C$

14. $\int_0^1 \sqrt{x^3}\,dx =$

a) 2/5 b) ½ c) 5/2 d) 3/2 + C e) 2/5 + C

15. Estimate the value of $\int_0^1 \sin(x)dx$ using a Riemann sum with four subintervals on which the height of the curve is measured at the midpoint. What is the percent error?

a) between 0% and 1% b) between 1% and 2% c) between 2% and 3%

d) between 3% and 4% e) between 4% and 5%

16. $\lim_{z\to -2} \frac{z^3+8}{z+2} =$

a) 0 b) 4 c) 10/0 d) 12 e) undefined

17. $\lim_{\theta \to \pi} \frac{\sin(\theta)}{\theta} =$

a) 0 b) 1 c) $\pi/2$ d) $2/\pi$ e) undefined

18. At what point is the curve $y = x^2 - 2x + 1$ tangent to the line $y = 4x - 8$?

a) (4, 9) b) (4, 16) c) (3, 4) d) (2, 9) e) (0, 1)

19. Which of the following curves has vertical asymptotes at x = 2 and x = 3 ?

a) $y = \frac{x-2}{(x-3)(x-2)}$ b) $y = \frac{x+2}{(x+3)(x+2)}$ c) $y = \frac{x-3}{(x-3)(x-2)}$

d) $y = \frac{x+2}{(x-3)(x-2)}$ e) $y = \frac{x-2}{(x+3)(x+2)}$

20. Use the first two terms of the Maclaurin series for sin(x) to estimate the value for sin(2). What is the percent error of this estimate?

a) less than 1% b) between 1.1% and 5% c) between 5.1% and 10%

d) between 10.1% and 20% e) more than 20%

21. The derivative of $y = x\sin(x)$ is

a) cos(x) b) sin(x) + cox(x) c) $\sin^2(x) + \cos^2(x)$

d) x cos(x) + x sin(x) e) x cos(x) + sin(x)

22. The derivative of $y = \sin(\sin(x))$ is

a) $\cos(\cos(x))$ b) $\cos(\sin(x))$ c) $\sin(\cos(x))$

d) $\cos(\sin(x)) + \sin(\cos(x))$ e) $\cos(\sin(x))(\cos(x))$

23. The derivative of $y = \frac{e^x}{\ln(x)}$ is

a) $\frac{e^x}{1/x}$ b) $\frac{e^x \ln(x) - \frac{e^x}{x}}{(\ln(x))^2}$ c) $\frac{e^x(\ln(x) - x^{-1})}{\ln(x^2)}$

d) $\frac{e^x(\frac{1}{x} - \ln(x))}{(\ln(x))^2}$ e) xe^x

24. An object is thrown upward from the ground with an initial velocity of 10 m/sec. How long before it hits the ground? Approximate g by 10 m/sec^2.

a) .5 sec b) 1 sec c) 1.5 sec d) 1.78 sec e) 2 sec

25. An object is thrown upward with an initial velocity of 3 m/sec from an initial height of 7 m. How fast will it be moving when it hits the bottom of a 4 m deep hole directly under it? Use g = 9.8 m/sec^2.

a) about 15 m/sec b) about 15 meters c) about 1.8 m/sec

d) about 1.8 m e) about 1.8 sec

26. Find the highest point on the graph e^{-x^2}.

a) (e, 0.0006) b) (e, 6000) c) (e, 1)

d) (0, 1) e) (1, 0)

27. $\int x\sin(x)dx =$

a) $\cos(x) + C$ b) $\frac{1}{2}x^2\cos(x)$ c) $-\frac{1}{2}x^2\cos(x)$

d) $-\frac{1}{2}x^2\cos(x) + C$ e) $\sin(x) - x\cos(x) + C$

28. Use the method of partial fractions to express $\frac{x+1}{(x-3)(x-2)}$ as the sum of two fractions with linear denominators. The numerators will be

a) 3 and 4 b) 3 and – 4 c) ½ and – ½

d) – 3 and 4 e) – 3 and – 4

29. Find $\int_0^{\infty} e^{-x}\,dx$

a) *e* b) 1 c) 1/*e* d) –1 e) undefined

30. What is the longest ladder than can be carried horizontally around a right angle corner between a corridor three feet wide and a corridor four feet wide? Neglect the width of the ladder. Answer to the nearest tenth of a foot.

a) 4.5 feet b) 5.4 feet c) 9.5 feet d) 9.9 feet e) 10 feet

31. Which of the following are true? Assume all limits exist.

#1: $\lim_{x\to a}[f(x)+g(x)] = [\lim_{x\to a} f(x)] + [\lim_{x\to a} g(x)]$

#2: $\lim_{x\to a}[f(x)\times g(x)] = [\lim_{x\to a} f(x)] \times [\lim_{x\to a} g(x)]$

#3: $\lim_{x\to a} f(g(x)) = [\lim_{x\to a} f(g(x))] \times [\lim_{x\to a} g'(x)]$

a) only #1 b) only #2 c) only #3

d) only #1 and #2 e) only #1 and #3

32. $\lim_{x \to a} f(x) = L$ if and only if $\forall \varepsilon > 0 \quad \exists \delta > 0$ such that

a) If x is not equal to 0, and x > a then f(x) > L.

b) If x is not equal to 0, and the distance from x to delta is greater than 0, then the distance from delta to L is less than f(x).

c) If x is not equal to 0, and the distance from x to a is greater than delta, then the distance from f(x) to L is greater than epsilon.

d) If x is not equal to a, and the distance from x to a is less than epsilon, then the distance from f(x) to L is less than delta.

e) If x is not equal to a, and the distance from x to a is less than delta, then the distance from f(x) to L is less than epsilon.

33. The line tangent to the curve $y = x^2$ at (3, 9) is also tangent to

a) $y - 12 = 6(x-2)^2$ b) $y - 12 = (x+2)^2$ c) $y - 12 = (x-2)^2$

d) $y - 12 = (x-6)^2$ e) $y - 12 = 6(x-3)^2$

34. The derivative of $y = x^{(x^x)}$ with respect to x is

a) $[x^{(x^x)}(\ln(x^{(x^x)}))][1 + \ln(x) + \frac{1}{\ln(x)}]$ b) $[x^{(x^x)}(\ln(x^{(x^x)}))][\ln(x) + \frac{1}{x\ln(x)}]$

c) $[x^{(x^x)}(\ln(x^{(x^x)}))][1 + \ln(x) + \frac{1}{x\ln(x)}]$ d) $[x^{(x^x)}(\ln(x))][1 + \ln(x) + \frac{1}{x\ln(x)}]$

e) $[x(\ln(x^{(x^x)}))][1 + \ln(x) + \frac{1}{x\ln(x)}]$

35. Estimate $\lim_{x \to 0} \frac{\sin(x)}{x}$ using $x = 0.01$. The percent error is

a) 0.17% b) 0.017% c) 0.0017% d) 0.00017% e) 0.000017%

36. Given that $\ln(xe^y) = e^{x\ln(y)}$ find $\frac{dy}{dx}$. Assume x and y are both positive.

a) $\frac{y^x \ln(y^x) - x}{x - x^2 y^x}$ b) $\frac{y^x \ln(y^x) - 1}{x - x^2 y^{x-1}}$ c) $\frac{y^x \ln(y^x) - 1}{1 - xy^{x-1}}$

d) $\frac{y^x \ln(y^x) - 1}{1 - x^2 y^{x-1}}$ e) $\frac{y^x \ln(y^x) - x}{x - x^2 y^{x-1}}$

37. First three terms of the Maclaurin series for sin(cos(x)) are

a) $\sin(1) + 0\,x - \frac{1}{2}\cos(1)\,x^2$ b) $\sin(1) - \cos(1)\,x + x^2$

c) $\sin(1) - \cos(1)\,x + \frac{1}{2}\,x^2$ d) $\sin(1) + 0\,x + \sin^3(1)\,x^2$

e) $\sin(1)\,x - \sin^2(2)\,x^2/2! + \sin^3(3)\,x^3/3!$

38. Represent polynomials as infinite dimensional column vectors of their coefficients, in which all but finitely many components are zero. For example $3 + 2x + x^2$ is $(3, 2, 1, 0, \ldots)^T$. Then the derivative operator can be represented by an infinite matrix. The first three entries in the first row of this matrix are

a) 0 1 2 b) 1 2 3 c) 0 2 0 d) 1 0 0 e) 0 1 0

39. The graph of cosh(x) – sinh(x) is always

a) increasing b) decreasing c) negative

d) concave down e) zero

40. Use the first two non-zero terms of the Maclaurin series for sin(x) to estimate the value of $\sin(i)$ where $i = \sqrt{-1}$.

a) $1.16667\,i$ b) 1.16667 c) $-1.16667\,i$

d) $0.833333\,i$ e) 0.833333

Extra problems

A. The derivative $f'(t)$ represents

a) the rate of change b) the area under a curve c) the volume of a solid

d) the distance to the x-axis e) the distance to the y-axis

B. All but one of the following symbols is sometimes used for the derivative. Which symbol is not used for the derivative?

a) y' b) $\otimes$ c) $\frac{dy}{dx}$ d) $f'(x)$ e) D_x

C. The integral sign was originally

a) the symbol of a snake b) a caduceus c) the Greek letter sigma

d) the symbol for the element mercury e) an elongated S

Answer key:

1 c 2 b 3 e 4 b 5 a 6 d 7 d 8 a 9 d 10 c

11 c 12 a 13 e 14 a 15 a 16 d 17 a 18 c 19 d 20 e

21 e 22 e 23 b 24 e 25 a 26 d 27 e 28 d 29 b 30 d

31 d 32 e 33 c 34 c 35 c 36 b 37 a 38 e 39 b 40 a

A a B b C e