TENNESSEE MATHEMATICS TEACHERS ASSOCIATION

SIXTY-SEVENTH ANNUAL MATHEMATICS CONTEST

2025

Calculus

Scoring Formula: $4 \times$ (Number Right) $-$ (Number Wrong) $+ 40$

Directions:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. What is the value of the following limit?

$$\lim_{x \to 0} \frac{\sin(3x)}{4x}$$

(a) $\frac{3}{4}$
(b) 0
(c) $\frac{4}{3}$
(d) ∞
(e) 1

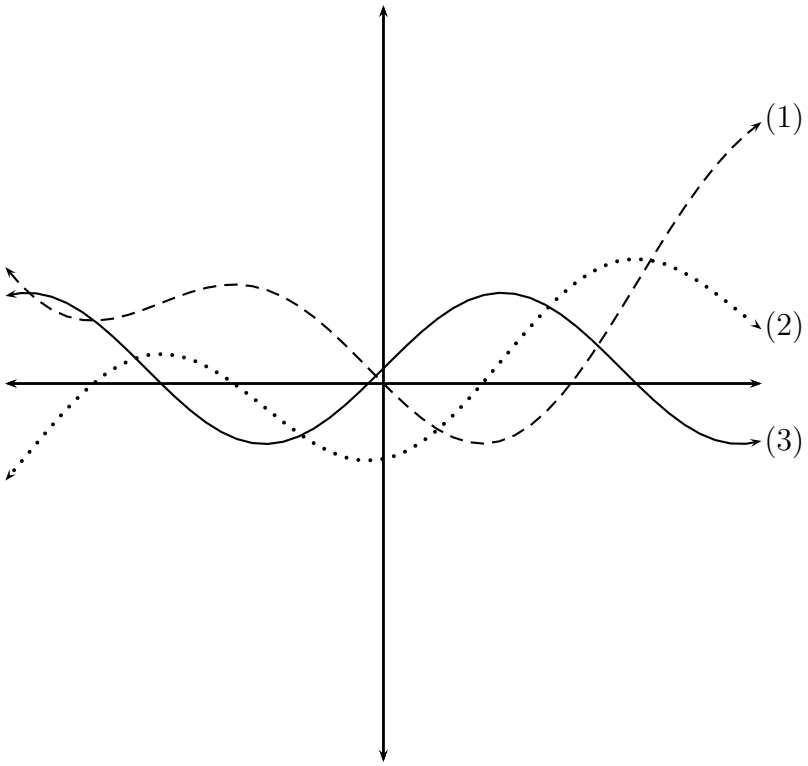
2. What is the equation of the tangent line to the curve $y = 3x^2 - x$ at the point $(1, 2)$?

(a) $y = x + 1$
(b) $y = 5x - 3$
(c) $y = 4x - 2$
(d) $y = 6x - 4$
(e) $y = 3x - 1$

3. Let $f(x) = \int_0^{x^2} e^{-t}\, dt$. What is $f'(x)$?

(a) $-2xe^{-x^2}$
(b) $2xe^{-x^2}$
(c) $2e^{-x^2}$
(d) e^{-x^2}
(e) $2xe^{x^2}$

4. The following set of axis has the graphs of the functions $f(x)$, $f'(x)$, and $f''(x)$. Which curves correspond to which functions?



(a) $f(x)$ is curve (1), $f'(x)$ is curve (2), $f''(x)$ is curve (3).
(b) $f(x)$ is curve (3), $f'(x)$ is curve (1), $f''(x)$ is curve (2).
(c) $f(x)$ is curve (2), $f'(x)$ is curve (3), $f''(x)$ is curve (1).
(d) $f(x)$ is curve (2), $f'(x)$ is curve (1), $f''(x)$ is curve (3).
(e) $f(x)$ is curve (1), $f'(x)$ is curve (3), $f''(x)$ is curve (2).

5. Which of the following functions is continuous but not differentiable at $x = 0$?

(a) $y = \dfrac{x}{x^2 + 1}$
(b) $y = \dfrac{1}{x}$
(c) $y = \sqrt[3]{x}$
(d) $y = \ln(x^2 + 1)$
(e) $y = \dfrac{|x|}{x}$

6. What is the average value of the function $f(x) = \dfrac{1}{1+x^2}$ on the interval $[-1, 1]$?

(a) $\dfrac{\pi}{3}$
(b) $\dfrac{\pi}{4}$
(c) $\dfrac{\pi}{6}$
(d) 0
(e) $\dfrac{\pi}{2}$

7. Which of the following is NOT an antiderivative of $\sin x \cos x$?

(a) $-\dfrac{1}{2}\cos(2x)$
(b) $\dfrac{1}{4}(\sin^2 x - \cos^2 x)$
(c) $\dfrac{1}{2}\sin^2 x$
(d) $-\dfrac{1}{4}\cos(2x)$
(e) $-\dfrac{1}{2}\cos^2 x$

8. What is the area of the region bounded between the curve $y = 4 - 2^x$, the x-axis, and the y-axis?

(a) $\dfrac{17\pi}{9}$
(b) 5
(c) $8 - 3\ln 2$
(d) $8 - \ln 2$
(e) $8 - \dfrac{3}{\ln 2}$

9. Let $y = e^x \cos x$. What is y'?

(a) $-e^x \sin x$
(b) $-xe^x \sin x$
(c) $e^x(\cos x + \sin x)$
(d) $e^x \sin x$
(e) $e^x(\cos x - \sin x)$

10. What are all critical numbers of the function $f(x) = \dfrac{1}{x^2 - 3x - 4}$?

(a) 1.5
(b) -1 and 4
(c) -4 and 1
(d) -4, 1.5, and 1
(e) -1, 1.5, and 4

11. The velocity of a particle moving along the number line is given by $v(t) = 12 - 3t^2$. What is the total distance traveled by the particle between $t = 0$ and $t = 4$ seconds?

(a) 60 units
(b) 54 units
(c) 16 units
(d) 32 units
(e) 48 units

12. What is the value of the following limit?

$$\lim_{x \to \infty} \tan^{-1} x$$

(a) $\dfrac{\pi}{4}$
(b) 1
(c) ∞
(d) $\dfrac{\pi}{3}$
(e) $\dfrac{\pi}{2}$

13. Let $f(x)$ be a differentiable function where $f'(1) = 5$ and $f(1) = -2$. If $g(x) = \dfrac{1}{f(x)}$, what is the value of $g'(1)$?

(a) 2.5
(b) -1.25
(c) 0.2
(d) -0.2
(e) 1.25

14. Consider the polynomial function $f(x) = x^3 + cx^2 + cx - 2$. For what values of c does the function f have a local minimum?

(a) all real numbers c
(b) $c < 0$ or $c > 3$
(c) $0 < c < 3$
(d) $c \leq 1$ or $c \geq 2$
(e) $1 \leq c \leq 2$

15. What is the value of the following sum?

$$\sum_{n=0}^{100} (-1)^n \binom{100}{n}$$

(a) 2^{100}
(b) 0
(c) -2^{50}
(d) -2^{100}
(e) 2^{50}

16. Consider the equation of the unit circle: $x^2 + y^2 = 1$. What is $\dfrac{dy}{dx}$?

(a) $\dfrac{dy}{dx} = -1$
(b) $\dfrac{dy}{dx} = \dfrac{y}{x}$
(c) $\dfrac{dy}{dx} = -\dfrac{x}{y}$
(d) $\dfrac{dy}{dx} = -\dfrac{y}{x}$
(e) $\dfrac{dy}{dx} = \dfrac{x}{y}$

17. Let $f(x) = (x^5 + x^2 + 1)^4$. What is $f'(x)$?

(a) $f'(x) = 4(x^5 + x^2 + 1)^3$
(b) $f'(x) = (5x^4 + 2x)^4$
(c) $f'(x) = 4(x^5 + x^2 + 1)^3(5x^4 + 2x)$
(d) $f'(x) = 4(5x^4 + 2x)^3$
(e) $f'(x) = 4(5x^4 + 2x + 1)^3$

18. What is the value of the following limit?

$$\lim_{n\to\infty}\left(\frac{n-1}{n}\right)^n$$

(a) 0
(b) 1
(c) e
(d) $\dfrac{1}{\sqrt{e}}$
(e) $\dfrac{1}{e}$

19. Let $y = \sqrt[4]{x}$. Using the tangent line to the curve at the point $(81, 3)$ to approximate, what is the approximate value of $\sqrt[4]{80}$? Round to the nearest hundred thousandth.

(a) 2.99972
(b) 2.99068
(c) 2.99066
(d) 2.99074
(e) 2.99070

20. Let f be a continuous function on $(-\infty, \infty)$. If $\displaystyle\int_1^3 f(x)\,dx = 5$, $\displaystyle\int_3^7 f(x)\,dx = -3$, and $\displaystyle\int_5^7 f(x)\,dx = 2$, what is $\displaystyle\int_1^5 f(x)\,dx$?

(a) 7
(b) 10
(c) 4
(d) 2
(e) 0

21. Consider the region bounded between the curves $y = \sqrt{x}$ and $y = x^2$. What is the volume of the solid generated by revolving the region about the line $y = 1$?

(a) $\dfrac{11\pi}{30}$
(b) $\dfrac{4\pi}{15}$
(c) $\dfrac{3\pi}{10}$
(d) $\dfrac{\pi}{3}$
(e) $\dfrac{2\pi}{5}$

22. What is the approximate value of $\displaystyle\int_0^4 \sqrt{x}\,dx$ using the midpoints of 4 subintervals? Round to the nearest thousandth.

(a) 5.383
(b) 6.146
(c) 5.146
(d) 4.146
(e) 5.252

23. How many the inflection points does the curve $y = \dfrac{1}{1+e^{-x}}$ have?.

(a) 4
(b) 0
(c) 1
(d) 3
(e) 2

24. Let $f(x) = \dfrac{1}{2}(e^x + e^{-x})$ and $g(x) = \dfrac{1}{2}(e^x - e^{-x})$. Which of the following is equivalent to the expression $\big(f(x)\big)^2 - \big(g(x)\big)^2$?

(a) $e^{2x} + e^{-2x}$
(b) $f(2x)$
(c) $g(2x)$
(d) 1
(e) 0

25. Let $f(x) = \dfrac{e^x + e^{-x}}{2}$ where $x > 0$. What is the inverse function $f^{-1}(x)$?

(a) $\ln(x + \sqrt{x^2 - 1})$

(b) $\ln(\sqrt{x^2 + 1} + x)$

(c) $\ln(x - \sqrt{x^2 - 1})$

(d) $\dfrac{2}{e^x + e^{-x}}$

(e) $\ln(\sqrt{x^2 + 1} - x)$

26. Let $f(x) = 2^x - x$. At what value for x does f attain an absolute minimum?

(a) $x = \dfrac{1 - \ln 2}{2}$

(b) $x = \dfrac{1 + \ln 2}{3}$

(c) $x = 2 - 2\ln 2$

(d) $x = -\dfrac{\ln(\ln 2)}{\ln 2}$

(e) $x = \ln 2$

27. A particle moving along a number line has acceleration $a(t) = \sqrt{t}$ at t seconds for $t \geq 0$. If the particle begins 0 and has an initial velocity of 5 units per second in the negative direction, what is the particle's position at 9 seconds?

(a) $\dfrac{49}{5}$

(b) -45

(c) $\dfrac{99}{5}$

(d) $-\dfrac{99}{5}$

(e) $\dfrac{549}{5}$

28. Let $y = \tan(x)$. What is y''?

(a) $\sec^2 x \tan^2 x$

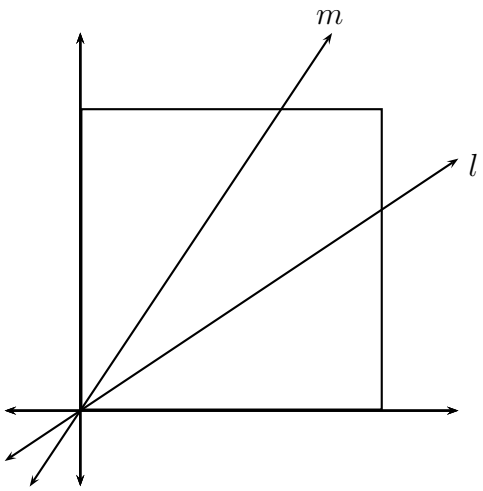
(b) $2 \sec x \tan^2 x$

(c) $2 \sec^2 x \tan x$

(d) $2 \sec x$

(e) $\sec^2 x$

29. In the picture below, lines l and m divide the square into three regions with equal area. What is the slope of line l?



(a) $\frac{2}{3}$

(b) $\frac{5}{9}$

(c) $\frac{3}{4}$

(d) $\frac{3}{5}$

(e) $\frac{4}{7}$

30. Let n be a positive integer, how many inflection points does the graph of $y = \sin(nx)$ have on the interval $(0, \pi)$?

(a) $n + 2$

(b) n

(c) $n - 1$

(d) $n - 2$

(e) $n + 1$

31. Where is the function $f(x) = \dfrac{x}{x^2+4}$ increasing?

(a) $(2, \infty)$
(b) $(0, \infty)$
(c) $(-2, 2)$
(d) $(-1, 1)$
(e) $(-\infty, \infty)$

32. What is the area under the curve $y = \dfrac{1}{\sqrt{1-x^2}}$ and above the x-axis where $0 \leq x \leq \dfrac{1}{2}$?

(a) $\dfrac{\pi}{4}$
(b) $\dfrac{\sqrt{2}}{2}$
(c) $\dfrac{\pi}{6}$
(d) $\sqrt{2} - 1$
(e) $\dfrac{\pi}{3}$

33. What is the coefficient of x^2y^{12} in the expansion of $(2x - y)^{14}$?

(a) 2916
(b) 91
(c) 182
(d) 364
(e) 28

34. What is the general antiderivative of $\dfrac{x}{x^2+4}$?

(a) $\ln(x^2 + 4) + C$
(b) $\ln|x + 4| + C$
(c) $\ln(\sqrt{x^2 + 4}) + C$
(d) $\dfrac{3x^2}{2x^3 + 24x} + C$
(e) $\dfrac{1}{2}x^2 \ln(x^2 + 4) + C$

35. What is the value of $\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5+\cdots}}}}$?

(a) $5-\dfrac{\sqrt{30}}{5}$

(b) $\dfrac{2+\sqrt{13}}{2}$

(c) $\dfrac{1+\sqrt{21}}{2}$

(d) $\dfrac{14}{5}$

(e) $\dfrac{8\pi}{9}$

36. What point on the line with equation $y=4x+3$ is closest to the point $(6,2)$?

(a) $(0,3)$

(b) $\left(\dfrac{1}{10},\dfrac{17}{5}\right)$

(c) $\left(\dfrac{2}{17},\dfrac{59}{17}\right)$

(d) $\left(-\dfrac{1}{4},2\right)$

(e) $\left(\dfrac{1}{9},\dfrac{31}{9}\right)$

37. A torus is formed by rotating region bounded by the circle $(x-a)^2+y^2=1$, where $a>1$, about the y-axis. What is the volume of the torus in terms of a?

(a) $4\pi a^2$

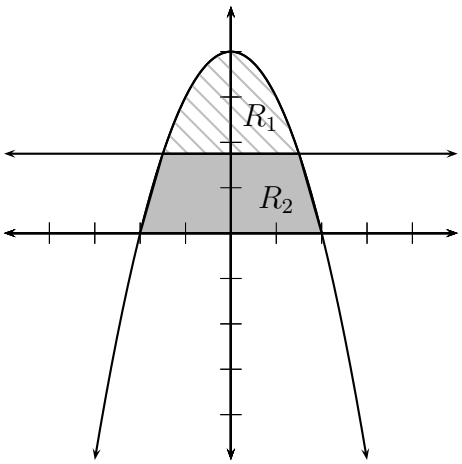
(b) πa^2

(c) $4\pi a$

(d) $2\pi a^2$

(e) $2\pi a$

38. The picture below depicts the curve $y = 4 - x^2$ and horizontal line $y = k$. For what value of k do the regions R_1 and R_2 have equal area?



(a) $k = 2 - \dfrac{\sqrt{2}}{2}$

(b) $k = \sqrt{2}$

(c) $k = \sqrt[3]{4}$

(d) $k = \sqrt[4]{8}$

(e) $k = 4 - 2\sqrt[3]{2}$

39. What is the value of the following limit?

$$\lim_{n\to\infty} \frac{\pi}{3n} \sum_{k=1}^{n} \sin\left(\frac{k\pi}{3n}\right)$$

(a) $\dfrac{\sqrt{2}}{2}$

(b) $\dfrac{\sqrt{3}}{2}$

(c) $\dfrac{1}{2}$

(d) 1

(e) 0

40. Let line l be the tangent line to the curve $y = x^3$ at the point $(1, 1)$ and line m be the tangent line to the curve $y = x^5$ at the point $(1, 1)$. To the nearest degree, what is the measure of the acute angle formed by the intersection of lines l and m at $(1, 1)$?

(a) $8°$

(b) $4°$

(c) $7°$

(d) $5°$

(e) $6°$