Calculus and Advanced Topics 2013

1. Let $\lim_{x\to 10} f(x) = 4$ and $\lim_{x\to 10} g(x) = -6$. Find $\lim_{x\to 10}[f(x)\cdot g(x)]$.
   a. -2
   b. -6
   c. 10
   d. 24
   e. -24

2. Decide if the limit exists. If the limit exists, find its value.
$$\lim_{x\to 0}\frac{\frac{1}{x+7}-\frac{1}{7}}{x}$$
   a. 0
   b. $\frac{1}{49}$
   c. Does not exist
   d. $-\frac{1}{49}$
   e. 1

3. Find the equation of the secant line through the points where x has the given values.
$$f(x) = 3\sqrt{x}\,;\; x = 9\,, x = 25$$
   a. $y = -\frac{3}{8}x + \frac{45}{8}$
   b. $y = \frac{3}{8}x - \frac{45}{8}$
   c. $y = \frac{3}{2\sqrt{x}}$
   d. $y = -\frac{8}{3}x + 33$
   e. $y = \frac{3}{8}x + \frac{45}{8}$

4. Find the equation of the tangent line to the curve when x has the given value.
$$f(x) = \frac{18}{x}\;\;;\;\; x = 3$$
   a. $y = -2x + 6$
   b. $y = -2x$
   c. $y = -2x + 12$
   d. $y = -4x + 18$
   e. $y = \frac{1}{2}x - \frac{9}{2}$

5. Find $f'(x)$ when x has the given value.

$$f(x) = 2x^2 - 3x\,;\, x = 13$$

a. $52x - 3$
b. 4
c. 55
d. 1211.1667
e. 49

6. The profit from the expenditure of x thousand dollars on advertising is given by $P(x) = 950 + 25x - 3x^2$. Find the marginal profit when the expenditure is $x = 9$.
a. 950 thousand dollars
b. 171 thousand dollars
c. -29 thousand dollars
d. 225 thousand dollars
e. -79 thousand dollars

7. If the price of a product is given by $P(x) = \frac{1024}{x} + 2000$, where x represents the demand for the product, find the rate of change of price when the demand is 8.
a. 4
b. 128
c. -128
d. 16
e. -16

8. Use the product rule to find the derivative of $f(y) = (y^{-2} + y^{-1})(3y^{-3} - 7y^{-4})$
a. $f'(y) = \frac{42+16y-3y^2}{y^7}$
b. $f'(y) = \frac{42-20y+12y^2}{y^7}$
c. $f'(y) = \frac{42+20y-12y^2}{y^7}$
d. $f'(y) = \frac{42+50y-12y^2}{y^7}$
e. $f'(y) = \frac{42+20y+12y^2}{y^7}$

9. Use the quotient rule to find the derivative of $f(x) = \frac{x^{1.7}+3}{x^{3.1}+1}$

   a. $f'(x) = \frac{-1.4x^{3.8}+1.7x^{0.7}-9.3x^{2.1}}{(x^{3.1}+1)^2}$

   b. $f'(x) = \frac{-1.4x^{3.8}-1.4x^{0.7}-9.3x^{2.1}-9.3}{(x^{3.1}+1)^2}$

   c. $f'(x) = \frac{-1.4x^{3.8}+1.7x^{0.7}-3.1x^{1.7}-9.3x^{2.1}-9.3}{x^{3.1}+1}$

   d. $f'(x) = \frac{-1.4x^{3.8}+1.7x^{0.7}-3.1x^{1.7}+3x^{2.1}-9.3}{(x^{3.1}+1)^2}$

   e. $f'(x) = \frac{-1.4x^{3.8}+1.7x^{0.7}+9.3x^{2.1}}{(x^{3.1}+1)^2}$

10. At what points on the graph of $f(x) = 2x^3 - 9x^2 - 27x$ is the slope of the tangent line -3?
    a. (1,-34), (16,-164)
    b. (-1,16), (4,-124)
    c. (-1,16), (1,-34)
    d. (0,0), (4,-144)
    e. (1, -34) (-4, -164)

11. The formula $E = 1000(100 - T) + 580(100 - T)^2$ is used to approximate the elevation (in meters) above sea level at which water boils at a temperature of *T* (in degrees Celsius). Find the rate of change of *E* with respect to *T* for a temperature of 75°C.
    a. 30,000 m/°C
    b. -73,500 m/°C
    c. -30,000 m/°C
    d. -29,000 m/°C
    e. 28,000 m/°C

12. The median weight, w (in pounds), of a girl between the ages of 0 and 36 months can be approximated by the function $w(t) = 0.0006t^3 - 0.048t^2 + 1.61t + 7.60$, where *t* is measured in months. For a girl of median weight, find the rate of change of weight with respect to time at age 20 months.
    a. 0.086 lb/mo
    b. 0.410 lb/mo
    c. 1.362 lb/mo
    d. 0.882 lb/mo
    e. Cannot be determined since w(t) is not differentiable at t = 20 months

13. When a particular circuit containing a resistor, an inductor, and a capacitor in series is connected to a battery, the current *i* (in amperes) is given by $i = 24e^{-3t}(e^{2.6t} - e^{-2.6t})$ where *t* is the time (in seconds). Find the time at which the maximum current occurs. Round to the nearest tenth of a second.
    a. 0.6 sec
    b. 1.4 sec
    c. 1.5 sec
    d. 0.5 sec
    e. There is no absolute maximum value.

14. Using a graphing calculator, find the values of x for which $f'(x) = 0$, to three decimal places.

$$f(x) = \frac{x^2 - 5}{x^4 + 4}$$

    a. -3.162, 3.162
    b. 0
    c. 0, -3.223, 3.223
    d. 0, -3.096, -0.646, 0.646, 3.096
    e. There are not real values of x for which $f'(x) = 0$

15. Find the derivative of $y = (2x^2 + 2)\ln(x + 8)$
    a. $4x \ln(x + 8)$
    b. $\frac{4x}{x+8}$
    c. $\frac{2x^2+2}{\ln(x+8)} + 4x\ln(x + 8)$
    d. $\frac{2x^2+2}{x+8} - 4x\ln(x + 8)$
    e. $\frac{2x^2+2}{x+8} + 4x\ln(x + 8)$

16. Students in a math class took a final exam. They took equivalent forms of the exam in monthly intervals thereafter. The average score $S(t)$, in percent, after *t* months was found to be given by $S(t) = 78 - 17\ln(t + 1), t \geq 0$. Find $S'(t)$.
    a. $S'(t) = \frac{17}{t+1}$
    b. $S'(t) = 78 - \frac{17}{t+1}$
    c. $S'(t) = -\frac{17}{t+1}$
    d. $S'(t) = -17\ln\left(\frac{1}{t+1}\right)$
    e. $S'(t) = -\frac{17}{(ln10)(t+1)}$

17. During a one-hour race, the velocities of two cars are $v_1(t) = 52(1 - \cos(\pi t))$ and $v_2(t) = 99t$, where $0 \leq t \leq 1$. If, at the beginning of the race, both cars were at the mile 0 mark, which car travelled the farthest during the hour?
    a. Car 2
    b. Car 1
    c. It was a tie.
    d. The winner cannot be determined because neither $v_1(t)$ nor $v_2(t)$ is differentiable on the interval (0,1).
    e. The winner cannot be determined because neither $v_1(t)$ nor $v_2(t)$ is integrable on the interval [0,1].

18. Find the integral. $\int \sin x \cos^7 x \ dx$
    a. $7 \sin^7 x + C$
    b. $-7 \cos^7 x + C$
    c. $\frac{1}{8} \sin^8 x + C$
    d. $-\frac{1}{8} \cos^8 x + C$
    e. $\frac{1}{8} cos^8 x + C$

19. Find the derivative of the function $y = x^3 \cos 3x^2$
    a. $\frac{dy}{dx} = -6x^4 \sin 3x^2 + 3x^2 \cos 3x^2$
    b. $\frac{dy}{dx} = 6x^4 \sin 3x^2 + 3x^2 \cos 3x^2$
    c. $\frac{dy}{dx} = \sin 3x^2 + 3x^2 \cos 3x^2$
    d. $\frac{dy}{dx} = -6x^4 \sin 3x^2$
    e. $\frac{dy}{dx} = -x^3 \sin 3x^2 + 3x^2 \cos 3x^2$

20. Use n = 4 to approximate the value of the integral $\int_1^5 9x\sqrt{2x-1}\, dx$ by Simpson's rule.
    a. 253.2
    b. 256.8
    c. 28.5
    d. 236.7
    e. 258.8

21. The number of mosquitoes in a lake area after an insecticide spraying decreases at a rate of $M'(t) = -2800e^{-0.2t}$ mosquitoes per hour. If there were 14,000 mosquitoes initially, how many will there be after 4 hours?

a. $M(4) \approx 12{,}742$ mosquitoes
b. $M(4) \approx 31{,}158$ mosquitoes
c. $M(4) \approx 14{,}000$ mosquitoes
d. $M(4) \approx 764{,}374$ mosquitoes
e. $M(4) \approx 6291$ mosquitoes

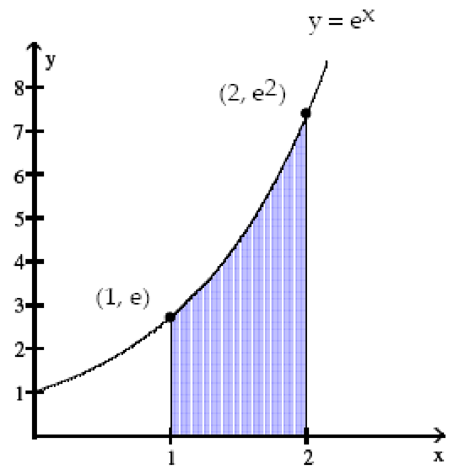
22. Find the equation of the tangent line at $x = 1$ on the curve $\frac{y}{4}(1-x) + x\sqrt{y} + 2x = 5$.

a. $y = -\frac{33}{2}x + 8$
b. $y = -\frac{33}{8}x + \frac{105}{8}$
c. $y = -\frac{33}{2}x + \frac{51}{2}$
d. $y = -\frac{11}{6}x + \frac{65}{6}$
e. $y = \frac{33}{2}x - 9$

23. Suppose that the acceleration of an object is given by $a(t) = 5t^{2/3} + 2e^{-t}$. The object's initial velocity, $v(0)$, is 12 and the object's initial position, $s(0)$, is -3. Find $s(t)$.

a. $s(t) = \frac{200t^{8/3}}{9} + 2e^{-t} + 10t - 5$
b. $s(t) = 3t^{5/3} - 2e^{-t} + 14$
c. $s(t) = \frac{9t^{8/3}}{8} - 2e^{-t} + 14t - 3$
d. $s(t) = \frac{9t^{8/3}}{8} + 2e^{-t} + 12t - 3$
e. $s(t) = \frac{9t^{8/3}}{8} + 2e^{-t} + 14t - 5$

24. Find the area of the shaded region.



a. $e^2 - 1$
b. $e^2 + e - 1$
c. $e^2 - e + 1$
d. $e^2 - e$
e. $e^2 + e$

25. The percent of concentration of a certain drug in the bloodstream *x* hours after the drug is administered is given by $K(x) = \frac{5x}{x^2+9}$. At what time is the concentration the maximum?

a. 3 hr
b. 0.9 hr
c. 0.5 hr
d. 0.8 hr
e. 5 hr

26. Find the derivative of $y = \frac{\ln(5x+5)}{e^{5x+5}}$

a. $\frac{1-5[\ln(5x+5)]^2}{\ln[5x+5]e^{(5x+5)}}$
b. $\frac{-5}{(x+1)e^{5x+5}}$
c. $\frac{1}{(5x+5)e^{(5x+5)}}$
d. $\frac{5-(25x+25)\ln(5x+5)}{(5x+5)e^{(5x+5)}}$
e. $\frac{1-(5x+5)\ln(5x+5)}{(5x+5)e^{(5x+5)}}$

27. Find the values of x and y that maximize $Q = xy^2$, where x and y are positive numbers such that $x + y^2 = 7$.

   a. $x = 0, y = \sqrt{7}$
   b. $x = \sqrt{\frac{7}{2}}, y = \frac{7}{2}$
   c. $x = \frac{7}{2}, y = \sqrt{\frac{7}{2}}$
   d. $x = 1, y = \sqrt{6}$
   e. $x = \frac{7}{2}, y = \sqrt{\frac{21}{2}}$

28. Evaluate the definite integral. $\int_4^9 \frac{t^2+1}{\sqrt{t}}\,dt$

   a. $\frac{472}{5}$
   b. $\frac{432}{5}$
   c. 212
   d. $\frac{516}{5}$
   e. $\frac{447}{5}$

29. The pH scale is used by chemists to measure the acidity of a solution. It is base 10 logarithmic scale. The pH, P, of a solution and its hydronium ion concentration in moles per liter, H are related as follows:

$$H = 10^{-P}$$

   Find the formula for the rate of change $\frac{dH}{dP}$.

   a. $\frac{dH}{dP} = (\ln 10)10^{-P}$
   b. $\frac{dH}{dP} = -(\ln 10)10^{-P}$
   c. $\frac{dH}{dP} = -P(10^{-P-1})$
   d. $\frac{dH}{dP} = -\frac{10^{-P}}{\ln 10}$
   e. $\frac{dH}{dP} = -(\ln P)10^{-P}$

30. The correlation between respiratory rate and body mass in the first three years of life can be expressed by the function

$$\log R(w) = 1.87 - 0.35\log w$$

where w is the body weight (in kg) and R(w) is the respiratory rate (in breaths per minute). Find R'(w) .

a. $R'(w) = -25.95w^{-0.35}$
b. $R'(w) = -25.95w^{-1.35}$
c. $R'(w) = 74.13w^{-1.35}$
d. $R'(w) = -25.95w^{-0.65}$
e. $R'(w) = -74.13w^{-1.35}$

31. Find the integral $\int (1-6x)e^{3x-9x^2}dx$
a. $(x-3x^2)e^{3x-9x^2} + C$
b. $\frac{1}{3}e^{3x-9x^2} + C$
c. $3(1-6x)e^{3x-9x^2} + C$
d. $\frac{1}{3}(1-6x)e^{3x-9x^2} + C$
e. $3e^{3x-9x^2} + C$

32. Find two numbers x and y such that their sum is 480 and x^2y is maximized.
a. $x = 360, y = 120$
b. $x = 240, y = 240$
c. $x = 160, y = 320$
d. $x = 320, y = 160$
e. $x = 120, y = 360$

33. Use the properties of limits to help decide whether the limit exists. If the limit exists, find its value.

$$\lim_{x\to\infty} \frac{5x^3 + 2x}{-3x^4 + 3x^3 + 6}$$

a. $-\frac{5}{3}$
b. 1
c. $\frac{5}{18}$
d. 0
e. Does not exist

34. Find the integral $\int \frac{t^4+2}{t^5+10t+9} dt$

a. $\frac{ln|t^5+10t+9|}{5} + C$

b. $\frac{\frac{1}{5}t^5+2t}{\frac{1}{6}t^6+5t^2+9t} + C$

c. $-\frac{1}{5(t^5+10t+9)^2} + C$

d. $-\frac{5}{(t^5+10t+9)^2} + C$

e. $5\ln|t^5 + 10t + 9| + C$

35. Find the derivative of $y = \frac{5}{\sin x} + \frac{1}{\cot x}$

a. $\frac{dy}{dx} = 5\csc x\cot x - \sec^2 x$

b. $\frac{dy}{dx} = 5\csc x\cot x - \csc^2 x$

c. $\frac{dy}{dx} = 5\cos x - \csc^2 x$

d. $\frac{dy}{dx} = -5\csc \mathrm{x}\cot x + \sec^2 x$

e. $\frac{dy}{dx} = \frac{5}{\cos x} - \frac{1}{\cos^2 x}$

36. Evaluate the definite integral $\int_0^1 5x\sqrt[5]{1+x^2}\, dx$

a. $\frac{25}{6}\left(2^{6/5} - 1\right)$

b. $\frac{25}{12}\left(2^{6/5} - 1\right)$

c. $\frac{5}{2}$

d. $\frac{5}{2}\left(2^{6/5} - 1\right)$

e. $\frac{25}{6}\sqrt[5]{2}$

37. Find the integral $\int \frac{\log_7 x}{x} dx$

a. $\frac{(\ln x)(\log_7 x)^2}{2} + C$

b. $\frac{2x+14}{7x^3}$

c. $\frac{(\ln 7)\,(\log_7 x)^2}{2} + C$

d. $\frac{(\log_7 x)^2}{2} + C$

e. $\frac{(\log_7 x)^2}{2\ln 7} + C$

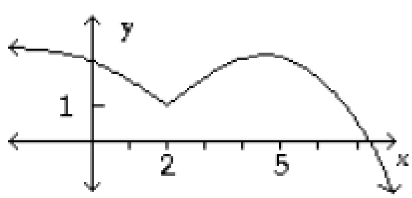
38. The number of ducks (in thousands) counted at a certain checkpoint in their migration is given by $O(t) = 5 + 5\cos\left(\frac{\pi t}{6}\right)$, where t is time in months and t = 0 is October 1. Find the number of ducks passing the checkpoint between October 1and April 1.

   a. 29,000
   b. 29,500
   c. 30,000
   d. 29,965
   e. 35,000

39. Find the integral. $\int \frac{x}{5} \tan\left(\frac{x}{5}\right)^2 dx$

   a. $-\frac{1}{10}x^2 \ln\left|\cos\left(\frac{x}{5}\right)^2\right| + C$
   b. $\frac{5}{2}\ln\left|\sin\left(\frac{x}{5}\right)^2\right| + C$
   c. $\frac{5}{2}\sec^2\left(\frac{x}{5}\right)^2 + C$
   d. $-\frac{5}{2}\ln\left|\cos\left(\frac{x}{5}\right)^2\right| + C$
   e. $\frac{x}{5}\tan\frac{x}{5} + \ln\left|\cos\frac{x}{5}\right| - \frac{x^2}{50} + C$

40. Find the x-values where the function does not have a derivative.



   a. $x = 7.2$
   b. $x = 5$
   c. $x = 2$
   d. $x = 2, x = 5$
   e. Exists at all points