TMTA Calculus and Advanced Topics Test 2010

1. Evaluate $\lim\limits_{x \to -1} \dfrac{x^2 - x - 2}{x + 1}$

    a) Does not exist
    b) -3
    c) -1
    d) 1
    e) 0

2. Find the period of $y = -2A\cos(4Bx + \pi) + 6D$, where $A > 0, B > 0, D > 0$

    a) $2A$
    b) 2π
    c) $6D$
    d) $\dfrac{B}{\pi}$
    e) $\dfrac{\pi}{2B}$

3. Solve the given equation for x: $\ln x^2 = 2\ln 4 - 4\ln 2$
    a) { }
    b) {-1}
    c) {0}
    d) {1}
    e) {1 , -1}

4. Which of the following is <u>NOT</u> equivalent to $\sin\left(60^\circ\right)$

    a) $2\sin\left(30^\circ\right)$

    b) $2\sin\left(30^\circ\right)\cos\left(30^\circ\right)$

    c) $\sin\left(45^\circ\right)\cos\left(15^\circ\right) + \cos\left(45^\circ\right)\sin\left(15^\circ\right)$

    d) $\sin\left(90^\circ\right)\cos\left(30^\circ\right) - \cos\left(90^\circ\right)\sin\left(30^\circ\right)$

    e) $\sqrt{\dfrac{1 - \cos\left(120^\circ\right)}{2}}$

5. Find an equation of the tangent line to the curve $y = 4\sin^2 x$ at the point $\left(\dfrac{\pi}{6}, 1\right)$

a) $y - 1 = 4\sqrt{3}\left(x - \dfrac{\pi}{6}\right)$

b) $y - 1 = 2\sqrt{3}\left(x - \dfrac{\pi}{6}\right)$

c) $y - 1 = 4\left(x - \dfrac{\pi}{6}\right)$

d) $y - 1 = \dfrac{2}{3}\left(x - \dfrac{\pi}{6}\right)$

e) $y - 1 = x - \dfrac{\pi}{6}$

6. Find the area of the region bounded by the given curves $y = x^2$ and $y = 4x - x^2$

a) $\dfrac{10}{3}$

b) $\dfrac{8}{3}$

c) $-\dfrac{10}{3}$

d) $\dfrac{5}{3}$

e) $\dfrac{4}{3}$

7. The graph of $2y^2 = 4x^2 + 4x + 3$ is which type of conic section?
    a) parabola opening up
    b) parabola opening down
    c) circle
    d) ellipse
    e) hyperbola

8. Find the slope of the tangent line to the curve $x^2y + 3x^2 = 12$ at the point (-1, 2)

   a) -3
   b) 3
   c) 0
   d) 10
   e) $-\dfrac{1}{10}$

9. Find $f''(x)$ if $f(x) = x \sin x$
   a) $-x \sin x + 2 \cos x$
   b) $x \cos x + \sin x$
   c) $-x \cos x + \sin x$
   d) $\cos x (2 - x)$
   e) $\cos x (2 + x)$

10. Solve graphically in the interval $[-2\pi, 2\pi]$. Round to the nearest hundredth.
    $$\sin(x - 0.5) = e^x + 0.5$$
    a) $\{\approx -3.21, \approx 0.54\}$
    b) $\{\approx -2.75\}$
    c) $\{\approx -5.78, \approx -2.75\}$
    d) $\{\approx -5.25, \approx -3.21\}$
    e) $\varnothing$

11. In how many ways can 7 toys be divided between 3 children if the youngest gets 3 toys and each of the others get 2 toys?

    a) 2,520
    b) 6
    c) 35
    d) 12
    e) 210

12. Integrate: $\int \dfrac{3x-6}{\sqrt{x^2-4x+11}}\,dx$

a) $3\sqrt{x^2-4x+11}$

b) $\sqrt{x^2-4x+11}$

c) $3\sqrt{x^2-4x+11}+c$

d) $\sqrt{x^2-4x+11}+c$

e) $2\sqrt{x^2-4x+11}+c$

13. Find the values of x such that the vectors $\langle 8,x,5x\rangle$ and $\langle -3,x,2\rangle$ are orthogonal.

a) $x=-12,\ x=2$

b) $x=3,\ x=8$

c) $x=4,\ x=6$

d) $x=-1,\ x=0$

e) $x=-10,\ x=-2$

14. A ladder 25 feet long is leaning against the wall of a building. The base of the ladder is pulled away from the wall at a rate of 4 feet per second. At what rate is the angle between the ladder and the wall changing when the base of the ladder is 15 feet from the wall?

a) $\dfrac{1}{5}$ rad/s

b) $\dfrac{\pi}{6}$ rad/s

c) $\dfrac{2}{3}$ rad/s

d) $\dfrac{\pi}{12}$ rad/s

e) $\dfrac{3}{2}$ rad/s

15. A motorboat leaves a dock and travels 1370 m due west, then turns 24.0° to the south and travels another 1520 m to a second dock. What is the displacement of the second dock from the first dock?

a) 2890 m, 24.0° S of W

b) 2756 m, 18.4° S of W

c) 2827 m, 12.6° S of W

d) 2779 m, 14.1° S of W

e) 2715 m, 21.7° S of W

16. The position function of a particle is given by $\mathbf{r}(t) = \left\langle 4t^2, t, 4t^2 - 48t \right\rangle$. When is the speed a minimum?
    a) $t = 6$
    b) $t = 0$
    c) $t = 1$
    d) $t = 4$
    e) $t = 3$

17. Find the coordinates of the point on the curve $y = x^2 - x$ where there is a tangent line which is perpendicular to the line $y = -\dfrac{1}{3}x + 4$

    a) $(1, 0)$
    b) $\left(\dfrac{2}{3}, -\dfrac{2}{9} \right)$
    c) $\left(\dfrac{1}{3}, -\dfrac{2}{9} \right)$
    d) $(-1, 2)$
    e) $(2, 2)$

18. The velocity function for a particle moving along a line is given by $v(t) = 2t - 3$. Find the distance traveled by the particle during the time interval $0 \le t \le 4$.
    a) 0.5
    b) 4
    c) 6.25
    d) 13
    e) 8.5

19. Find an equation of the tangent plane to the surface $z = 4x^2 - 9xy + 2y^2$ at the point (3, 2, -10).
    a) $6x - 19y + z = 66$
    b) $19x - 6y - z = -10$
    c) $6x - 19y - z = -10$
    d) $19x + 6y + z = 10$
    e) $6x + 19y - z = 66$

20. Find the maximum rate of change of $f(x, y) = \ln(x^2 + y^2)$ at the point (5, 1).

a) $\dfrac{\sqrt{26}}{13}$

b) $\ln 26$

c) $\dfrac{2}{13}$

d) $13\sqrt{2}$

e) $2\ln 13$

21. If $f(x) = \displaystyle\int_0^x x^3\sqrt{1+t^2}\ dt$, find $f'(x)$.

a) $\dfrac{x^4}{4}\sqrt{1+t^2}$

b) $3x^2\sqrt{1+x^2}$

c) $x^3\sqrt{1+x^2}$

d) $\dfrac{tx^3}{\sqrt{1+t^2}} + 3x^2\sqrt{1+t^2}$

e) $x^3\sqrt{1+x^2} + 3x^2\displaystyle\int_0^x\sqrt{1+t^2}\ dt$

22. A music class of 5 girls and 4 boys is having a recital. If each child is to perform once, how many ways can the program be arranged if the first two performers must be girls?

a) 362,880

b) 100,800

c) 2,880

d) 15,876

e) 181,440

23. Evaluate the integral $\displaystyle\int \frac{\sqrt{x-1}}{x}\ dx$

a) $\dfrac{2}{3}(x-1)^{3/2} + c$

b) $2\left(\sec\sqrt{x-1}\right)^2 + c$

c) $\dfrac{x(x-1)-1}{x^2\sqrt{x-1}} + c$

d) $2\sqrt{x-1} - 2\tan^{-1}\sqrt{x-1} + c$

e) $2\sec^2 x\tan x + c$

24. Evaluate $\lim\limits_{x\to 0} \pi x^2 (\csc x)^2$

    a) 0

    b) $\pi - 1$

    c) $\dfrac{\pi^2}{2}$

    d) $\dfrac{\pi}{2}$

    e) π

25. Solve the differential equation $\dfrac{dy}{dx} + 3y = e^{2x}$; $y(0) = 1$

    a) $y = e^{2x} - e^{-3x} + 1$

    b) $y = e^{3x} + e^{-2x} - 1$

    c) $y = \dfrac{1}{5}e^{2x} + \dfrac{4}{5}e^{-3x}$

    d) $y = \dfrac{12}{5}e^{2x} - \dfrac{7}{5}e^{-3x}$

    e) $y = \dfrac{3}{5}e^{3x} + \dfrac{2}{5}e^{2x}$

26. Find the value of k if $x - 2$ is a factor of $3x^4 + kx^3 + kx^2 - 7x - 10$.

    a) -3
    b) -2
    c) -1
    d) 0
    e) 1

27. If $f(3) = -3$, $f'(3) = 6$, $f''(3) = -4$, $g(3) = 3$, $g'(3) = 2$, find $\dfrac{d}{dx}\big[f(g(3))\big]$

    a) -24
    b) 12
    c) -18
    d) 15
    e) 18

28. Simplify $\sin(x+h) - \sin(x-h)$

    a) 0
    b) $2\sin(h)$
    c) $2\sin(x)\cos(h)$
    d) $2\cos(x)\sin(h)$
    e) $2\sin(x)\cos(h) + 2\cos(x)\sin(h)$

29. In a certain college, 5% of the men and 1% of the women are taller than 180 cm. Also, 60% of the students are women. If a student is selected at random and found to be taller than 180 cm, what is the probability that this student is a woman?

    a) $\dfrac{1}{13}$
    b) $\dfrac{1}{3}$
    c) 0
    d) $\dfrac{3}{13}$
    e) 1

30. Find the area enclosed by the graphs $y = 7 - x^2$ and $y = 3|x-1|$.

    a) 10.5
    b) 11.5
    c) 12.5
    d) 13.5
    e) 14.5

31. Find the area of the region under the curve $y = \dfrac{1}{x^2}$ for $x \geq 2$

    a) The area is infinite
    b) 2
    c) 1
    d) $\dfrac{1}{2}$
    e) $\dfrac{1}{3}$

32. Find the volume of the solid formed by rotating the region bounded by $y = x^2$ and $y = x^3$ about the x-axis.

a) $\dfrac{3\pi}{4}$

b) $\dfrac{\pi}{12}$

c) $\dfrac{2\pi}{35}$

d) $\dfrac{\pi}{3}$

e) $\dfrac{7\pi}{12}$

33. Find the value of the real numbers a and b if $8x - 4y = 7$ is a tangent to the curve $y = \dfrac{ax+b}{(x+1)^2}$ at x = 1.

a) a = -19 and b = 5
b) a = -9 and b = 8
c) a = 9 and b = 8
d) a = 9 and b = -8
e) a = -9 and b = -8

34. The volume of a cube is increasing at a rate of 10 cm^3/min. How fast is the surface area increasing when the length of an edge is 30 cm?

a) $\dfrac{3}{4}$ cm^2/min

b) $\dfrac{4}{3}$ cm^2/min

c) $\dfrac{3}{5}$ cm^2/min

d) $\dfrac{5}{3}$ cm^2/min

e) $\dfrac{4}{5}$ cm^2/min

35. Find the Cartesian equation for the curve represented by the polar equation
$r^2 + 8r\cos\theta - 6r\sin\theta + 16 = 0$

a) $\dfrac{(x-4)^2}{4} + \dfrac{(y+3)^2}{3} = 1$

b) $(x+4)^2 + (y-3)^2 = 9$

c) $y = x^2 + 4$

d) $(x-4)^2 + (y+3)^2 = 16$

e) $\dfrac{(x+4)^2}{8} - \dfrac{(y+3)^2}{6} = 1$

36. An object is projected directly upward from the ground with an initial velocity of 96 feet per second. Its height, s, at the end of t seconds is given by $s(t) = 96t - 16t^2$. What is the velocity of the object when it hits the ground?
a) 128 ft/s
b) 80 ft/s
c) -112 ft/s
d) -96 ft/s
e) -6 ft/s

37. Find the interval of convergence for the power series
$\dfrac{x+2}{1\cdot 3} - \dfrac{(x+2)^2}{2\cdot 4} + \dfrac{(x+2)^3}{3\cdot 5} - \dfrac{(x+2)^4}{4\cdot 6} + \ldots$

a) $0 \le x \le 2$
b) $0 < x < 2$
c) $-2 < x < 0$
d) $-3 \le x < -1$
e) $-3 \le x \le -1$

38. An airplane is climbing at a 12° angle to the horizontal. How fast, to the nearest mile per hour, is it gaining altitude if its speed is 325 miles per hour?
a) 87 mi/h
b) 318 mi/h
c) 68 mi/h
d) 181 mi/h
e) 129 mi/h

39. Determine the values of the number a for which the function $f(x)$ has no critical numbers:

$$f(x) = (a^2 - 7a + 12)\cos 2x + (a - 3)x + 1$$

a) $a < 3.5$

b) $3.5 < a < 4.5$

c) $3 \le a \le 5$

d) $a > 5$

e) $3 \le a \le 4.5$

40. Find the sum of the following series $\dfrac{1}{2!} + \dfrac{x}{3!} + \dfrac{x^2}{4!} + \dfrac{x^3}{5!} + ...$

a) $\dfrac{e^x - (1 + x)}{x^2}$

b) $\dfrac{e^x - x}{x^2}$

c) $\dfrac{e^x + x}{x^2}$

d) $\dfrac{e^{-x}}{x^2}$

e) $\dfrac{xe^x + 1}{x^2}$

1. b
2. e
3. e
4. a
5. b
6. b
7. e
8. d
9. a
10. d
11. e
12. c
13. a
14. a
15. c
16. e
17. e
18. e
19. c
20. a
21. e
22. b
23. d
24. e
25. c
26. b
27. b
28. d
29. d
30. a
31. d
32. c
33. d
34. b
35. b
36. d
37. e
38. c
39. b
40. a