TENNESSEE MATHEMATICS TEACHERS ASSOCIATION

SIXTY-SIXTH ANNUAL MATHEMATICS CONTEST

2024

Algebra I/Integrated Math I

Scoring Formula: $4 \times$ (Number Right) $-$ (Number Wrong) $+ 40$

Directions:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. Which of the following expressions is a correct simplification of the expression $\dfrac{xy - x^2}{xy}$?

(a) $y - x$ (b) $1 - x^2$ (c) $-x^2$ (d) $1 - \dfrac{x}{y}$ (e) $-\dfrac{x}{y}$

2. What is the slope of the line containing the points $(3, 4)$ and $(-5, -2)$?

(a) -1 (b) $\dfrac{9}{5}$ (c) $\dfrac{5}{9}$ (d) $\dfrac{4}{3}$ (e) $\dfrac{3}{4}$

3. What is the solution set of the equation $-2|x - 3| = -6$?

(a) $\{6\}$ (b) $\{0\}$ (c) $\{3, 6\}$ (d) $\{0, 3\}$ (e) $\{0, 6\}$

4. Which of the following is an equation of the line with slope $-\dfrac{1}{2}$ and x-intercept 4?

(a) $y = -\dfrac{1}{2}x + 4$ (b) $y = \dfrac{1}{2}x + 2$ (c) $y = \dfrac{1}{2}x - 2$ (d) $y = \dfrac{1}{2}x + 4$ (e) $y = -\dfrac{1}{2}x + 2$

5. What is the next term in the sequence $3, 4, 7, 11, 18, 29, \ldots$?

(a) 42 (b) 43 (c) 45 (d) 47 (e) 51

6. For $x \neq 0$, which equation is equivalent to $\dfrac{1}{x} = x - 1$?

(a) $x^2 + x + 1 = 0$
(b) $x^2 - x + 1 = 0$
(c) $x^2 + x - 1 = 0$
(d) $x^2 - x - 1 = 0$
(e) $x^2 + x = 0$

7. Which of the following equations is that of a line that is parallel to the line with equation $2x - 4y = 5$?

(a) $y = 2x$ (b) $y = -2x$ (c) $y = \frac{1}{2}x$ (d) $y = -\frac{1}{2}x$ (e) $y = -\frac{5}{4}x$

8. Charlotte buys a 16-ounce slushy that is part blue raspberry and part red cherry. If blue raspberry costs 12 cents per ounce and red cherry costs 14 cents per ounce, how many ounces of blue raspberry did she get if the slushy cost \$2.14?

(a) 4 ounces (b) 5 ounces (c) 6 ounces (d) 7 ounces (e) 8 ounces

9. Which of the following is an equation of the parabola that contains the points $(-1, 1)$, $(2, 4)$, and $(3, -3)$?

(a) $y = x^2$

(b) $y = -x^2 + x + 3$

(c) $y = -x^2 + 2x + 4$

(d) $y = -2x^2 + 3x + 6$

(e) $y = 2x^2 - x - 2$

10. A rectangle has a perimeter of 70 meters. If the length of the rectangle is 7 meters longer than its width, what is the area of the rectangle?

(a) 228 m^2 (b) 260 m^2 (c) 294 m^2 (d) 300 m^2 (e) 330 m^2

11. Which of the following best describes the solutions to the equation $2x^2 - 3x = 4$?

(a) There is exactly one rational solution.

(b) There are two rational solutions.

(c) There is exactly one irrational solution.

(d) There are two irrational solutions.

(e) There are two non-real solutions.

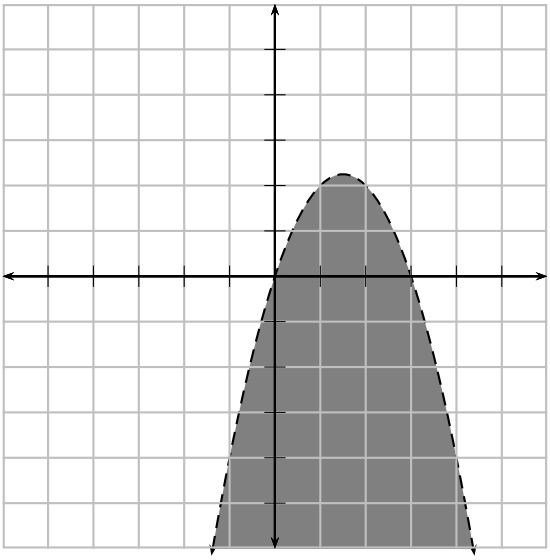
12. What is the maximum value of the function $f(x) = 6x - x^2 + 4$?

(a) 3 (b) 4 (c) 12 (d) 13 (e) 15

13. If $3x - 2y = 5k$ and $x - y = 2k$, what is y as a function of x?

(a) $y = x$ (b) $y = 2x$ (c) $y = -x$ (d) $y = -2x$ (e) $y = 5x$

14. Which of the following inequalities has the solution set shown as the shaded region below?



(a) $y < -x^2 - 3x$
(b) $y > -x^2 - 3x$
(c) $y < x^2 - 3x$
(d) $y > -x^2 + 3x$
(e) $y < -x^2 + 3x$

15. Amazon has discounted a new computer graphics card 23% off the retail price. If the discounted price is \$57.78, what is the retail price?

(a) \$251.22 (b) \$102.27 (c) \$80.78 (d) \$75.04 (e) \$71.07

16. Let $f(x,y) = \dfrac{2xy}{x^2 - y^2}$. What is $f(2,-3)$?

(a) $-\dfrac{12}{13}$ (b) $\dfrac{2}{5}$ (c) $\dfrac{12}{5}$ (d) -2 (e) $\dfrac{12}{13}$

17. If x and y satisfy the linear system below, what is the value of $x + y$?

$$3x - 5y = -8$$
$$4x + 2y = 11$$

(a) 3 (b) 4 (c) 5 (d) 6 (e) 7

18. A 5-pound container of cashews and peanuts has 25% cashews and 75% peanuts by weight. Robert only likes cashews and eats 6 ounces of cashews from the container. What is the percentage (by weight) of each nut is in the container now?

(a) 15% cashews and 85% peanuts
(b) 16% cashews and 84% peanuts
(c) 17% cashews and 83% peanuts
(d) 18% cashews and 82% peanuts
(e) 19% cashews and 81% peanuts

19. Which of the following is $(\sqrt{27} - \sqrt{3} + \sqrt{x})^2$ simplified?

(a) $24 + x$
(b) $30 + x$
(c) $12 + 4\sqrt{3x} + x$
(d) $24 + 2\sqrt{6x} + x$
(e) $12 + 12\sqrt{x} + x$

20. Which of the following is a factor of $2x^3 + 3x^2 + 30x + 45$?

(a) $x + 3$ (b) $x + 5$ (c) $2x + 3$ (d) $2x + 5$ (e) $2x + 15$

21. What is the solution set of the inequality $|2x - 3| < 5$?

(a) $\{x : x < -1 \text{ or } x > 4\}$

(b) $\{x : -1 < x < 4\}$

(c) $\{x : -4 < x < 1\}$

(d) $\{x : x < -4 \text{ or } x > 1\}$

(e) $\emptyset$

22. What is the quotient $\dfrac{x^2 - 4x - 5}{x^2 - 7x + 10} \div \dfrac{x^2 - 1}{x^2 - 4}$ simplified?

(a) -2 (b) $\dfrac{x+2}{x-1}$ (c) $\dfrac{x-2}{x+1}$ (d) $\dfrac{x+2}{x+1}$ (e) $\dfrac{x-2}{x-1}$

23. Let $f(x) = \dfrac{x}{x^2+1}$. What is the value of $f(-2)$?

(a) $\dfrac{2}{3}$ (b) $\dfrac{2}{5}$ (c) $-\dfrac{2}{3}$ (d) $-\dfrac{2}{5}$ (e) $\dfrac{1}{2}$

24. If the points $(0, 0)$ and $(2, 0)$ are two of the vertices of an equilateral triangle, what are all possible options for the coordinates of the third vertex?

(a) $(1, 1)$

(b) $(1, -1)$ and $(1, 1)$

(c) $(1, \sqrt{3})$

(d) $(1, \sqrt{3})$ and $(1, -\sqrt{3})$

(e) $(1, -1)$, $(1, 1)$, $(1, -\sqrt{3})$, and $(1, \sqrt{3})$

25. Which of the following quadratic equations has exactly one solution?

(a) $x^2 + 9 = 0$ (b) $x^2 = 9$ (c) $x^2 + 6x = 9$ (d) $x^2 + 9 = 6x$ (e) $x^2 = 6x + 9$

26. Ann is twice is old as Beth. Beth is 6 years older than Carlie. Ann is three times as old as Carlie. How many years older is Ann than Beth?

(a) 6 years (b) 9 years (c) 12 years (d) 15 years (e) 18 years

27. What is the simplified form of the expression $\left(\dfrac{xy}{x^{-1}+y^{-1}}\right)^{-1}$?

(a) $\dfrac{x+y}{x^2y^2}$ (b) $\dfrac{x+y}{xy}$ (c) $\dfrac{xy}{x+y}$ (d) $\dfrac{1}{x^2+y^2}$ (e) $\dfrac{1}{xy(x+y)}$

28. What is the coefficient of the x^3 term in the expansion of the product $(x^2+x+1)(x+2)^2$?

(a) 6 (b) 5 (c) 4 (d) 2 (e) 1

29. A right triangle has a leg that is 5 cm and a hypotenuse that is 10 cm. What is the perimeter of the right triangle?

(a) $15+5\sqrt{3}$ cm (b) 25 cm (c) $15+5\sqrt{2}$ cm (d) $\dfrac{25\sqrt{2}}{2}$ cm (e) $\dfrac{25\sqrt{3}}{2}$ cm

30. Your total bill at a restaurant (including 7% tax) is \$39.59. If you want to give a 16% gratuity (tip) on the pre-tax amount, how much would the gratuity (tip) be?

(a) \$5.78 (b) \$5.92 (c) \$6.08 (d) \$6.23 (e) \$6.33

31. What is the product of $(2x^2+x-3)$ and $(4x^2-x+5)$?

(a) $8x^4-x^2-15$

(b) $6x^2+2$

(c) $8x^4+2x^3-3x^2+8x-15$

(d) $8x^4+2x^3+3x^2+8x-15$

(e) $8x^4-2x^3-3x^2+8x-15$

32. Which of the following best describes the relationship between x and y where y is a function of x that contains the points $(1, 17)$, $(3, 11)$, $(6, 2)$, $(2, 14)$, and $(5, 5)$?

(a) linear (b) exponential (c) quadratic (d) logarithmic (e) cubic

33. Kevin has 8 algebra tests in a semester. If the average (mean) of his first five test scores is 87, what average (mean) score does he need on the last three tests to have a test average (mean) of 90?

(a) 93 (b) 94 (c) 95 (d) 96 (e) 97

34. What is the x-intercept of the line that is perpendicular to the line $y = 4x$ and contains the point (a, b)?

(a) $a + 4b$ (b) $4a + b$ (c) $a - 4b$ (d) $4a - b$ (e) $4a - 4b$

35. The formula to compute the harmonic mean of two numbers x and y is $\dfrac{2}{x^{-1} + y^{-1}}$. What is the harmonic mean of 12 and 20 rounded to the nearest tenth?

(a) 13.5 (b) 14 (c) 15 (d) 15.5 (e) 16

36. What is the solution set to the inequality $|3x - 2| \leq x$?

(a) $\{x : 0 \leq x \leq 1\}$
(b) $\{x : 0 \leq x \leq 0.5\}$
(c) $\{x : x \geq 0.5\}$
(d) $\{x : 0.5 \leq x \leq 1\}$
(e) $\emptyset$

37. Given the equation $\dfrac{2x + 3}{y + 4} = \dfrac{3x - 1}{y + 3}$, what is y in terms of x?

(a) $y = \dfrac{6x + 13}{x - 4}$ (b) $y = \dfrac{6x + 13}{4 - x}$ (c) $y = \dfrac{6x - 13}{x + 4}$ (d) $y = \dfrac{6x - 13}{x - 4}$ (e) $y = \dfrac{6x - 13}{4 - x}$

38. What is the y-intercept of the line with slope 2 that intersects the parabola with equation $y = 8x - x^2$ in exactly one point?

(a) -9 (b) -5 (c) 1 (d) 5 (e) 9

39. Let $A = \begin{bmatrix} -3 & -5 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 1 \\ -1 & 4 \end{bmatrix}$. What is the product AB?

(a) $\begin{bmatrix} -21 & -5 \\ -2 & -4 \end{bmatrix}$

(b) $\begin{bmatrix} -19 & -36 \\ 11 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -16 & -23 \\ 15 & -2 \end{bmatrix}$

(d) $\begin{bmatrix} -26 & -23 \\ -6 & -9 \end{bmatrix}$

(e) $\begin{bmatrix} -26 & -6 \\ -23 & -9 \end{bmatrix}$

40. A brother and sister travel together to lunch in a nearby town. The brother drives to the restaurant and the sister drives back home, so both drive exactly the same distance, d. The sister drives the trip 10 minutes faster than the brother. If x is the average speed in miles per hour for the brother, what is the average speed for the sister in terms of x and d?

(a) $\dfrac{6xd}{6d - x}$ (b) $\dfrac{6xd + d}{6d + x}$ (c) $\dfrac{6xd + d}{6d - x}$ (d) $\dfrac{6xd + x}{6d + x}$ (e) $\dfrac{6xd + x}{6d - x}$