

FORTY-FIFTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Advanced Topics I 2001

Prepared by:

Mathematics Department
Lipscomb University
Nashville, TN

Reviewed by:

Mathematics Faculty
Austin Peay State University
Clarksville, TN 37044

Coordinated by: Randy Bouldin

Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

Dr. Hal Ramer, President, Volunteer State Community College, Gallatin, Tennessee
Donnelley Printing Company, Gallatin, Tennessee
TRW Commercial Steering Division, Lebanon, Tennessee
Wright Industries, Inc., Nashville, Tennessee

ADVANCED TOPICS I

1. Rationalize the denominator of $\frac{\sqrt{x}}{3\sqrt{x}-\sqrt{y}}$.

a. $\frac{3x + \sqrt{xy}}{9x + y}$

b. $\frac{3x - \sqrt{xy}}{3x + y}$

c. $\frac{3x + \sqrt{xy}}{3x + y}$

d. $\frac{3x + \sqrt{xy}}{3x - y}$

e. $\frac{3x + \sqrt{xy}}{9x - y}$

2. If $f(x) = \frac{1}{2} \sin(bx - \frac{\pi}{2})$, what is the value for b that would result in a phase shift of 3?

a. $\frac{3\pi}{2}$

b. $\frac{\pi}{6}$

c. $-\frac{\pi}{6}$

d. $\frac{2\pi}{3}$

e. 3

3. Factor $x^2y - 25y + 3x^2 - 75$.
- a. $(y+3)(x+5)^2(x-5)$
 - b. $(y+3)(x+5)^2(x-5)^2$
 - c. $(y-3)(x+5)(x-5)$
 - d. $(y+3)(x+5)(x-5)$
 - e. $(y-3)(x+5)^2(x-5)^2$
4. Which of the following pairs of vectors is orthogonal?
- a. $4i - 6j, 5i + j$
 - b. $2i + 3j, 6i - 4j$
 - c. $i + 2j, -7i - 2j$
 - d. $5i + 10j, 2i + 4j$
 - e. $2i + j, 8i - 7j$
5. A telephone company charges 50¢ for a long-distance call that does not exceed two minutes, and 30¢ for each additional minute. Find the cost of a 15-minute call.
- a. \$4.40
 - b. \$4.50
 - c. \$4.60
 - d. \$4.70
 - e. None of the above

6. Let $f(x)$ be a polynomial of degree 3 with real coefficients. Two of the roots or zeroes of $f(x)=0$ are $-3+5i$ and -1 . We also know $f(1)=4$. Express $f(x)$ as a product of a linear and quadratic factors with real coefficients.

a. $f(x)=(x+1)(x^2+6x+34)$

b. $f(x)=\frac{2}{41}(x-1)(x^2+6x+34)$

c. $f(x)=\frac{-2}{9}(x+1)(x^2+6x-16)$

d. $f(x)=\frac{2}{41}(x+1)(x^2+6x+34)$

e. $f(x)=\frac{2}{41}(x-1)(x^2-6x+34)$

7. One card is selected at random from 50 cards numbered 1 through 50. What is the probability that the number of the card is a prime number?

a. .30

b. .32

c. .50

d. .70

e. .28

8. In how many ways can a committee of three be chosen from a group of ten people?

a. 13

b. 30

c. 52

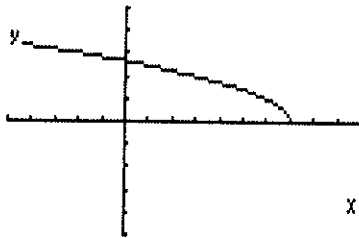
d. 120

e. 210

9. Find the complex conjugate of the complex number $\frac{1}{4+7i}$.

- a. $4-7i$
- b. $4+7i$
- c. $\frac{4}{65} + \frac{7}{65}i$
- d. $\frac{1}{7+4i}$
- e. $\frac{4}{65} - \frac{7}{65}i$

10. What function f has the following graph?



- a. $f(x) = \sqrt{7-x}$
- b. $f(x) = \sqrt{7+x}$
- c. $f(x) = \sqrt{-7+x}$
- d. $f(x) = \sqrt{-7-x}$
- e. $f(x) = -\sqrt{7+x}$

11. In triangle ABC the length of \overline{BC} is 12.4 ft., the length of \overline{AC} is 8.7 ft. and the measure of angle B = 36.7° . What is the largest measure angle A can have (to the nearest tenth of a degree)?

- a. 58.4°
- b. 84.9°
- c. 121.6°
- d. 21.7°
- e. Triangle doesn't exist.

12. Find the solution(s) to the following equation in the interval $[0, 2\pi)$.

$$\sin\left(2x - \frac{\pi}{4}\right) = 1$$

- a. $\frac{3\pi}{8}$
- b. $\frac{\pi}{8}$
- c. $\frac{\pi}{2}$
- d. $\frac{3\pi}{8}, \frac{7\pi}{8}$
- e. $\frac{3\pi}{8}, \frac{11\pi}{8}$
13. Environmentalists have accused a large company in the eastern United States of dumping nuclear waste material into a local river. The probability that either the fish in the river or the animals that drink from the river will die is $\frac{11}{21}$. The probability that the fish will die is $\frac{1}{3}$ and the probability that the animals that drink from the river will die is $\frac{2}{7}$. What is the probability that both the fish and the animals that drink from the river will die?
- a. $\frac{2}{21}$
- b. $\frac{11}{21}$
- c. $\frac{4}{7}$
- d. $\frac{22}{21}$
- e. none of these

14. A golfer, centered in a 30 yard wide straight fairway, hits a ball 280 yards. Approximate the largest angle the drive can have from the center of the fairway if the ball is to stay in the fairway.
- 3.07°
 - 6.14°
 - $.05^\circ$
 - $.1^\circ$
 - 30°
15. A cat falls down a well 20 feet deep. The cat crawls up 3 feet a day and down 2 feet a night. How long will it take the cat to get out of the well?
- 18 days
 - 19 days
 - 20 days
 - 21 days
 - 22 days
16. Bud Adams, owner of the Tennessee Titans, gives \$1,000,000 to help out the homeless of Nashville. If the entire amount is invested at 7% compounded continuously, how many years will it take for the investment to equal \$1,500,000?
- 1.5 yrs.
 - 3.0 yrs.
 - 5.0 yrs.
 - 5.8 yrs.
 - 6.2 yrs.
17. Which pair of functions are not inverse functions?
- $f(x)=\frac{1}{x}$; $g(x)=\frac{1}{x}$
 - $f(x)=3x-4$; $g(x)=\frac{x+4}{3}$
 - $f(x)=x^3-2$; $g(x)=\sqrt[3]{x+2}$
 - $f(x)=\frac{1}{3x-2}$; $g(x)=\frac{2x+1}{3x}$
 - $f(x)=2-x^3$; $g(x)=\sqrt[3]{x-2}$

18. Two dice are rolled. What is the probability that the sum of the dots appearing on both dice together is 9 or 11?
- $1/20$
 - $1/5$
 - $1/6$
 - $1/4$
 - $7/80$
19. A bicycle rider coasts downhill, traveling 4 feet the first second. In each succeeding second, the rider travels 5 feet farther than in the preceding second. If the rider reaches the bottom of the hill in 11 seconds, find the total distance traveled.
- 314 feet
 - 316 feet
 - 319 feet
 - 321 feet
 - 323 feet
20. A parallelogram has sides of length 30 centimeters and 70 centimeters and one angle of measure 65° . Approximate the length of the longest diagonal to the nearest centimeter.
- 63 cm
 - 73 cm
 - 90 cm
 - 60 cm
 - 87 cm
21. Solve the following system of equations for x and y :

$$\begin{aligned} 2^x + 3^{y+1} &= 10 \\ 2^{x+1} - 3^y &= 5 \end{aligned}$$

- $x = \frac{25}{7}$ $y = \frac{15}{7}$
- $x = \log_2\left(\frac{25}{7}\right)$ $y = \log_3\left(\frac{15}{7}\right)$
- $x = \log_2\left(\frac{15}{7}\right)$ $y = \log_3\left(\frac{25}{7}\right)$
- $x = \log_3\left(\frac{25}{7}\right)$ $y = \log_2\left(\frac{15}{7}\right)$
- $x = \log_{25/7}(2)$ $y = \log_{15/7}(3)$

22. At a reception attended by 25 people, 10 people drank tea, 12 drank coffee and 3 drank both coffee and tea. How many drank neither coffee nor tea?
- a. 0
 - b. 1
 - c. 3
 - d. 6
 - e. 8

23. This expression $\frac{1 + \csc 3x}{\sec 3x} - \cot 3x$ can be written as:

- a. $\tan 3x$
- b. $\cos 3x$
- c. $\sec 3x$
- d. $\sin 3x$
- e. $\csc 3x$

24. If $\tan \theta = \frac{8}{7}$ and θ is acute, find $\csc(\theta + \pi)$.

- a. $-\frac{1}{8}$
- b. $\frac{1}{8}$
- c. $\frac{7}{8}$
- d. $-\frac{\sqrt{113}}{8}$
- e. $\frac{\sqrt{113}}{8}$

25. $(1+i)^{20} =$

- a. $724.1 + i(724.1)$
- b. $\sqrt{2} + i\sqrt{2}$
- c. $1+i$
- d. -1024
- e. 2

26. The sum of an infinite geometric series equals 5 and the first term of the series equals 2. Find the common ratio, r , of the geometric series.
- a. 0.30
 - b. 0.25
 - c. 0.60
 - d. 0.50
 - e. 0.40
27. A light pole is 30 feet tall. How long is the shadow cast by a woman 6 feet tall who is standing 8 feet from the pole?
- a. 1.6 ft.
 - b. 2 ft.
 - c. 10 ft.
 - d. 16 ft.
 - e. 3 ft.
28. The amount of carbon 14 in the bones changes according to $A(t) = A_0 e^{Rt}$, where A_0 is the initial amount. The bone of a prehistoric man in the desert of New Mexico contains approximately 5% of the original amount of carbon 14. If the half-life of carbon is 5600 years, approximately how long ago did the man die?
- a. 112,000 years
 - b. 1,296 years
 - c. 280 years
 - d. 24,203 years
 - e. 5,895 years
29. In October, when the crowds are the greatest, a tourist is driving 20 m/hr. directly toward a mountain in the Smokies. Because the traffic is so slow, the tourist has time to notice that between 12:00 A.M. and 12:15 A.M. the angle of the elevation to the top of the mountain changes from 15° to 45° . Approximate the height of the mountain if the road is level.
- a. ≈ 0.06 miles
 - b. ≈ 1.34 miles
 - c. ≈ 1.83 miles
 - d. Not sufficient information to solve
 - e. None of the above

30. The number of miles M that a certain car can travel on one gallon of gasoline is related to its speed v (in mi/hr) by $M = -\frac{1}{100}v^2 + v$ for $0 < v < 80$. For what speeds will M be greater than 21?
- $v \in (0, 30)$
 - $v \in (0, 70)$
 - $v \in (0, 80)$
 - $v \in (30, 70)$
 - $v \in (30, 80)$
31. With a regular deck of 52 cards what is the probability of being dealt a royal flush: the ace, king, queen, jack, and ten of the same suit?
- .0052
 - .00004
 - .00001385
 - .00000154
 - .00000987
32. On the Richter scale, the magnitude R of an earthquake of intensity I is given by $R = \log \frac{I}{I_0}$, where I_0 is a certain minimum intensity. If the "Survivor" island has an earthquake of magnitude $R=5$ and Gilligan's island has an earthquake of magnitude $R=7$, how many more times intense was the intensity of the "Gilligan" earthquake than the intensity of the "Survivor" earthquake?
- 2 times
 - 20 times
 - 100 times
 - 200 times
 - None of the above
33. The graph of $x^2 + 2y^2 + 2x - 20y = -43$ is
- an ellipse with center (1,5)
 - an ellipse with center (-1,5)
 - a hyperbola with center (1,5)
 - a hyperbola with center (-1,5)
 - a parabola with vertex (-1,5)

34. What is the diameter of the largest circular metal plate (so thin that we can ignore thickness) which will fit through a rectangular hole that is 3.2 inches wide and 8.1 inches high?
- a. 8.70 in.
 - b. 8.71 in.
 - c. 8.1 in.
 - d. 3.2 in.
 - e. 3.3 in.

35. Consider the matrices $A = \begin{bmatrix} 7 & -1 \\ 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & 4 \\ 3 & x \end{bmatrix}$

Find the value of x such that the matrix $C = A - B$ will not have an inverse.

- a. 24
 - b. $-\frac{10}{13}$
 - c. $\frac{24}{13}$
 - d. $\frac{10}{13}$
 - e. 0
36. The International Widget Corporation produces two types of widgets – deluxe and mediocre. There are two phases to the widget manufacturing process: A and B. For the deluxe widgets, phase A requires 2 hours and phase B requires 1 hour. For the mediocre widgets, phase A requires 1 hour and phase B requires 1 hour. The profit for each deluxe widget is \$70 per widget and the profit for each mediocre widget is \$50 per widget. If 40 hours are available for phase A of the manufacturing process and 32 hours for phase B, how many widgets of each type should be produced to maximize profit?
- a. 0 deluxe, 30 mediocre
 - b. 8 deluxe, 24 mediocre
 - c. 24 deluxe, 8 mediocre
 - d. 20 deluxe, 0 mediocre
 - e. 0 deluxe, 40 mediocre

37. Find all solutions to the following equation in the interval $(-\infty, \infty)$
 $\tan^2 \mu = -\tan \mu$

a. $\mu = 0, \pi, 2\pi, \frac{3\pi}{4}, \frac{7\pi}{4}$

b. $\mu = \pi n$ and $\frac{3\pi}{4} + \pi n$, for all integers, n

c. $\mu = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

d. $\mu = \frac{\pi}{2} + \pi n$ and $\frac{3\pi}{4} + \pi n$, for all integers, n

e. no solutions exist

38. From an observation point A in the Rockies, a forest ranger sights a fire in the direction $S30^\circ 45' E$. From a point B in the Rockies, 10 miles due east of observation point A, another ranger sights the same fire in the direction $S59^\circ 15' W$. Approximate the distance of the fire from observation point A.

- a. 1.4 miles
- b. 3.6 miles
- c. 5.1 miles
- d. 6.4 miles
- e. 8.6 miles

39. If a boy and girl are 20 miles apart and if they are walking toward each other at rates of 7 m.p.h. and 3 m.p.h., respectively, how many miles will a fly fly who is flying 20 m.p.h. and who is flying in a straight line back and forth between the boy and girl?

- a. 10 miles
- b. 20 miles
- c. 30 miles
- d. 40 miles
- e. 50 miles

40. Let $f(x) = x^2 - 16$ and $g(x) = \sqrt{x}$.
The domain of $(f \circ g)$ is

- a. $(-\infty, \infty)$
- b. $[0, \infty)$
- c. $[-4, 4]$
- d. $(-\infty, -4] \cup [4, \infty)$
- e. none of these

