

FORTY-FOURTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Advanced Topics I 2000

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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Advanced Topics Test

1. Find all solutions in the interval $[0, 2\pi)$: $\csc^2 x - (\cos^4 x + \cos^2 x \sin^2 x + \sin^4 x) = 0$

(a) $\frac{\pi}{2}, \frac{3\pi}{2}$

(b) $0, \pi$

(c) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

(d) $\frac{\pi}{2}$

(e) none of these

2. A television antenna sits on a roof. Two 76-foot guy wires are positioned on opposite sides of the antenna. The angle of elevation each makes with the ground is 24° . How far apart are the ends of the two guy wires, to the nearest foot?

(a) 42 feet

(b) 139 feet

(c) 251 feet

(d) 18 feet

(e) none of these

3. Two forces, one of 120 pounds and the other of 200 pounds, act on the same object at angles 30° and -30° respectively, with the positive x-axis. Find the direction of the resultant of these two forces.

(a) 14°

(b) -8.2°

(c) 0°

(d) -18°

(e) none of these

4. Evaluate: $\left[4 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \right]^5$

(a) $512 - 512\sqrt{3}i$

(b) 1024

(c) $32 + 32\sqrt{3}i$

(d) $-32 + 32\sqrt{3}i$

(e) none of these

5. Find the value of c in the quadratic equation, $y = ax^2 + bx + c$, if its graph passes through the points $(1,0)$, $(-1,-6)$, and $(2,9)$

(a) -5
(b) -4
(c) 3
(d) 11
(e) none of these

6. Find the minimum value of the function $C = 10x + 12y$ subject to the following constraints:

$$x \geq 0$$

$$y \geq 0$$

$$x + y \leq 36$$

$$x - 2y \geq 0$$

(a) 0
(b) 360
(c) 384
(d) 432
(e) none of these

7. Find a formula for the n th term of the sequence. (Assume that n begins with 1.) $\frac{2}{1}, \frac{4}{1}, \frac{6}{2}, \frac{8}{6}, \frac{10}{24}, \dots$

(a) $\frac{2^n}{(n+1)!}$

(b) $\frac{3-2^n}{n(2^n)}$

(c) $\frac{2n}{2n-1}$

(d) $\frac{2n}{(n-1)!}$

(e) none of these

8. Find the sum of the first 50 terms of the arithmetic sequence:
 $25, 35, 45, 55, 65, \dots$

(a) $27,000$
(b) $13,750$
(c) $12,875$
(d) $13,500$
(e) none of these

9. Suppose in 1985 you accepted a job at \$17,000. If you receive a 5% raise in salary each year, what will be your salary in 1995?

- (a) \$25,116.74
- (b) \$26,372.58
- (c) \$27,691.21
- (d) \$29,075.77
- (e) none of these

10. If the probability of getting a rotten apple in a basket of apples is 12%, what is the probability of getting 3 good apples choosing one from each of three different baskets?

- (a) 0.9983
- (b) 0.0017
- (c) 0.8800
- (d) 0.6815
- (e) none of these

11. Write the polynomial as a product of factors irreducible over the real numbers: $x^4 - 3x^2 - 28$.

- (a) $(x^2 + 4)(x^2 - 7)$
- (b) $(x - 2i)(x + 2i)(x - \sqrt{7})(x + \sqrt{7})$
- (c) $(x^2 + 4)(x - \sqrt{7})(x + \sqrt{7})$
- (d) $(x - 2i)(x + 2i)(x^2 - 7)$
- (e) none of these.

12. The concentration of a mixture is given by $C = \frac{2x + 9}{3(x + 12)}$

Use a graphing utility using the indicated range setting to determine what the concentration approaches.

Xmin=0
Xmax=280
Xscale=50
Ymin=0
Ymax=1
Yscale=0.1

- (a) 33%
- (b) 67%
- (c) 50%
- (d) 75%
- (e) 45%

13. Write as a sum, difference, or multiple of logarithms: $\log \sqrt[3]{\frac{a^2 b}{c}}$

(a) $\sqrt[3]{\frac{2 \log a + \log b}{\log c}}$

(b) $\frac{1}{3} \left(\frac{2 \log a + \log b}{\log c} \right)$

(c) $\frac{1}{3} (2 \log a + \log b - \log c)$

(d) $\sqrt[3]{2 \log a + \log b - \log c}$

(e) None of these

14. Write as the logarithm of a single quantity: $\frac{1}{4} \log_b 16 - 2 \log_b 5 + \log_b 7$.

(a) $\frac{14}{25}$

(b) $\log_b \frac{2}{175}$

(c) 1

(d) $\log_b \frac{14}{25}$

(e) none of these

15. An initial deposit of \$2800 is made in a savings account for which the interest is compounded continuously. The balance will triple in eight years. What is the annual rate of interest for this account?

(a) 6.9%

(b) 13.7%

(c) 11.6%

(d) 9.9%

(e) none of these

16. Determine the period: $f(x) = \frac{-1}{2} \sin\left(\frac{3x}{2} - \frac{1}{2}\right)$

- (a) $\frac{1}{2}$
- (b) $\frac{1}{2}\pi$
- (c) $\frac{3\pi}{4}$
- (d) $\frac{4\pi}{3}$
- (e) none of these

17. An open box is to be made from a rectangular piece of material 9 inches by 12 inches by cutting equal squares from each corner and turning up the sides. Let x be the length of each side of the square cut out of each corner. Write the volume V of the box as a function of x .

- (a) $V = x^3$
- (b) $V = 108x$
- (c) $V = x(9 - x)(12 - x)$
- (d) $V = x(9 - 2x)(12 - 2x)$
- (e) none of these

18. Determine which function is neither even nor odd.

- (a) $f(x) = \tan x$
- (b) $f(x) = 3x^5 + 5x^3 + 1$
- (c) $f(x) = \frac{3}{x^2}$
- (d) $f(x) = \sqrt{x^2 + 1}$
- (e) both a and b

19. Describe the transformation needed to sketch the graph of $y = \frac{1}{x} + 2$ using the graph of $f(x) = \frac{1}{x}$.

- (a) Shift $f(x)$ two units to the right.
- (b) Shift $f(x)$ two units to the left.
- (c) Shift $f(x)$ two units upward.
- (d) Shift $f(x)$ two units downward.
- (e) Reflect $f(x)$ about the x-axis.

20. The table gives the unemployment rate r for the years 1985 through 1994 in the United States. The time t was measured in years, with $t=0$ corresponding to 1990. (Source: U.S. Bureau of Labor Statistics)

t	-5	-4	-3	-2	-1	0	1	2	3	4
r	7.2	7.0	6.2	5.5	5.3	5.5	6.7	7.4	6.8	6.1

Determine which of the following models best represents the data.

- (a) $r = 1.5 \cos(t + 3.9) + 6.37$
- (b) $r = 1.03 \sin(0.92t - 0.45) + 6.19$
- (c) $r = 1.05 \sin[0.95(t + 6.32)] + 6.20$
- (d) $r = 1.5 \sin[0.5(t + 2.8)] + 6.25$
- (e) None of these

21. Describe the shifts of the graph of g with respect to the graph of f :

$$g(x) = 1 + \cos\left(2x - \frac{\pi}{2}\right) \text{ and } f(x) = \cos 2x$$

- (a) $\frac{\pi}{2}$ right, down 1
- (b) $\frac{\pi}{2}$ right, up 1
- (c) $\frac{\pi}{4}$ right, up 1
- (d) $\frac{\pi}{4}$ right, down 1
- (e) none of these

22. Evaluate: $\cot\left(\arcsin\frac{1}{x-1}\right)$

(a) $2x - x^2$

(b) $x-1$

(c) $\frac{1}{\sqrt{x^2 + 2x + 2}}$

(d) $\sqrt{x^2 - 2x}$

(e) none of these

23. An airplane flying at 600 miles an hour has a bearing of S 34°E. After flying 3 hours, how far south has the plane traveled from its point of departure? (Round to the nearest mile)

(a) 1007 miles

(b) 1720 miles

(c) 1947 miles

(d) 1492 miles

(e) none of these

24. Simplify: $\frac{\cos^4 x - \sin^4 x}{\cos^2 x - \sin^2 x}$

(a) $1-2 \sin^2 x$

(b) $2 \cos^2 x - 1$

(c) 1

(d) -1

(e) none of these

25. For what values of x , $0 \leq x \leq 2\pi$ is it true that $\csc x = \sqrt{\cot^2 x + 1}$?

(a) $0 < x < \pi$

(b) $\pi < x < 2\pi$

(c) $\frac{\pi}{2} \leq x \leq 3\frac{\pi}{2}$

(d) $0 \leq x \leq \frac{\pi}{2}$

(e) none of these

26. Rewrite as a sum: $\sin 3x \cos 4y$

(a) $\frac{1}{2}[\sin(3x + 4y) + \sin(3x - 4y)]$

(b) $\frac{1}{2}[\sin(3x + 4y) - \sin(3x - 4y)]$

(c) $2[\cos(3x + 4y) + \cos(3x - 4y)]$

(d) $2[\sin(3x - 4y) + \cos(3x - 4y)]$

(e) none of these

27. Given a triangle with $a = 72$, $b = 51$, and $A = 27^\circ$, where A is angle opposite side a . Find the area of the triangle to the nearest tenth of a square unit.

- (a) 833.5 square units
- (b) 1315.3 square units
- (c) 1635.9 square units
- (d) 2630.6 square units
- (e) none of these

28. Given a triangle with $a=12$, $c=21$, and $B=72^\circ$. Find C , where C is angle opposite side c .

- (a) 69.2°
- (b) 74.6°
- (c) 81.0°
- (d) 33.4°
- (e) none of these

29. Find all solutions: $x^2 - 4i = 0$

- (a) $\sqrt{2} - \sqrt{2}i, -\sqrt{2} + \sqrt{2}i$
- (b) $2 + i, -2 - i$
- (c) $2 - i, 2 + i$
- (d) $\sqrt{2} + \sqrt{2}i, -\sqrt{2} - \sqrt{2}i$
- (e) none of these.

30. A small corporation borrowed \$900,000: some at 7%, some at 8%, and some at 11%. How much was borrowed at 8% if the annual interest was \$72,250 and the amount borrowed at 8% was \$50,000 more than the amount borrowed at 11%?

- (a) \$225,000
- (b) \$175,000
- (c) \$650,000
- (d) \$450,000
- (e) none of these.

31. Determine the seating capacity of an auditorium with 30 rows of seats if there are 15 seats in the first row, 18 seats in the second row, 21 seats in the third row, and so on.

- (a) 1635
- (b) 1792
- (c) 2055
- (d) 3125
- (e) none of these

32. Find the sum of the infinite geometric sequence, $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$

(a) $\frac{3}{2}$

(b) 3

(c) $\frac{5}{3}$

(d) $\frac{5}{2}$

(e) none of these

33. How many ways can an eight question multiple choice test be answered if each question has five possible answers?

(a) 390,625

(b) 4,838,400

(c) 40,320

(d) 120

(e) none of these

34. There are seven possible digits in a phone number. How many different phone numbers are possible if the first digit cannot be 0 and no digit can be used more than once?

(a) 128

(b) 181,440

(c) 544,320

(d) 5040

(e) none of these

35. Before an election, a sample of 120,000 people throughout the county showed that 79,386 people would vote for Candidate A. If a person from the sample is chosen at random, what is the probability that the person is one of the people who said that they would not vote for Candidate A?

(a) 0.66

(b) 0.34

(c) 0.47

(d) 0.53

(e) none of these

36. Determine which function is NOT one-to one:

(a) $y = \sqrt[3]{x^2 + 1}$

(b) $y = \frac{2}{x}$

(c) $y = 7x - 2$

(d) $y = \sqrt{2 - x}$

(e) None of these

37. Given $f(x) = 7x + 2$ find $f^{-1}(x)$.

- (a) $7x + 2$
- (b) $\frac{1}{7x + 2}$
- (c) $\frac{x - 2}{7}$
- (d) $\frac{x}{7} - 2$
- (e) None of these

38. Given $f(x) = \sqrt[3]{x}$ and $g(x) = 3 - x$ find $(f^{-1} \circ g^{-1})(-1)$.

- (a) -6
- (b) -4
- (c) 2
- (d) -8
- (e) None of these

39. Solve for x: $\frac{1}{x - 2} + \frac{1}{x + 3} = \frac{4}{x^2 + x - 6}$

- (a) $\frac{2}{3}$
- (b) 3
- (c) $\frac{3}{2}$
- (d) 1
- (e) None of these

40. Approximate the solutions of $(x^2 - 9x + 2)^{\frac{3}{2}} = 216$.

- (a) -0.5, 9
- (b) $\pm 3, 4.5$
- (c) -2.87, 11.87
- (d) 1.63, 16.37
- (e) None of these

