

THIRTY-SIXTH ANNUAL MATHEMATICS CONTEST  
sponsored by  
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Advanced Topics I 1992

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Scoring formula:  $4R - W + 40$

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

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Contributors to TMTA for Annual Mathematics Contest:

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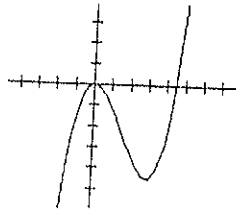
TRW, Ross Gear Division, Lebanon, Tennessee

NOTE: 1993 CONTEST DATE --APRIL 6

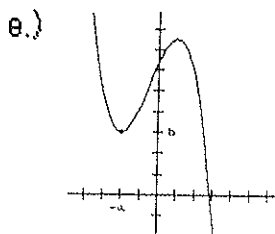
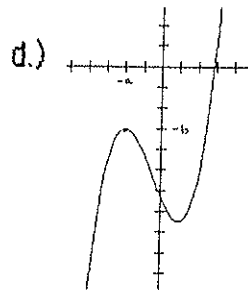
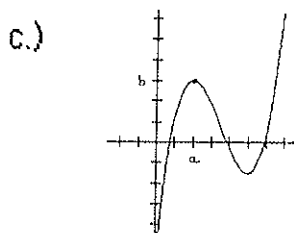
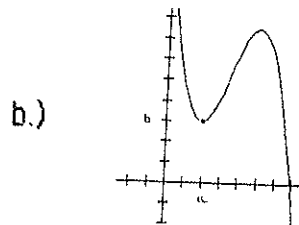
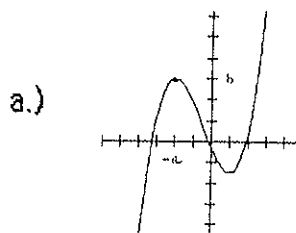


1. Find the point that is 4 units away from the point  $P(x,y)$  in the direction of vector  $\vec{a} = \langle 2, 1 \rangle = 2\vec{i} + 1\vec{j}$
- $(x + 8, y + 4)$
  - $(4x + 2, 4y + 1)$
  - $(x + 8\sqrt{5}/5, y + 4\sqrt{5}/5)$
  - $(6, 5)$
  - $(x + 2, y + 1)$

2. The graph of the function  $y = f(x)$  is given below.



Given positive constants  $a$  and  $b$ , identify the graph of  $y = b - f(x + a)$ .



3. In a  $3 \times 3$  matrix, 2 times the first row and 3 times the second row are added. The sum replaces the second row. The determinant has been multiplied by

- 1
- 2
- 3
- 6
- 5

4. Function  $g(x)$  has domain  $\{x \mid 5 < |x|\}$ . Given  $f(x) = 2x + 1$ . Find the domain of the composite function  $g \circ f$ .

- a.)  $\{x \mid 2 < x\}$
- b.)  $\{x \mid -5 < x < 5\}$
- c.)  $\{x \mid \text{any real number}\}$
- d.)  $\{x \mid x < -3 \text{ or } 2 < x\}$
- e.)  $\{x \mid x < -5 \text{ or } 5 < x\}$

5. Let  $A_n = \frac{5 + \sin(n)}{1 + (0.3)^n}$  where  $n$  is a positive integer.

Which of the following is FALSE ?

- a.)  $A_n \leq \frac{6}{1 + (0.3)^n}$
- b.)  $A_n \leq \frac{5 + \sin(n)}{1 + (0.2)^n}$
- c.)  $A_n \leq \frac{5 + |\sin(n)|}{1 + (0.3)^n}$
- d.)  $A_n \leq \frac{5 + \sin(n)}{1 + (0.4)^n}$
- e.)  $A_n \leq \frac{5 + \sin(n)}{1 + (0.3)^{n+1}}$

6. Find  $\sin(\tan^{-1} y)$ .

- a.)  $\frac{y}{y+1}$
- b.)  $\frac{\sqrt{y^2+1}}{y}$
- c.)  $\frac{\sqrt{y^2-1}}{y}$
- d.)  $\frac{y+1}{y}$
- e.)  $\frac{y\sqrt{y^2+1}}{y^2+1}$

7. A triangular-shaped lot lies between two streets which intersect at an angle of  $73^\circ$ . If the frontage of the lot on one of the streets is 115 meters and the area of the lot is 7500 square meters, what is the frontage on the other street (in meters)?

a.)  $115 \cos 73^\circ$       b.)  $\frac{3000}{23 \sin 73^\circ}$       c.)  $115 \sin 73^\circ$

d.)  $115 \tan 73^\circ$       e.)  $\frac{3000}{23 \cos 73^\circ}$

8. A rectangle is inscribed in the unit circle. The rectangle's diagonals intersect at the origin and one side is parallel to the x-axis. There is a positive  $x$  such that the point  $(x,0)$  is on one of the sides of the rectangle. What is the area of the rectangle?

a.)  $x \sqrt{1-x^2}$       b.)  $4x(1-x^2)$       c.)  $4x \sqrt{1-x^2}$

d.)  $2x \sqrt{1-x^2}$

9. Let  $Q = B e^t$  where  $Q$  and  $B$  are positive constants. Solve for  $t$ .

a.)  $\ln Q - \ln B$       b.)  $\frac{\ln(Q-B)}{e}$       c.)  $\frac{\ln Q}{\ln B} - e$

d.)  $\frac{\ln Q}{e \ln B}$       e.)  $\frac{\ln Q}{\ln B} - 1$

10. Let  $q = 27 - 18 \sin^2(t)$  and  $r = 12 - 15 \cos(t)$ . Find one value of  $t$  for which  $q = r$ .

a.)  $\pi$       b.)  $2\pi$       c.)  $3\pi/2$       d.)  $15/18$       e.) no solution

11. A standard deck of 52 playing cards is shuffled and a card is selected at random. If the card is not replaced and another card is selected, what is the probability that the first card is a heart and the second card is not a club?
- a.)  $\frac{2}{51}$     b.)  $\frac{19}{102}$     c.)  $\frac{19}{104}$     d.)  $\frac{1}{51}$     e.)  $\frac{51}{103}$
12. Find the 200<sup>th</sup> term of the following arithmetic sequence:  
5, 8, 11, 14, ...
- a.) 602    b.) 1000    c.) 756    d.) 456    e.) 896
13. If 10 numbers are multiplied by 5, the variance is multiplied by
- a.) 5    b.) 25    c.) 50    d.) 250    e.) 1
14. Evaluate  $\frac{\log_{49} 7 - \log_8 64}{\log_9 27 + \log_{10} 100}$
- a.)  $\frac{3}{4}$     b.)  $-\frac{3}{7}$     c.)  $-\frac{1}{4}$     d.)  $-\frac{1}{3}$     e.)  $-\frac{3}{4}$
15. A certain colony of bacteria increases in number by 10% per hour. After 6 hours, what is the percentage increase in the population over the initial population (to the nearest whole number)?
- a.) 60%    b.) 77%    c.) 95%    d.) 160%    e.) 177%
16. A pet shop has 9 puppies, 6 of them are males. There are 2 beagles (1 male), 2 cocker spaniels (1 male), and 5 poodles. They held a random drawing and notified you that you had won a poodle. What is the probability that your puppy is a male?
- a.)  $\frac{2}{3}$     b.)  $\frac{5}{6}$     c.)  $\frac{5}{9}$     d.)  $\frac{4}{5}$     e.)  $\frac{1}{2}$

17. At one hamburger shop, you can choose one-, two-, or three-patty hamburgers as well as cheese, relish, lettuce, tomato, mustard, and/or catsup. How many different kinds of hamburgers are possible?

a.) 9      b.) 15      c.) 96      d.) 128      e.) 192

18. In a triangle, a line segment joins the midpoint of a median to a vertex. Extend the line segment until it intersects the side opposite the vertex considered. This extended segment divides the side into the following ratio:

a.) 1:1      b.) 1:2      c.) 1:3  
d.) 1:4      e.) 1:5

19. Given the set  $I = \{ \dots -3, -2, -1, 0, 1, 2, 3, \dots \}$   
Which of the properties hold for multiplication?

1. Closure
2. Associative
3. Existence of an identity
4. Existence of inverses

a.) 1 only  
b.) 2 only  
c.) 1 and 2  
d.) 1, 2, and 3  
e.) all of them

20. Simplify  $\frac{i^{27}(i^6 - 6i^3)}{\sqrt{-4}}$

a.)  $\frac{1}{2}i - 3$       b.)  $\frac{1}{2} - 3i$       c.)  $\frac{7}{2}$   
d.)  $\frac{1}{2} + 3i$       e.)  $-\frac{5}{2}$

21. Soft drink cans are stacked on their sides. The pile is stacked with 30 cans on the bottom, 29 in the second row, 28 in the third row and so forth. How many cans would be needed for the stack to form an isosceles triangle?

a.) 465      b.) 450      c.) 420      d.) 434      e.) 480

22. Which of the following functions satisfy the property:

$$f(kx) = k f(x)$$

where  $k$  is any positive constant and  $x$  is in the domain of  $f$ .

- a.)  $f(x) = \sin(x)$
- b.)  $f(x) = \sqrt{x}$
- c.)  $f(x) = 3x^2 + 6$
- d.)  $f(x) = \log(x)$
- e.)  $f(x) = \frac{3x^2}{|2x|}$

23. For  $t$  a positive constant, let

$$\mathbf{U}(x - t) = \begin{cases} 0 & \text{for } 0 \leq x < t \\ 1 & \text{for } t \leq x \end{cases}$$

Let  $f$  be the function

$$f(x) = \begin{cases} 5 & \text{for } 0 \leq x < 3 \\ -20 & \text{for } 3 \leq x \end{cases}$$

Express  $f$  in terms of  $\mathbf{U}$ .

- a.)  $f(x) = 5 - 20 \mathbf{U}(x + 3)$
- b.)  $f(x) = 5 \mathbf{U}(x - 3) - 25$
- c.)  $f(x) = 5 - 20 \mathbf{U}(x - 3)$
- d.)  $f(x) = 5 - 25 \mathbf{U}(x - 3)$
- e.)  $f(x) = 5 \mathbf{U}(x + 3) - 20$

24. Given  $2 \sin^2(x) = 3 \cos(\pi/2 - x) - 1$ .

Find all solutions on  $[0, 2\pi)$ .

- a.)  $x = \pi/2, x = \pi/6$
- b.)  $x = 0, x = \pi/4, x = \pi/2$
- c.)  $x = \pi/2, x = \pi/6, x = 5\pi/6$
- d.)  $x = 0, x = \pi/3, x = 2\pi/3$
- e.)  $x = \pi/2, x = \pi/6, x = 11\pi/6$



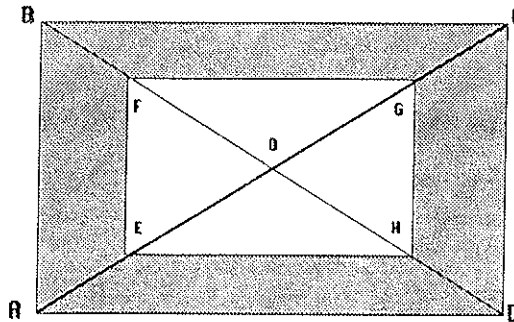
25. Which of the following is equivalent to the following intervals?  
 $1 \leq x < 5$  or  $5 < x \leq 9$

- a.)  $x \in (1, 5) \cup (5, 9)$
- b.)  $0 < |x - 5| \leq 4$
- c.)  $x \in [0, 9] \cap [1, 18]$
- d.)  $|x - 5| \leq 4$
- e.)  $1 \leq |x| < 5$  or  $5 < |x| \leq 9$

26. Simplify.  $\left(\sqrt[10]{\sqrt[5]{x^{15}}}\right)^5 \left(\sqrt[5]{\sqrt[10]{x^{15}}}\right)^5$

- a.)  $x$
- b.)  $x^{4/3}$
- c.)  $x^{5/6}$
- d.)  $x^3$
- e.)  $1$

27. In the diagram below, ABCD is a rectangle. The diagonals  $\overline{AC}$  and  $\overline{BD}$  meet at point O. Rectangle EFGH is formed by joining the midpoint of  $\overline{OA}$ ,  $\overline{OB}$ ,  $\overline{OC}$ , and  $\overline{OD}$ . The ratio of the shaded portion to the unshaded portion is



- a.) 1:1
- b.) 2:1
- c.) 3:1
- d.) 4:1
- e.) 5:1

28. In a list of numbers, the first number is 3, the second is 6, and each subsequent number is the sum of the preceding numbers in the list. If the 20<sup>th</sup> number in the list is  $x$ , what is the 25<sup>th</sup> number in the list?

- a.)  $9x$
- b.)  $16x$
- c.)  $32x$
- d.)  $45x$
- e.)  $144x$

29. If  $x - y = 6$  and  $2x^2 - xy - y^2 = 30$ , find  $4x + 2y$ .

- a.)  $4/3$
- b.)  $11/3$
- c.)  $5$
- d.)  $10$
- e.)  $44/3$

30. If the areas of three of the faces of a rectangular solid are 6, 8, and 12, what is the volume of the solid?

- a.) 24    b.) 48    c.) 96    d.) 184    e.) 576

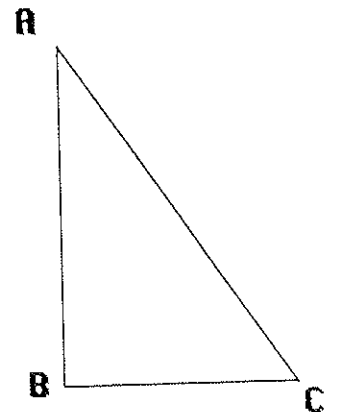
31. Given  $f(x) = \frac{5x^2 - 5x - 30}{x^3 - 8x^2 + 15x}$ , find all vertical asymptotes.

- a.)  $x = 0, x = 5, x = 3$   
b.)  $x = 0, x = 3$   
c.)  $x = 0, x = -5, x = -3$   
d.)  $x = 3, x = 5$   
e.)  $x = 0, x = 5$

32. Find the domain of  $f(x) = \frac{\sqrt{9 - x^2}}{2x}$

- a.)  $\{x \mid x \neq 0 \text{ and } -3 < x < 3\}$   
b.)  $\{x \mid x \leq -3 \text{ or } 3 \leq x\}$   
c.)  $\{x \mid x \neq 0 \text{ and } -3 \leq x \leq 3\}$   
d.)  $\{x \mid -3 \leq x \leq 3\}$   
e.)  $\{x \mid x \neq 0\}$

33. In the right triangle  $ABC$ ,  $\overline{AB}$  is divided into 6 congruent segments. Five lines parallel to  $\overline{BC}$  are drawn through these point. If  $BC$  is 8, find the sum of the lengths of the 5 parallel segments.



- a.) 28    b.) 22    c.) 24    d.) 20    e.) 18

34. Which of the following is not equivalent to the sum of these binomial coefficients?

$$\binom{9}{5} + \binom{9}{6}$$

- a.)  $\binom{9}{5} + \binom{9}{3}$       b.)  $\binom{9}{3} + \binom{8}{3} + \binom{8}{4}$       c.)  $\binom{10}{6}$   
 d.)  $\binom{9}{4} + \binom{9}{5}$       e.)  $\binom{9}{5} + \binom{8}{2} + \binom{8}{3}$

35. If  $M = \{ 2, 4, 6 \}$  and  $N = \{ x, y \}$  which of the following is a subset of Cartesian Product  $M \times N$

- a.  $(2,x)$       b.  $\{2,y\}$       c.  $\{(y,6)\}$       d.  $\{(4,x)\}$       e.  $(y,2)$

36. Given the nonparallel vectors  $\vec{u} = \langle 1, 2 \rangle$ ,  $\vec{v} = \langle -1/2, 3 \rangle$ , and  $\vec{w} = \langle 1, 1/2 \rangle$ , which of the following real numbers is the greatest.

- a.  $\|\vec{u} - 2\vec{v} + 2\vec{w}\|$   
 b.  $\|\vec{u}\| + \|-2\vec{v}\| + \|2\vec{w}\|$   
 c.  $\|\vec{u} - 2\vec{v}\| + \|2\vec{w}\|$   
 d.  $\|\vec{u} - 2\vec{v} - 2\vec{w}\|$   
 e.  $\|\vec{u}\| + \|\vec{v}\| - \|\vec{w}\|$

37. Which one of the following polynomials is irreducible over the field of rational numbers?

- a.)  $3x^3 + 2x^2 - 7x + 2$       b.)  $8 - 8x + 2x^2$   
c.)  $9x^2 - 4$       d.)  $8x^3 + 27$       e.)  $12x^2 - 28x - 15$

38. Find an equation of the perpendicular bisector of the segment joining A(-3,4) and B(7,8)

- a.)  $2x - 5y = -26$       b.)  $2x + 5y = -26$       c.)  $5x + 2y = 22$   
d.)  $5x - 2y = -2$       e.)  $5x - 2y = -22$

39. Which of the following relations is **not** a function?

- a.  $\{ (x,y) \mid y = x^2 + 2 \}$   
b.  $\{ (x,y) \mid y^2 = x + 4 \text{ and } y > 0 \}$   
c.  $\{ (x,y) \mid x^2 + y^2 = 25 \text{ and } xy > 0 \}$   
d.  $\{ (x,y) \mid y^2 = x + 4 \text{ and } x > -4 \}$   
e.  $\{ (x,y) \mid y = 3x + 2 \text{ and } x > 1 \}$

40. An even function is a function with the property  $f(x) = f(-x)$ . Which of the following functions is **not** even?

- a.)  $y = 2x^2 + 3$   
b.)  $y = \sin(x - \pi/2)$   
c.)  $y = \cos(x - \pi/2)$   
d.)  $y = |3x| + 5$   
e.)  $y = 5x \sqrt[3]{x}$