TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

SIXTY-SEVENTH ANNUAL MATHEMATICS CONTEST

2025

Algebra II/ Integrated Math III

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Scoring Formula: $4 \times (\text{Number Right}) - (\text{Number Wrong}) + 40$

Directions:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. Which of the following is a zero to the function $f(x) = x^4 - 3x^3 - 4x^2 - 12x$?
   a) -2
   b) 2
   c) -3
   d) 0
   e) -5

2. Which of the following is **not** a term in the expansion of $(2x - y)^9$?
   a) $-2304x^8y$
   b) $-5376x^6y^3$
   c) $-4032x^5y^4$
   d) $-2016x^4y^5$
   e) $672x^3y^6$

3. Which of the following matrices is equal to $4\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix} - 5\begin{bmatrix}7 & 5\\3 & 2\end{bmatrix}$?
   a) $\begin{bmatrix}-27 & -2\\5 & 5\end{bmatrix}$
   b) $\begin{bmatrix}-5 & -2\\2 & 5\end{bmatrix}$
   c) $\begin{bmatrix}15 & -12\\-8 & -5\end{bmatrix}$
   d) $\begin{bmatrix}-27 & -13\\5 & 18\end{bmatrix}$
   e) $\begin{bmatrix}-27 & -13\\2 & 5\end{bmatrix}$

4. What is the equation of a parabola in vertex form containing the coordinate points $(5, 8)$, $(7, -4)$, and $(10, 23)$?
   a) $y = (x - 7)^2 - 4$
   b) $y = (x + 7)^2 - 4$
   c) $y = 3(x + 7)^2 - 4$
   d) $y = 3(x + 7)^2 + 4$
   e) $y = 3(x - 7)^2 - 4$

5. If $\log_x 4 = 1.262$ and $\log_x 7 = 1.771$, what is the value of $\log_x \left(\frac{49}{16}\right)$?
   a) 1.963
   b) 1.403
   c) 1.544
   d) 1.018
   e) 1.969

6. Which of the following describes the transformation of the parent function $y = e^x$ to the function $y = 2e^{-2x-8} + 8$?
   a) Vertical stretch by a factor of 2, reflect about the y-axis, horizontal compression by a factor of ½, translated left 8 and up 8
   b) Vertical stretch by a factor of 2, reflect about the y-axis, horizontal compression by a factor of ½, translated left 4 and up 8
   c) Vertical stretch by a factor of 2, reflect about the y-axis, horizontal compression by a factor of ½, translated right 8 and up 8
   d) Vertical stretch by a factor of 2, reflect about the y-axis, horizontal compression by a factor of ½, translated right 8 and down 8
   e) Vertical stretch by a factor of 2, reflect about the y-axis, horizontal compression by a factor of ½, translated right 4 and up 8

7. Theoretically, scores on the math sections of the ACT and SAT college entrance examinations are normally distributed. For the ACT, $\mu = 18, \sigma = 6$ and for the SAT, $\mu = 500, \sigma = 100$. All scores on the ACT are recorded as whole numbers and all scores on the SAT are recorded as multiples of 10. If a student scores a 27 on the math section of the ACT, what would the corresponding score be on the SAT using equivalent z-scores?
   a) 450
   b) 570
   c) 590
   d) 650
   e) 710

8. For what length radius would the value of the surface area of a sphere (in square units) be triple the value of the volume (in cubic units) of the same sphere?
   a) $r = 1$
   b) $r = \frac{4}{9}$
   c) $r = 4$
   d) $r = 12$
   e) $r = \frac{4}{3}$

9. What is the smallest whole-number value of x where the value of 3^x exceeds $2x^3 + x^2 + x + 5$ and does for all other larger values of x?
   a) $x = -2$
   b) $x = -1$
   c) $x = 5$
   d) $x = 6$
   e) $x = 243$

10. For the restricted domain $[3, \infty)$, if $f(x) = x^2 - 6x + 5$, what is the inverse function $f^{-1}(x)$?
   a) $f^{-1}(x) = 3 + \sqrt{x + 4}$
   b) $f^{-1}(x) = 3 + \sqrt{x + 14}$
   c) $f^{-1}(x) = 6 + \sqrt{x + 5}$
   d) $f^{-1}(x) = -5 + \sqrt{x - 6}$
   e) $f^{-1}(x) = 6 + \sqrt{x + 31}$

11. An operation ∋ on real numbers a and b is defined as $a \ni b = \sqrt{a} + \sqrt[3]{b} - \sqrt[3]{a} - \sqrt{b}$. What is the value of $4096 \ni 1$?
   a) 48
   b) 80
   c) -16
   d) 16
   e) 2

12. What is the average rate of change for the piecewise-defined function:

$$f(x) = \begin{cases} x^2 - 3x & \text{if } x \leq 3 \\ 6x - 14 & \text{if } x > 3 \end{cases}$$

on the interval [2,4]?

a) 1
b) 4
c) 4.5
d) 6
e) 18

13. Let r_1, r_2, and r_3 be the roots of the equation $x^3 + x^2 - 86x - 240 = 0$. What is the value of $\frac{r_1 r_2 r_3}{r_1 + r_2 + r_3}$?

a) -120
b) 120
c) 240
d) -240
e) 0

14. What is the solution set to the equation $\sqrt{3x - 5} = |x - 5|$?

a) $\emptyset$
b) $\{10\}$
c) $\{3, 10\}$
d) $\{-10\}$
e) $\{-3, -10\}$

15. A triangle is formed with the vertices of three cities on a Tennessee map: Memphis, Nashville, and Chattanooga. According to Google Maps, Memphis and Nashville are separated by 212 miles, Nashville and Chattanooga are separated by 133 miles, and Memphis and Chattanooga are separated by 316 miles. What is the measurement of the angle where Nashville is the vertex, when rounded to the nearest whole degree?

a) 131°
b) 18°
c) 30°
d) 109°
e) 95°

16. What are the transformations from the parent graph $f(x) = \sqrt{x}$ to the transformed graph $g(x) = \sqrt{4x - 16}$?
    a) g is a vertical stretch by 4 and a rightward shift by 16
    b) g is a vertical stretch by 2 and a leftward shift by 16
    c) g is a vertical stretch by 4 and a leftward shift by 4
    d) g is a vertical stretch by 2 and a rightward shift by 4
    e) g is a vertical stretch by 2 and a leftward shift by 4

17. If 1 in $= 2.54$ cm, how many cubic centimeters are in one cubic inch, when rounded to the nearest hundredth?
    a) 2.54 cubic centimeters
    b) 7.62 cubic centimeters
    c) 1.36 cubic centimeters
    d) 0.85 cubic centimeters
    e) 16.39 cubic centimeters

18. How many three-digit base 10 even numbers are there?
    a) 450
    b) 1000
    c) 500
    d) 405
    e) 900

19. Which of the following best describes the graphical representation of the system of linear equations?

$$4x + z = 9$$
$$7x - y + 3z = 9$$
$$3x + 2y - z = 7$$

    a) Three parallel planes
    b) Three planes intersecting at the line $4x + z = 9$
    c) Three planes intersecting at the point $\left(\frac{20}{3}, \frac{-46}{3}, \frac{-53}{3}\right)$
    d) Two planes and a line coinciding with the line $4x + z = 9$
    e) Two planes and a line intersecting at the point $(2,0,1)$

20. How many intersection points does the parabola $y = (2x - 3)^2 + 4$ have with the circle $(x - 1)^2 + (y - 2)^2 = 9$?
    a) None
    b) One
    c) Two
    d) Three
    e) Four

21. Which of the best describes the real solutions to the equation $x^6 - 729 = 0$?
    a) The only real solution is x = 3
    b) The only real solution is x = -3
    c) The real solutions are 3 and -3, each with a multiplicity of two
    d) The real solutions are 3 and -3
    e) The real solutions are 3 and -3, each with a multiplicity of three

22. A professional hockey team has a roster of 12 forwards, 6 defense, and 2 goaltenders. A starting line-up has 3 forwards, 2 defense, and 1 goaltender. How many different combinations of starting line-ups are possible by choosing 3 out of 12 from the first group (forwards), 2 out of 6 from the second group (defense), and 1 out of 2 from the third group (goaltender)?
    a) 38,760
    b) 6,600
    c) 237
    d) 3,300
    e) 2,640

23. What is the product of the matrices $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \end{bmatrix}$?
    a) The product is not defined.
    b) $\begin{bmatrix} -9 & -12 & -15 \\ -19 & -26 & -33 \\ -29 & -40 & -51 \end{bmatrix}$
    c) $\begin{bmatrix} 8 & 20 & 36 \\ 48 & 120 & 216 \\ 120 & 300 & 540 \end{bmatrix}$
    d) $\begin{bmatrix} -5 & -11 \\ -14 & -32 \end{bmatrix}$
    e) $\begin{bmatrix} 4 & 24 \\ 40 & 240 \end{bmatrix}$

24. Money deposited in a savings account earns interest at an annual rate of 5.25%. After how many years would the initial contribution to the account double in value, assuming no withdrawals or additional deposits are made?

    a) About 2 years
    b) About 6 years
    c) About 8 years
    d) About 13 years
    e) About 19 years

25. If a set of data has a distribution that is highly skewed to the right, which inequality must be true?
    a) mean = median
    b) mean < median
    c) median < mean
    d) mean > range
    e) median > range

26. If $f(x) = 2x - 1$ and $g(x) = x^2 + 3x$, what is $g\big(f(x)\big) - f\big(g(x)\big)$?
    a) $2x^2 - 4x - 1$
    b) 0
    c) $2x^2 - 16x + 3$
    d) $x^2 + 6x - 2$
    e) $2x^2 + 6x - 1$

27. What is the domain of the function $f(x) = \frac{x-7}{\sqrt{x^2-x-30}}$?

    a) $(-\infty, -5) \cup (-5, 6) \cup (6, 7) \cup (7, \infty)$
    b) $(-\infty, -5) \cup (-5, 6) \cup (6, \infty)$
    c) $(-\infty, -6) \cup (-6, 5) \cup (5, \infty)$
    d) $(-\infty, -6) \cup (5, \infty)$
    e) $(-\infty, -5) \cup (6, \infty)$

28. What value of x satisfies the equation $\sqrt{5x+1}+\sqrt{2x-5}=5$?

a) $x=\frac{29}{7}$
b) $x=3$
c) $x=7$
d) $x=-2$
e) No real solution

29. What is the value of the interquartile range for the data set below?

23	12	40	1	40	32	6	7	38
15	33	2	26	17	34	19	5	24
16	36	11	2	15	30	9	24	14

a) 7
b) 10
c) 14
d) 23
e) 31

30. Which of these expressions is equal to $\sin 50°$?

a) $\sin 40°$
b) $\cos 40°$
c) $\tan 40°$
d) $\cos 50°$
e) $\tan 50°$

31. What is the solution set of the equation $\log_6(x^3+4x^2-9x)=2$?

a) $\{-2,2,3\}$
b) $\{-4,3,4\}$
c) $\{-4,-3,3\}$
d) $\{-4,-3,4\}$
e) $\{2,3,4\}$

32. Which of the following polynomials is equivalent to the result of transforming the graph of $f(x) = x^3$ by a vertical stretch of 2, translated left 2, and up 3?
    a) $y = x^3 - 6x^2 + 12x - 5$
    b) $y = x^3 + 6x^2 + 12x + 11$
    c) $y = 2x^3 - 12x^2 + 24x - 10$
    d) $y = 2x^3 - 12x^2 + 24x - 13$
    e) $y = 2x^3 + 12x^2 + 24x + 19$

33. What values of x satisfy the inequality: $\sqrt[3]{-11x + 4} > 5$?
    a) $x > 11$
    b) $x < -11$
    c) $x > -11$
    d) $x < 11$
    e) $x < 1$

34. Which of the following is equivalent to the expression $\dfrac{x^{3/5}\left(y^{2/3}\right)^2}{\left(x^{2/3}\right)^4 y^{5/6}}$?
    a) $\dfrac{\sqrt{y}}{x^2\left(\sqrt[15]{x}\right)}$
    b) $\dfrac{\sqrt[405]{x^{163}}}{\sqrt[18]{y^7}}$
    c) $\left(\sqrt[40]{x^9}\right)\left(y\left(\sqrt[5]{y^3}\right)\right)$
    d) $\left(x^3\left(\sqrt[80]{x^3}\right)\right)\left(\sqrt[15]{y^8}\right)$
    e) $\dfrac{1}{\left(\sqrt[15]{x^4}\right)\left(\sqrt[3]{y}\right)}$

35. What is the sum of the combinations $\binom{7}{4} + \binom{7}{5}$?
    a) $\binom{8}{4}$
    b) $\binom{8}{5}$
    c) $\binom{8}{6}$
    d) $\binom{14}{9}$
    e) $\binom{14}{5}$

36. What is the solution set to the equation $x^4 - 29x^2 + 100 = 0$?

a) $\{2, 5\}$
b) $\{-5, -2, 2, 5\}$
c) $\{4, 25\}$
d) $\{-25, -4, 4, 25\}$
e) $\emptyset$

37. Given the table below, what is the value of $f\big(g(3)\big)$?

x	1	2	3	4	5	6	7	8
$f(x)$	0	7	8	6	2	2	0	7
$g(x)$	6	8	4	1	2	8	2	7

a) 2
b) 6
c) 7
d) 8
e) 32

38. If $f(t) = t^2 - 4t + 6$, which of the following describes the average rate of change on the interval $[2, x]$, $\frac{f(x)-f(2)}{x-2}$?

a) $2x - 4$
b) $x + 2$
c) $x - 2$
d) $x^2 - 6$
e) $x^2 - 8$

39. Which of the coordinates would be a point on the graph of the inverse of the function $f(x) = x^3 - 2x - 9$?

a) $(0, -9)$
b) $\left(0, \sqrt[3]{-9}\right)$
c) $(0, 9)$
d) $\left(0, -\frac{1}{9}\right)$
e) $(-9, 0)$

40. What is the probability of guessing four out of five multiple-choice questions correctly, where each question has five possible answers (like the test you are taking now!)?
    a) $\frac{1}{5}$
    b) $\frac{4}{5}$
    c) $\frac{16}{3125}$
    d) $\frac{4}{3125}$
    e) $\frac{1}{3125}$