FORTY-SEVENTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 2003

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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High School Algebra II Math Test

1. Find the equation of the straight line which is perpendicular to the line 2x-3y = -5 and goes through the point (-5,7).

(a) 3x+2y=29 (b) 3x+2y= -1 (c) 3x+2y= 1 (d) 2x-3y= -31 (e) 15x-10y= -6

2. How many solutions does the following system of equations have?

$$\left.\begin{aligned} 2x-4y&=2\\ 3x-6y&=3 \end{aligned}\right\}$$

(a) None (b) 1 (c) 2 (d) 3 (e) infinitely many

3. The functions f(x) = x^2 and g(x) = $-x^2+2x+4$ intersect at:

(a) (-2,4) only (b) (2,4), (1,5) (c) (-1,1), (2,4) (d) (1,-1), (4,2) (e) none of these

4. The determinant of the matrix

$$\begin{pmatrix} 5 & -8 & 10\\ 4 & 2 & 8\\ 1 & 1 & 2 \end{pmatrix} \text{ is}$$

(a) –30 (b) –204 (c) –80 (d) 0 (e) –20

5. The product AB of matrices

$$A=\begin{pmatrix} 1 & 2\\ 2 & -1 \end{pmatrix} \text{ and } B=\begin{pmatrix} 3 & 0\\ -1 & 2 \end{pmatrix} \text{ is:}$$

(a) $\begin{pmatrix} 3 & 0\\ -2 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 4\\ 7 & -2 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 2\\ 5 & -2 \end{pmatrix}$ (d) $\begin{pmatrix} 5 & 2\\ 7 & 2 \end{pmatrix}$ (e) None of these

6. Solve the following system of equations

$$\left.\begin{aligned} 2x-y+2z&=6\\ x-3y+z&=-2\\ -x+2y-3z&=2 \end{aligned}\right\}$$

(a) (5, 2, 1) (b) (-5, -2, 1) (c) (5, 2, -1) (d) infinitely many solutions (e) no solution

7. The solution set for the inequality $|-2x+3| > 5$ is

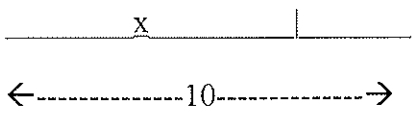
(a) $\{x \mid -1 < x < 4\}$ (b) $\{x \mid -1 \le x \le 4\}$ (c) $\{x \mid x>4 \text{ and } x<-1\}$ (d) $\{x \mid x<-1 \text{ or } x > 4\}$

(e) None of these

8. When the real roots of the equation $x^4 - x^2 - 2 = 0$ are summed and that sum is then squared, one gets

(a) 8 (b) 4 (c) 1 (d) 0 (e) None of these

9. The line segment below is of length 10 units and is cut at a point x units from the left end of the segment such that the entire length of the segment is to the longer part x as the longer part is to the shorter part. Find this positive value of x



(a) $5\sqrt{5}+5$ (b) $5\sqrt{5}-5$ (c) $-5\sqrt{5}+5$ (d) $5\sqrt{3}+5$ (e) $5\sqrt{3}-5$

10. Given that the quadratic equation $ax^2+bx+c=0$ has imaginary roots, which of the following is true?

(a) $4ac-b^2<0$ (b) $4ac-b^2=0$ (c) 2+0i and 3+0i are both roots (d) $b^2-4ac>0$ (e) None of these

11. The exponential growth model $A = 208e^{0.008t}$ describes the population of the United States, in millions, t years after January 1, 1970. In what year will the population of the United States be 300 million?

(a) 2018 (b) 2005 (c) 2010 (d) 2015 (e)2016

12. Simplify: $\ln(A+B) - \ln(A^{-1} + B^{-1})$

(a) ln AB
(b) $\ln(A^2 + B^2)$
(c) $\ln\left(\frac{1}{A}+\frac{1}{B}\right)$
(d) ln 0
(e) $\frac{1}{A}+\frac{1}{B}$

13. A radioactive substance undergoes exponential decay. If forty percent of the original amount decays in five years, by what percent does the substance decay each year?

(a) 8.3%
(b) 125%
(c) 8%
(d) 51%
(e) 9.71%

14. Solve for x: $x = 1 - \sqrt{2 - \frac{x}{2}}$

(a) $2, -\frac{1}{2}$
(b) 2
(c) $-\frac{1}{2}$
(d) 0
(e) No real solution

15. Find a formula for the exponential function f(x) = a b^x satisfying the given conditions:

$$f(3) = -\frac{3}{8} \qquad f(-2) = -12$$

(a) $f(x) = -12\left(\frac{3}{8}\right)^x$
(b) $f(x) = 3\left(\frac{1}{2}\right)^x$
(c) $f(x) = 1.5^x$
(d) $f(x) = -1.5^x$
(e) $f(x) = -3(.5)^x$

16. Solve for x: $\log(1-x) - \log(1+x) = 2$

(a) -1
(b) +1
(c) $\frac{101}{99}$
(d) $-\frac{99}{101}$
(e) -.99

17. Find the domain of: $f(x) = \frac{5}{6x^2 + x - 2}$

(a) $x \neq 2, \ -1$ (b) All real numbers (c) $x \neq \frac{1}{2}, \ -\frac{2}{3}$ (d) $x \neq -\frac{1}{2}, \ \frac{2}{3}$ (e) $-\frac{1}{2} < x < \frac{2}{3}$

18. Find $(f \circ g)(-x)$ *if* $f(x) = \dfrac{1}{x+1}$ *and* $g(x) = x^2 - 5$

(a) $\dfrac{-5x^2 - 4}{(x+1)^2}$

(b) $\dfrac{1}{x^2 - 4}$

(c) $\dfrac{-4}{(x+1)^2}$

(d) $\dfrac{x^2 - 5}{x+1}$

(e) $\dfrac{-5x^2 - 10x - 4}{x^2 + 2x + 1}$

19. Find the inverse of the function $f(x) = \dfrac{2x-3}{x+1}$

(a) $\dfrac{2x-3}{x+1}$

(b) $\dfrac{x+1}{2x-3}$

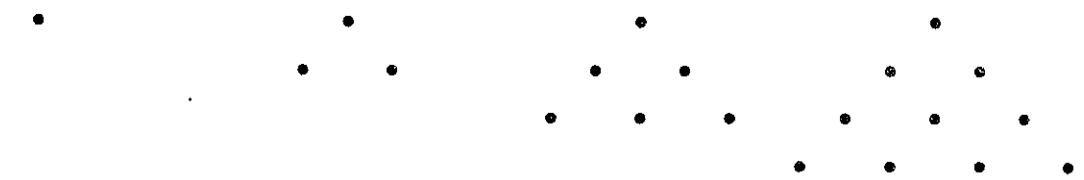
(c) $\dfrac{3+x}{2-x}$

(d) $\dfrac{2x+3}{x-1}$

(e) The inverse does not exist.

20. The diagram below shows the first four triangular numbers. How many dots are in the triangle when n = 20?

(a) 20! (b) 210 (c) 190 (d) 100 (e) 400



n=1 n=2 n=3 n=4

21. Given $f(x) = 2x - 6$ and $f(g(x)) = g(f(x)) = x$
Find the point of intersection of the two functions f and g.

(a) (3, 3)
(b) (1.2, -3.6)
(c) (0, 3)
(d) (6, 6)
(e) no intersection

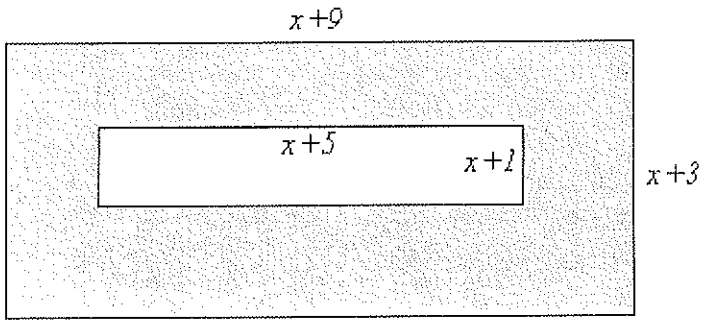
22. The remainder when $5x^3 - 6x^2 + 3x + 11$ is divided by $x - 2$ is

(a) $5x + 11$ (b) 33 (c) -59 (d) $5x^2 + 4x + 11$ (e) 0

23. The set of all real-number solutions to the equation $2x^3 + x^2 - 4x = 2$ is

(a) $\{\pm\sqrt{2}, \frac{1}{2}\}$ (b) $\{2, -2, \frac{1}{2}\}$ (c) $\{\pm\sqrt{2}, -\frac{1}{2}\}$ (d) $\{2, -2, -\frac{1}{2}\}$ (e) $\{0, -2, 1\}$

24. The area of the shaded region below is:



(a) 6 (b) 22 (c) $18x + 32$ (d) $-6x - 22$ (e) $6x + 22$

25. A value of k so that $x + 3$ is a factor of $3x^2 + kx + 21$ is

(a) 16 (b) $\frac{1}{7}$ (c) 7 (d) 2 (e) 21

26 The rational expression $\dfrac{\dfrac{x}{x-2}+1}{\dfrac{3}{x^2-4}+1}$ simplifies to

(a) $\dfrac{x^2+2x+3}{3}$ (b) $\dfrac{2x^2(4-x)}{12-x^2}$ (c) $\dfrac{2(x-1)^2(x+1)}{(x-2)^2(x+2)}$ (d) $\dfrac{2(x+2)}{x+1}$ (e) $\dfrac{2(x-1)}{3(x-2)}$

27. The missing expression in $\dfrac{3x}{x-5}+\dfrac{?}{5-x}=\dfrac{7x+1}{x-5}$ is

(a) $-4x-1$ (b) $10x+1$ (c) $4x+1$ (d) $-10x-1$ (e) $-10x+1$

28. The value of x that satisfies the equation $\dfrac{3}{x+3}-\dfrac{1}{x-2}=\dfrac{5}{2x+6}$ is

(a) $\dfrac{13}{4}$ (b) -8 (c) 4 (d) 5 (e) no solution

29. The center of a circle with equation $x^2+y^2+8x-2y-8=0$ is

(a) (8,-2) (b) (0,0) (c) (-4,1) (d) (64,-4) (e) (5,5)

30 One point of intersection of the line $y=3$ and the hyperbola $\dfrac{(x-2)^2}{16}-\dfrac{(y-3)^2}{9}=1$ is

(a) (2,3) (b) $\left(\dfrac{1}{2},3\right)$ (c) (5,3) (d) (3,2) (e) (6,3)

31. The maximum y value of any point on the parabola $(x+2)^2=-4(y+5)$ is

(a) -2 (b) -5 (c) -4 (d) 20 (e) 5

32. The general term for a sequence is $a_n=(-1)^{n+1}(n+4)$. The fourth term in the sequence is

(a) 4 (b) 7 (c) -48 (d) -8 (e) 3

33. A horizontal asymptote of $f(x) = \dfrac{-3x^2}{6x^2 + 7x - 10}$ is

(a) $y = \dfrac{3}{10}$ (b) $x = 2$ (c) $x = \dfrac{5}{6}$ (d) $y = \dfrac{-1}{2}$ (e) no horizontal asymptote

34. If sin A = -12/13, cos B = 3/5, and A and B are in the same quadrant, then tan (A+B) is equal to

(a) 56/33 (b) - 16/53 (c) -56/33 (d) 56/63 (e) 16/53

35. The period of the sinusoidal function f(x) = 5 sin(4x + π /4) + 10 is equal to

(a) 5 (b) $\pi/2$ (c) $\pi/4$ (d) 2π (e) $2\pi + 10$

36. How far does the tip of a minute hand of a clock move in 35 minutes if the hand is 6 inches long?

(a) 7.5 π in. (b) 6.5 π in. (c) 8 π in. (d) 7 π in. (e) 21 in.

37. A coin is altered so that the probability it comes up heads is 0.4 and the probability it comes up tails is 0.6. The coin is tossed 5 times. Which of the following events is most likely to occur?

(a) 0 Heads 5 Tails (b) 1 Head 4 Tails (c) 2 Heads 3 Tails (d) 3 Heads 2 Tails (e) 4 Heads 1 Tail

38. The Supreme Court of the United States has nine justices. On a certain case they voted 5 to 4 in favor of the defendant. In how many ways could this have happened, given that Justice 'Connor voted with the minority?

(a) 126 (b) 336 (c) 16 (d) 56 (e) 256

39. The reference angle for an angle A = 23 $\pi/5$ (radians) is:

(a) π (b) $\pi/5$ (c) 2 $\pi/5$ (d) 3 $\pi/5$ (e) 4 $\pi/5$

40. A baseball coach is determining the batting order for the team. The team has 9 members, but the coach does not want the pitcher to be one of the first four to bat. How many batting orders are possible?

(a) 362,880 (b) 40,320 (c) 50,400 (d) 320,000 (e) 201,600