

THIRTIETH ANNUAL MATHEMATICS CONTEST  
Sponsored by  
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA II 1986

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Scoring formula:  $4R - W + 40$

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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TRW, Ross Gear Division, Lebanon, Tennessee



1. If  $f(x) = x^2 - x + 2$ , then  $f[f(3)] =$
- a) 64                                  d) 8  
b) 16                                  e) 24  
c) 58
2. If  $2^x = 3$  and  $2^y = 5$ , then  $2^{3x-y} =$
- a) 22                                  d) 2  
b) 16                                  e)  $27/5$   
c) 4
3. If  $2\log_5 x + \log_5 2 = \log_5 (5x+3)$ , then  $x =$
- a)  $-1/3$                               d) 3  
b)  $-1/2$                               e)  $(5+\sqrt{29})/2$   
c) 2
4. The solutions to the inequality  $|x-1| < 2$  are
- a)  $3 < x < -1$                       d)  $x > -1$   
b)  $-1 < x < 3$                       e)  $x < -1$  or  $x > 3$   
c)  $x > 3$
5. If  $x$  and  $y$  satisfy both of the equations  $x + 2y = 1$  and  $x + 4y = 3$ , then  $x + y =$
- a) -1                                  d) 2  
b) 0                                      e) 4  
c) 1
6. If  $\theta = 30^\circ$ , then  $\sec \theta + \cos \theta =$
- a) 1                                      d)  $\sqrt{3}/2 + 2$   
b) 0                                      e)  $\sqrt{3}/2 + 2/\sqrt{3}$   
c)  $\sqrt{2} + 1/\sqrt{2}$

7. The roots of the equation  $x^2 - 17x + 60 = 0$  include

- a) -17
- b) -3
- c) -12
- d) 5
- e) 20

8. If two fair, six-sided dice are tossed, then the probability of the sum being six is

- a)  $5/36$
- b)  $1/6$
- c)  $1/12$
- d)  $1/18$
- e)  $1/36$

9. An automobile travels from point A to point B at a constant speed of 120 miles per hour, and then travels from B to A at a constant speed of 140 miles per hour. What is the average speed for the entire trip, in miles per hour and rounded off to the nearest integer?

- a) 128
- b) 129
- c) 130
- d) 131
- e) 132

10. How many noncongruent triangles can be constructed that satisfy both of the following conditions?

- i) one side has length  $\sqrt{3}$  and serves as a side of a  $30^\circ$  angle
- ii) one side has length 1 and does not serve as a side of the  $30^\circ$  angle

- a) 0
- b) 1
- c) 2
- d) 3
- e) 4

11. The first five prime numbers are 2, 3, 5, 7, and 11. If a prime number is chosen at random from the first ten primes, then the probability that the number exceeds 20 is

- a)  $1/10$
- b)  $2/10$
- c)  $3/10$
- d)  $4/10$
- e)  $5/10$

12. When  $2x^3 - x^2 + 7x + 2$  is divided by  $x^2 + 1$ , the remainder is

- a)  $9x + 1$
- b)  $2x + 1$
- c)  $2x - 1$
- d)  $5x + 1$
- e)  $5x + 3$

13. Let A be the set of all real numbers  $x$  such that  $|x - 2| > 10$  and let B be the set of all real numbers  $x$  such that  $|x| > -4$ . Then  $A \cap B$  is

- a) the set of all real numbers satisfying:  $x < 8$  and  $x > 12$
- b) the set of all real numbers satisfying:  $x < -4$  or  $x > 4$
- c) the set of all real numbers satisfying:  $x < -8$  or  $x > 12$
- d) the empty set
- e) the set of all real numbers

14. The coefficient of the  $x^2y^4$  term in the expansion of  $(2x - 3y^2)^4$  is

- a) 6
- b) 36
- c) -6
- d) -36
- e) 216

15. The complex number  $(3 + i)^2$  equals

- a)  $8 + 6i$
- b)  $6 + 2i$
- c) 8
- d)  $6i$
- e) -1

16. The binary representation of the decimal number 14 is

- a) 1100
- b) 1101
- c) 1001
- d) 1111
- e) 1110

17. The number of solutions of the equation  $|x| = \left(\frac{x}{3}\right) + 1$  is

- a) 0
- b) 1
- c) 2
- d) 3
- e) infinite

18. If matrix  $A = \begin{bmatrix} -4 & 1 \\ 1 & 5 \end{bmatrix}$  and matrix  $B = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$ , then  $B \times A =$

- a)  $\begin{bmatrix} -8 & -13 \\ 2 & -2 \end{bmatrix}$
- b)  $\begin{bmatrix} -5 & 17 \\ -1 & -5 \end{bmatrix}$
- c)  $\begin{bmatrix} -8 & 3 \\ 0 & -5 \end{bmatrix}$
- d)  $\begin{bmatrix} -2 & 4 \\ 1 & 4 \end{bmatrix}$
- e)  $\begin{bmatrix} 42 \end{bmatrix}$

19. How many seven digit phone numbers (e.g., 382-1609) can be formed if the only restriction is that the first digit can not be 0, 1, or 9?

- a) 67
- b) 10,000,000
- c) 9,999,997
- d) 1,000,006
- e) 7,000,000

20. The pull of gravity between two bodies is inversely proportional to the square of the distance between them. If the distance between two asteroids in space is multiplied by 3, then the gravitational force between them is multiplied by

- a) 3
- b) 6
- c) 9
- d)  $\frac{1}{3}$
- e)  $\frac{1}{9}$

21. The solutions of  $x^2 + 2x + 1 \leq 25$  are

- a)  $x \leq 4$  and  $x \leq -6$
- b)  $x \leq 4$  or  $x \leq -6$
- c)  $x \leq 4$  and  $x \geq -6$
- d)  $x \leq 4$  or  $x \geq -6$
- e)  $x \geq 4$  and  $x \geq -6$

22. The numbers 143 and 32 are already written in base 8. The product written in base 8 is

- a) 2574
- b) 416
- c) 4576
- d) 5016
- e) 2176

23. The graph of the equation  $4x^2 + 11 = y^2 + 2y + 16x$  is

- a) a line
- b) a point
- c) a hyperbola
- d) an ellipse
- e) a parabola

24. How many ways could a 40 question multiple choice test be answered if each question has 5 choices for the answer?

- a) 200
- b)  $5^{40}$
- c)  $40^5$
- d)  $40!/5!$
- e)  $40!/35!$

25. The number of solutions of the equation  $|x| = x^2$  is

- a) 0
- b) 1
- c) 2
- d) 3
- e) 4

26. The value of the determinant  $\begin{vmatrix} 3 & -2 \\ -1 & 4 \end{vmatrix}$  is

- a) -10
- b) 14
- c) 4
- d) 10
- e) -14

27. The solution set of the system of linear equations  $\begin{cases} ax + by = e \\ cx + dy = f \end{cases}$  can be any of the following (depending on the values of  $a, b, c, d, e,$  and  $f$ ) except

- a) the set of all points in the plane
- b) a set of all points on a line in the plane
- c) a set of exactly two points in the plane
- d) a set of exactly one point in the plane
- e) the empty set

28. The sum of the two roots of the equation  $x^2 + bx - 2 = 0$  is 1 if  $b =$

- a) -1
- b) 3
- c) 1
- d) 0
- e) -3

29. If  $\log_a b = 5$ , then  $\log_b a =$

- a)  $5b/a$
- b) -5
- c) 5
- d)  $1/5$
- e)  $-1/5$

30. The number of real solutions of the equation  $\sqrt{x+2} = -x$  is

- a) 0
- b) 1
- c) 2
- d) 3
- e) 4

31. The equation  $x^2 + y^2 - 2x + 4y = 4$  describes a circle of radius

- a) 2
- b) 4
- c) 9
- d) 3
- e) 1



32. The number  $\log_2 16$  equals

- a) 16
- b) 8
- c) 4
- d) 2
- e) 1

33. A man working alone can paint a house in six days and his son working alone can paint the house in twelve days. How many days would it take them to paint the house if they work together?

- a) 1
- b) 2
- c) 3
- d) 4
- e) 5

34. A chemist has two saline solutions; solution A contains 15% salt (by weight) and solution B contains 40% salt. How much of each must be mixed in order to make a 100 ml solution containing 20% salt? (A milliliter is a unit of volume and is abbreviated ml.)

- a) 40 ml of A, 60 ml of B
- b) 55 ml of A, 45 ml of B
- c) 73 ml of A, 27 ml of B
- d) 75 ml of A, 25 ml of B
- e) 80 ml of A, 20 ml of B

35. If  $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$ , then  $\begin{vmatrix} a & b & c & 2 \\ a & b & c & 5 \\ d & e & f & 0 \\ g & h & i & 0 \end{vmatrix} =$

- a) 21
- b) -9
- c) 3
- d) 30
- e) 9

36. The number of rational roots of the equation  $x^5 - 6x^2 - 5x + 2 = 0$  is

- a) 0
- b) 1
- c) 2
- d) 3
- e) 4

37. The solutions of the inequality  $\frac{x-2}{x-1} > 3$  are

- a)  $x < 1/2$
- b)  $x > 1/2$
- c)  $1/2 < x < 1$
- d)  $x < 1/2$  or  $x > 1$
- e)  $x > 1$

38. One of the three real roots of the equation  $6x^3 - 29x^2 + 36x - 9 = 0$  is 3. The sum of the other two roots is

- a)  $7/6$
- b) 11
- c)  $5/3$
- d) 2
- e)  $11/6$

39. The number of solutions of the system  $\begin{cases} x + y + z = 3 \\ 3x - 2y + 7z = 3 \end{cases}$  is

- a) 0
- b) 1
- c) 2
- d) 3
- e) infinite

40. An equilateral triangle has height 6 inches (and hence area  $12\sqrt{3}$  square inches). If this triangle is cut down by trimming a 1 inch wide border from each of its three sides, the area of the resulting triangle, in square inches, is

- a)  $6\sqrt{3}$
- b)  $12\sqrt{3} - 6$
- c)  $10\sqrt{3} - 6$
- d)  $3\sqrt{3}$
- e)  $5\sqrt{3} - 3$



