

TWENTY-FOURTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA II TEST 1980

Scoring Formula: $4R - W + 40$

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Austin Peay State University, Clarksville,
Tennessee

This test was prepared from a list of Algebra II questions submitted by
Maryville College.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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1. x^4 divided by $\frac{1}{2}x^{-4}$ is equal to
- (a) 2
 - (b) $\frac{1}{2}$
 - (c) $\frac{1}{2}x^8$
 - (d) $2x^{-8}$
 - (e) $2x^8$
2. The expression $\frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}}$ is equal to
- (a) $2x + 2\sqrt{x^2 - 1}$
 - (b) $x + 2\sqrt{x^2 - 1}$
 - (c) x
 - (d) $x + \sqrt{x^2 - 1}$
 - (e) 0
3. $(\frac{1}{x} - \frac{1}{x+1}) \div (1 + \frac{1}{x})$ is equal to
- (a) $-\frac{1}{x+1}$
 - (b) $\frac{1}{(x+1)^2}$
 - (c) $-(1+x)$
 - (d) $\frac{1}{x^2}$
 - (e) none of the above
4. Which of the following is a factor of $x^4 - 8x$?
- (a) x^2
 - (b) $x + 2$
 - (c) $x^2 - 2x + 4$
 - (d) $x^2 + 2x + 4$
 - (e) $x^2 + 4$

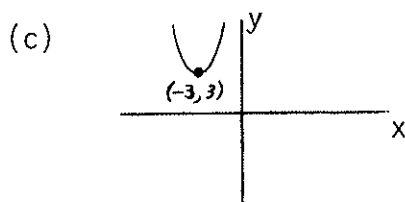
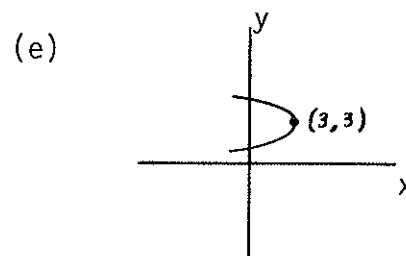
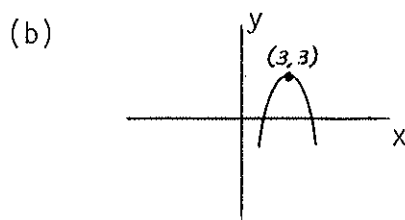
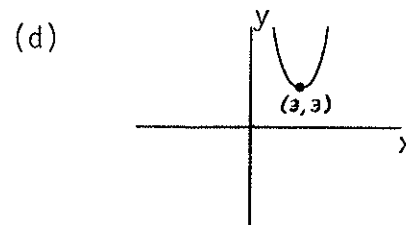
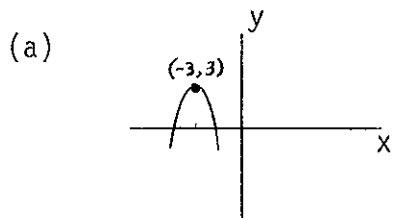
5. If $x = a$, $y = b$, and $z = c$ is the solution to $\begin{cases} x - y - z = 0 \\ x - y + z = 4 \\ x + y + z = 10 \end{cases}$, then $a^2b + c$ is equal to
- (a) 77
 - (b) 17
 - (c) 73
 - (d) -73
 - (e) none of the above
6. If $3(2 + \sqrt{x}) = 2(3\sqrt{2}) + \sqrt{x}$ then x is equal to
- (a) $6(\sqrt{2} - 1)$
 - (b) $36(3 - 4\sqrt{2})$
 - (c) $36(3 - 2\sqrt{2})$
 - (d) $36(3 - \sqrt{2})$
 - (e) none of the above
7. The graph of $f(x) = ax^2 + bx + c$ ($a \neq 0$) does not intersect the x -axis when $b^2 - 4ac$ is
- (a) positive
 - (b) zero
 - (c) negative
 - (d) a perfect square
 - (e) a prime number
8. $\log_{125}5 - \log_{36}216$ is equal to
- (a) $-\frac{7}{6}$
 - (b) $-\frac{1}{3}$
 - (c) 1
 - (d) $\frac{11}{6}$
 - (e) none of the above

9. An equation for the perpendicular bisector of the line segment joining the two points $P(1,3)$ and $Q(-3, -5)$ is
- (a) $y + 2x = -3$
 - (b) $2y + x = 3$
 - (c) $2y + x = -3$
 - (d) $y + 2x = 3$
 - (e) none of the above
10. The cube root of three times the sum of the roots of $|2x - 3| = |2 - x|$ is equal to
- (a) $\sqrt[3]{5}$
 - (b) $\sqrt[3]{6}$
 - (c) -2
 - (d) 2
 - (e) 24
11. If $\sum_{i=1}^n i = \frac{n(n+1)}{2}$ for all positive integers n , then $\sum_{i=1}^{100} (3i + 1)$ is equal to
- (a) 301
 - (b) 15151
 - (c) 30101
 - (d) 15250
 - (e) none of the above
12. If $\log_{10} 9 = 0.954243$ and $\log_{10} 2 = 0.301030$, then $\log_{10} 60$ (correct to four decimal places) is equal to
- (a) 1.2553
 - (b) 1.6000
 - (c) 1.7782
 - (d) 1.9085
 - (e) none of the above

13. If $x = a$ is the solution to $9^x = 3^{3x+1}$ and $x = b$ is the solution to $3^x = 9^{3x+1}$, then ab is equal to
- (a) -1
 - (b) $-\frac{2}{5}$
 - (c) 0
 - (d) 1
 - (e) $\frac{2}{5}$
14. The expression $\sqrt[m]{\frac{4^m \cdot 6}{4^{2m+1} + 2^{4m+1}}}$ is equal to
- (a) $\frac{1}{2^m}$
 - (b) $\frac{1}{4}$
 - (c) $\frac{1}{4^m}$
 - (d) 1
 - (e) none of the above
15. Water is added to 40 liters of a solution that is 30% acid to obtain a 20% acid solution. How many liters of water are added?
- (a) 20
 - (b) 2
 - (c) $\frac{40}{3}$
 - (d) 10
 - (e) none of the above
16. Last month a small business mailed out 160 letters, some requiring 15 cents postage and the rest requiring 30 cents postage. If the total cost for postage was \$38.40, what percentage of the letters required 15 cents postage?
- (a) 10%
 - (b) 20%
 - (c) 30%
 - (d) 40%
 - (e) 50%

17. When the polynomial $x^4 + 2x^3 - 3x^2 + 5x - 4$ is divided by $x + 3$, the remainder is equal to
- (a) 0
 - (b) 19
 - (c) -19
 - (d) 119
 - (e) none of the above
18. If $f(x) = 3x + 1$ and $g(x) = \sqrt{3x - 3}$ represent two functions, then $(f + g)(4)$ is equal to
- (a) 16
 - (b) 39
 - (c) 22
 - (d) 117
 - (e) 0
19. If the point (x, y) is 2 units from the point $(-1, 1)$ then
- (a) $x^2 + y^2 = 4$
 - (b) $(x - 1)^2 + (y + 1)^2 = 4$
 - (c) $(x + 1)^2 + (y - 1)^2 = 2$
 - (d) $(x - 1)^2 + (y + 1)^2 = 2$
 - (e) $(x + 1)^2 + (y - 1)^2 = 4$
20. The solution set of $6 + x - x^2 \geq 0$ is equal to
- (a) $\{x \mid -2 < x < 3\}$
 - (b) $\{x \mid 2 \leq x \leq 3\}$
 - (c) $\{x \mid -3 \leq x \leq -2\}$
 - (d) $\{x \mid -3 \leq x \leq 2\}$
 - (e) $\{x \mid -2 \leq x \leq 3\}$

21. If $f(x) = x^2 + 1$, then $\frac{f(x+h) - f(x)}{h}$ is equal to
- $2x + h + \frac{2}{h}$
 - 1
 - $2x + h$
 - $2x$
 - none of the above
22. If $f(x) = \frac{3x\sqrt{x-2}}{x-5}$ defines a real-valued function, then the largest possible domain of f is equal to
- $\{x|x \neq 5\}$
 - $\{x|x \geq 2 \text{ or } x \neq 5\}$
 - $\{x|x \geq 2\}$
 - $\{x|x \neq 5 \text{ and } x \geq 2\}$
 - none of the above
23. The graph of $f(x) = 3 - (x - 3)^2$ is



24. The number of roots of the equation $x - 1 = \sqrt{x + 5}$ is equal to
- (a) 0
 - (b) 1
 - (c) 2
 - (d) 3
 - (e) 4
25. Firewood is stacked in 25 layers. There are 3 logs in the top layer and each succeeding layer has 2 more logs than the layer above it. How many logs are in the stack?
- (a) 624
 - (b) 675
 - (c) 325
 - (d) 700
 - (e) none of the above
26. If $i = \sqrt{-1}$ then $(i^{54} - i^{80})^7$ is equal to
- (a) -1
 - (b) 128
 - (c) -128
 - (d) 0
 - (e) none of the above
27. If $y - 2x = -4$ and $y + x = 5$ are the equations of lines P and Q respectively, then an equation for the line passing through the intersection of lines P and Q and perpendicular to line P is
- (a) $2y + x = 13$
 - (b) $2y + x = 7$
 - (c) $2y + x = -1$
 - (d) $y - 2x = -4$
 - (e) none of the above

28. If $f(x) = \frac{3}{2}x - 3$ defines a function, then the formula for the inverse of f is
- $g(x) = \frac{2(x + 3)}{3}$
 - $g(x) = \frac{3}{2}(x - 3)$
 - $g(x) = \frac{2x + 3}{3}$
 - $g(x) = \frac{2}{3}x - \frac{1}{3}$
 - none of the above
29. The solution set of $1 < |2x - 2| \leq 4$ is
- $\{x | -1 \leq x < \frac{1}{2} \text{ or } \frac{3}{2} < x \leq 3\}$
 - $\{x | -1 < x \leq \frac{1}{2} \text{ or } \frac{3}{2} \leq x < 3\}$
 - $\{x | -1 \leq x \leq 3\}$
 - $\{x | x < \frac{1}{2} \text{ or } x > \frac{3}{2}\}$
 - none of the above
30. If $f(x) = \frac{|x|}{x}$ and $g(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$, then which of the following is a true statement?
- $f(x) = g(x)$ for all x in the domain of g
 - $f(x) = g(x)$ for all x in the domain of f
 - $f(x) \neq g(x)$ for some x in the domain of f
 - $f(x)$ and $g(x)$ are inverse functions
 - $f(x)$ and $g(x)$ are the same function
31. If A is the 2×2 matrix $\begin{bmatrix} a & 1 \\ 1 & d \end{bmatrix}$ and I is the 2×2 matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A^2 = I$, then
- $a = 1$ and $d = 1$
 - $a = 0$ and $d = 0$
 - $a = 0$ and $d = 1$
 - $a = 1$ and $d = 0$
 - a and d can be any real numbers such that $a = -d$

32. A company can now ship a cargo of 200 tons of fertilizer at a profit of \$10 per ton. However, by waiting, the company can add 10 tons per week to the shipment, but the profit on the entire shipment will be reduced by 20 cents per ton per week. When should the shipment be made to realize the maximum profit?
- (a) now
 - (b) in 10 weeks
 - (c) in 15 weeks
 - (d) in 35 weeks
 - (e) none of the above
33. The solution set of $|2 - 5x| = -1$ is
- (a) $\{\frac{3}{5}\}$
 - (b) $\{\frac{1}{5}, \frac{3}{5}\}$
 - (c) $\{-\frac{3}{5}\}$
 - (d) $\{-\frac{1}{5}, -\frac{3}{5}\}$
 - (e) \emptyset (the empty set)
34. The sum of the factors of $(2a + 3b)^2 - (a - b)^2$ is equal to
- (a) 0
 - (b) $a + 2b$
 - (c) $a + 4b$
 - (d) $4a + 4b$
 - (e) $4a + 6b$
35. The product of the roots of $x^4 - 2x^2 - 3 = 0$ is equal to
- (a) 3
 - (b) -3
 - (c) $\sqrt{3}i$
 - (d) $-\sqrt{3}i$
 - (e) $-3i$

36. The numerical coefficient of x^4y^{18} in the binomial expansion of $(3x - y^3)^{10}$ is equal to
- (a) 210
 - (b) -210
 - (c) 17010
 - (d) -17010
 - (e) none of the above
37. How many different 7-letter "words" can be formed using the letters in the word ALGEBRA? (A 7-letter "word" is any sequence of 7 letters).
- (a) 2520
 - (b) 5040
 - (c) $\frac{7!}{2!5!}$
 - (d) $2(7!)$
 - (e) none of the above
38. The number of real roots of the equation $x^4 - 2x^2 - 1 = 0$ is equal to
- (a) 0
 - (b) 1
 - (c) 2
 - (d) 3
 - (e) 4
39. The determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & x & x^2 \\ 1 & 0 & x \end{vmatrix}$ is equal to 24. The sum of the two possible values of x is equal to
- (a) 1
 - (b) -1
 - (c) 0
 - (d) 4
 - (e) none of the above

40. In a group of 5 persons, the height of the shortest person is 150 centimeters and the height of the tallest person is 180 centimeters. Which of the following could be the average (mean) height, in centimeters, of the 5 people?
- (a) 152
 - (b) 157
 - (c) 175
 - (d) 178
 - (e) all of the above