

TWENTY-THIRD ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA II TEST

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Scoring Formula: $4R - W + 40$

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This test was prepared from a list of Algebra II questions submitted by Memphis State University.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

- (a) $2/3$ (b) 3 (c) 6
(d) $3/2$ (e) none of these

A B C D E
1.

The correct answer for the sample problem is $3/2$, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. In simplest form $4\sqrt{12} + 2\sqrt{12}$ is
 - (a) $6\sqrt{12}$
 - (b) $6\sqrt{24}$
 - (c) $12\sqrt{3}$
 - (d) 36
 - (e) none of these

2. In simplified form $\sqrt{(-4d)^2}$ is
 - (a) $-4d$
 - (b) $4|d|$
 - (c) $4d$
 - (d) $-2d$
 - (e) none of these

3. Compute the sum $\frac{3}{4} + \frac{a}{b}$, $b \neq 0$.
 - (a) $\frac{3+a}{4b}$
 - (b) $\frac{3+4a}{b}$
 - (c) $\frac{3b+a}{b}$
 - (d) $\frac{3b+4a}{4b}$
 - (e) none of these

4. Solve $3(y - 1) - 2(y + 2) = 0$ for y . Then
 - (a) $y < 0$
 - (b) $-1 < y < +1$
 - (c) $y < 10$
 - (d) $0 < y < 7$
 - (e) none of these

5. Which of the following is not a term in the result obtained by expanding $-2x^2yz^2(3xy - 2x^3z + 2y^2z^3) + x^3yz$?
- (a) $-4x^2y^3z^5$
 - (b) $-6x^3y^2z^2$
 - (c) $6x^2y^2z^3$
 - (d) $4x^5yz^3$
 - (e) x^3yz
6. If $r = 7$ and $s = -9$ then $|s - r| + r - s =$
- (a) 32
 - (b) 18
 - (c) 0
 - (d) $2r - 2|s|$
 - (e) none of these
7. If $f(x) = x^2 - x$ then $f(2 - a)$ is
- (a) $(a - 1)(a - 2)$
 - (b) $2 + a - a^2$
 - (c) $2 - a$
 - (d) $2 + a + a^2$
 - (e) none of these
8. If $\frac{x}{1/4} = 25$, then
- (a) $x = 100$
 - (b) $4x = 100$
 - (c) $.25x = 25$
 - (d) $x = 6.25$
 - (e) none of these

9. Solve $\frac{A}{e} = \frac{r + R}{R}$ for r .
- (a) $r = \frac{A}{e} - 1$
 - (b) $r = R(\frac{A}{e} - 1)$
 - (c) $r = \frac{A}{e}$
 - (d) $r = \frac{AR}{e}$
 - (e) none of these
10. Which of the following is a factor of $(x - y)^2 - 25$?
- (a) $x - y$
 - (b) $x + 5$
 - (c) $(x - y)5$
 - (d) $x - y - 5$
 - (e) none of these
11. The difference of the two factors of $4x^2 + 12x + 9$ is
- (a) 6
 - (b) $4x$
 - (c) $x + 1$
 - (d) 0
 - (e) none of these
12. Solve the system of equations $5x + 3y = 2$. Then $x + y$ is
 $3x + 5y = -2$
- (a) 2
 - (b) -2
 - (c) 4
 - (d) -4
 - (e) none of these

13. $\frac{x^2 - 4x - 45}{4xy + 20y} \div \frac{x^2 - 81}{3x + 27}$ is equal to
- (a) $3/(4y)$
 (b) $\frac{(x - 9)^2}{12y}$
 (c) $\frac{x^2 - 4x - 45}{3(4xy - 20x)}$
 (d) $\frac{x + 9}{4y(x + 5)}$
 (e) none of these
14. If $x \neq 0$, then $\frac{x - \frac{a^2}{x}}{1 + \frac{a}{x}} =$
- (a) $\frac{x - a^2}{1 + a}$
 (b) $x - a$
 (c) $\frac{x - a}{2}$
 (d) $1 - a$
 (e) none of these
15. Compute the product $(\frac{x}{3} + \frac{y}{5})(\frac{x}{3} + \frac{y}{5})$ and write the result using the least common denominator. Which of the following is not in your final result?
- (a) 25
 (b) y^2
 (c) 225
 (d) x^2
 (e) none of these
16. A straight line has equation $\begin{vmatrix} 1 & -2 & x \\ 2 & 0 & y \\ 2 & -4 & 1 \end{vmatrix} = 0$. The slope of this line is
- (a) 8
 (b) $\frac{1}{2}$
 (c) 2
 (d) undefined or infinite
 (e) none of these

17. Solve $(x + 2)^2 + (x - 3)^2 = 15$ and compute the product of the two roots. The result is
- (a) -2
 (b) 2
 (c) -1
 (d) 1
 (e) none of these
18. Divide $4x^3 - 3x^2 + x + 7$ by $x - 2$ and subtract the quotient from the remainder. The result is
- (a) $4x^2 + 5x - 18$
 (b) $18 - 4x^2 - 5x$
 (c) $-62 - 4x^2 + 11x$
 (d) $4x^2 - 11x + 62$
 (e) none of these
19. The denominator of a fraction is 8 less than the numerator. If 4 is added to the numerator, the value of the fraction becomes five. What is the numerator of the fraction?
- (a) 15
 (b) 11
 (c) 3
 (d) 7
 (e) none of these
20. $\left[\frac{.15x^{2.5} y^{-3.4}}{.3x^{-3.5} z^{-1.6}} \right]^{-.7}$ is
- (a) not defined
 (b) $\frac{y^2}{.5x^4z}$
 (c) $\frac{2.7y^{2.38}}{x^{4.2}z^{1.12}}$
 (d) in simplest form
 (e) none of these

21. Write a formula for the total time, t , required by an automobile to travel n miles at the rate of r miles per hour and return the same distance at the rate of $(r + 8)$ miles per hour.
- (a) $t = \frac{n}{r}$
- (b) $t = \frac{2n}{2r + 8}$
- (c) $t = 2\left(\frac{n}{r + 8}\right)$
- (d) $t = \frac{2n}{r} + \frac{n}{8}$
- (e) none of these
22. If $\log_2(x^2 - 64) - \log_2(x + 8) = 3$ then $\log_2 x$ is equal to
- (a) 4
- (b) 16
- (c) 2
- (d) 8
- (e) none of these
23. Solve $\sqrt{7 - 4x} - \sqrt{3 - 2x} = 1$. The sum of the solutions is
- (a) $3/2$
- (b) 1
- (c) 2
- (d) $-1/2$
- (e) none of these
24. How many elements are in the solution set for the system $\frac{1}{x^2} + \frac{1}{y^2} = 13$
 $\frac{1}{xy} = 6$
- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) none of these

25. The distance between the minimum point on the graph of $y = x^2 - 2x - 3$ and the point where the curve intersects the positive x -axis is
- (a) 4.5
 - (b) 4
 - (c) $2\sqrt{5}$
 - (d) $4\sqrt{2}$
 - (e) none of these
26. Which of the following conditions must hold for $\sqrt{x^2 + y^2} = \sqrt{x^2} + \sqrt{y^2}$ to be true?
- (a) $x = y$
 - (b) $xy = 0$
 - (c) $x \geq 0$ and $y \geq 0$
 - (d) $\pm\sqrt{x} = \sqrt{y}$
 - (e) none of these
27. A man wants to fence in a rectangular area next to a river. He does not want to put a fence along the side next to the river. He has 100 m. of fence. What is the largest area he can fence in?
- (a) 625 m.²
 - (b) 1111.11 m.²
 - (c) 1875 m.²
 - (d) 1250 m.²
 - (e) none of these
28. What is the smallest value of x for which $|x^2 - 8x| > (x + 4)^2$ is not true?
- (a) -2
 - (b) -4
 - (c) 4
 - (d) 2
 - (e) none of these

29. $\frac{1}{3+4i} + \frac{3}{5} - \frac{4i}{5}$ is equal to

(a) $\frac{6-8i}{25}$

(b) $\frac{18-24i}{25}$

(c) $\frac{4i+2}{5}$

(d) $\frac{24+18i}{25}$

(e) none of these

30. Which of the following inequalities is not always true when $a > 0$ and $b > 0$?

(a) $\frac{a}{b} + \frac{b}{a} \geq 2$

(b) $a^3b + ab^3 \geq 2a^2b^2$

(c) $a^3 - b^3 > a^2b - ab^2$ if $a > b$

(d) $a + \frac{1}{a} \geq 0$

(e) none of these

31. Given that $x_{ki} = \begin{cases} a & \text{if } k = i \\ b & \text{if } k < i \\ c & \text{if } k > i \end{cases}$ then $\sum_{k=1}^4 \prod_{i=1}^3 x_{ki}$ is equal to

(a) $ab(b+c) + c^2(a+c)$

(b) $4abc$

(c) $a^4b^4c^4$

(d) $a^3bc + ab^3c + abc^3$

(e) none of these

32. If $x > 0$ then $\log_e \left[\frac{x}{\sqrt{\frac{y}{\sqrt{e^{-1}}}}} \right]^\pi$ is equal to

(a) $\frac{-\pi}{xy}$

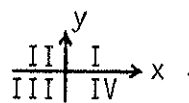
(b) $-\pi xy$

(c) $e^{\pi xy}$

(d) $e^{\pi/xy}$

(e) none of these

33. Let the quadrants of the rectangular coordinate system be identified by

 Then the solution set for the system $9 \leq x^2 + y^2 \leq 16$ $-1 \leq x - y \leq 1$ is found in

- (a) I only
 (b) III only
 (c) II and III
 (d) I and III
 (e) none of these
34. Which of the following is a factor of $x^3 - x^2 + 4x - 4$?
- (a) $x + 2$
 (b) $x^2 - x + 4$
 (c) $x - 2i$
 (d) $x + 1$
 (e) none of these
35. Which of the following is the equation of an asymptote of the graph of $\left\{ (x, y) \mid y = \frac{x^2 - 4}{x - 1} \right\}$?
- (a) $x = 4$
 (b) $y = x - 2$
 (c) $y = x + 1$
 (d) $y = x + 2$
 (e) none of these
36. If $a > b > 0$ then $a^a b^b$ is greater than
- (a) a^{ab} (d) b^b
 (b) $a^b b^a$ (e) none of these
 (c) a^a

37. Let $h(x) = \begin{cases} 2x + 5 & \text{if } x > 9 \\ x^2 - |x| & \text{if } -9 \leq x \leq 9 \\ x - 4 & \text{if } x < -9 \end{cases}$ and $g(x) = \begin{cases} |x| & \text{if } -1 < x < 1 \\ \sqrt[3]{x^3} & \text{otherwise.} \end{cases}$

Then $h(g(x))$ is equal to

- (a) $g(x)$
 (b) $h(x) + g(x)$
 (c) $g(h(x))$
 (d) $h(x)$
 (e) none of these
38. Find the sum of the roots of $3^x + 3^{-x} = 3$.
- (a) $\log_3 2$
 (b) $2 \log_3 4$
 (c) 2
 (d) $\log_3 1$
 (e) none of these
39. Define $f(x) = [x]$ to mean the greatest integer not greater than x . Which of the following equations correctly relates the cost C of mailing a letter and the weight of the letter in ounces x when the postage is d cents per ounce or fraction thereof?
- (a) $C = d[x + 1]$ (d) $C = -d[-x], x > 0$
 (b) $C = d[x]$ (e) none of these
 (c) $C = d[x] + d$
40. Let A and B be 2×2 matrices. Define an operation $*$, by $A * B = AB - BA$. If $A * B = 0$, then which of the following must be true?
- (a) $A = 0$ or $B = 0$
 (b) $A = B$
 (c) $*$ is commutative
 (d) $*$ is associative
 (e) none of these

