

TWENTY-SECOND ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA II TEST

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Scoring Formula: $4R - W$

This test was prepared from a list of Algebra II questions submitted by Shelby State Community College.

DIRECTION:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

- (a) $2/3$. (b) 3. (c) 6.
(d) $3/2$ (e) none of the above

1. A B C D E

The correct answer for the sample problem is $3/2$, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is compiled.

When told to do so, open your test booklet to page 1 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. If T varies directly as B and B varies directly as D , what effect on T will the doubling of D have?
- (a) none
 - (b) quadruple T
 - (c) halve T
 - (d) square T
 - (e) double T
2. The solution to $\frac{2}{x^2} = \left| \frac{3}{x} \right|$ is
- (a) $x = 0, x = \frac{2}{3}$
 - (b) $x = \frac{2}{3}$
 - (c) $x = \frac{2}{3}, x = -\frac{2}{3}$
 - (d) $x = 0, x = \frac{2}{3}, x = -\frac{2}{3}$
 - (e) $x = \sqrt{6}, x = \frac{2}{3}$
3. Solve
$$\begin{aligned}x - z &= 2 \\x + y &= 3 \\2x - y + z &= 3\end{aligned}$$
- (a) $x = 0 \quad y = 3 \quad z = -2$
 - (b) $x = 1 \quad y = 2 \quad z = -1$
 - (c) $x = 3 \quad y = 0 \quad z = 1$
 - (d) $x = 2 \quad y = -5 \quad z = 2$
 - (e) $x = 2 \quad y = 1 \quad z = 0$
4. If $\log_e a = 1 - y$, then $e^y =$
- (a) a
 - (b) $\frac{e}{a}$
 - (c) ae
 - (d) $\log_a e$
 - (e) e^a

5. If $x = 49$, then $\sqrt{1 - \frac{13}{x}} =$
- (a) $\frac{6}{7}$
 - (b) $\frac{1}{2}$
 - (c) $\frac{1}{2i}$
 - (d) $\frac{\sqrt{13}}{4\sqrt{3}}$
 - (e) none of the above
6. One of the factors of the polynomial, $1 - x - y - z + xy + xz + yz - xyz$, is
- (a) $x - y$
 - (b) $1 + z$
 - (c) $z + x$
 - (d) $1 - y$
 - (e) $x - y - z$
7. Evaluate $(.0049)^{\frac{3}{2}} + (.2)^4 + (.04)^{\frac{-1}{2}}$.
- (a) 5.001943
 - (b) .78875
 - (c) 5.000503
 - (d) .201943
 - (e) none of the above
8. On migration, a Canadian Goose flew 280 miles in $3\frac{1}{2}$ hours on Tuesday and 315 miles in 5 hours on Wednesday. What was his average flying speed for the two days?
- (a) 35 mph
 - (b) 70 mph
 - (c) 145 mph
 - (d) 72.5 mph
 - (e) 18 mph

9. A man drives a car at 30 mph for 1 mile of a 2-mile course. At what rate of speed must he drive the last mile to average 60 mph for the entire 2-mile course?
- (a) 120 mph
 - (b) Impossible to accomplish
 - (c) 90 mph
 - (d) 105 mph
 - (e) none of the above
10. The distance between (7, -3) and (-4, 2) is
- (a) $\sqrt{146}$
 - (b) $4\sqrt{6}$
 - (c) $\sqrt{122}$
 - (d) $\sqrt{10}$
 - (e) 16
11. The sum $\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \dots$ is equal to
- (a) 1
 - (b) 2
 - (c) 1.9987
 - (d) 1.5
 - (e) Infinite
12. A cowboy puts, at random, two bullets into his empty six shooter. What is the chance that the six shooter will not fire the first time he pulls the trigger?
- (a) 1
 - (b) $\frac{1}{6}$
 - (c) $\frac{2}{3}$
 - (d) $\frac{1}{2}$
 - (e) $\frac{5}{6}$

13. Solve the following equation for t .

$$w = \frac{a - 7\sqrt{t}}{\sqrt{t}}$$

(a) $\frac{a^2}{w^2 - 14w + 49}$

(b) $\frac{a^2}{w^2 + 49}$

(c) $a^2 - 14a + \frac{49}{w}$

(d) $\frac{a^2}{w^2 + 14w + 49}$

(e) $a^2 + 14a + \frac{49}{w}$

14. If the function f is defined by $f(x) = \frac{1}{\sqrt{3x - 12}}$, then the set of all real numbers included in the domain of f is

(a) $\{x \in \mathbb{R} \mid x \geq 4\}$

(b) $\{x \in \mathbb{R} \mid x > -4\}$

(c) $\{x \in \mathbb{R} \mid x > 4\}$

(d) \emptyset

(e) all real numbers except 4

15. The equation of the line through $(3,2)$ and $(-2,-2)$ is

(a) $5y - 4x = -2$

(b) $5y - 4x = 7$

(c) $5y - 4x = 2$

(d) $y = 3$

(e) none of the above

16. The inequality $|x + 1| > \sqrt{x^2 + 1}$ is valid when

(a) $x > 0$

(b) $x < 0$

(c) $x < -1$

(d) $-1 < x < 1$

(e) never valid

17. The coefficient of x^5y^3 in the expansion of $(\frac{x}{4} - 2y)^8$ is

- (a) 56
- (b) -28
- (c) $\frac{-1}{128}$
- (d) 64
- (e) $\frac{-7}{16}$

18. Consider the following proposed solution of $2(x - 2)(x + 7) = 2x - 4$. In which of the steps has an incorrect procedure been followed.

Step A: $2(x - 2)(x + 7) = 2x - 4$

Step B: $(2x - 4)(x + 7) = 2x - 4$

Step C: $\frac{(2x - 4)(x + 7)}{2x - 4} = \frac{2x - 4}{2x - 4}$

Step D: $x + 7 = 1$

Step E: $x = -6$

- (a) Step A, B and C
- (b) Steps B and C
- (c) Step B only
- (d) Steps A and B
- (e) Step C only

19. The equation $\begin{vmatrix} 1 & 0 & 1 \\ 0 & x & y \\ 4 & y & x \end{vmatrix} = 0$ defines a set of points whose locus is a

- (a) straight line
- (b) ellipse
- (c) hyperbola
- (d) parabola
- (e) none of the above

20. If $i = \sqrt{-1}$, then the value of $\frac{5i^3 + 2i}{3i^3} - i^6$ is
- (a) $\frac{10}{3}$
 - (b) $\frac{4}{3}$
 - (c) -2
 - (d) 0
 - (e) none of the above
21. Consider the two relations defined by $(x - 2)^2 + (y - 1)^2 = 4$ and $y = \frac{1}{3}x - 1$. Their graphs intersect in how many points?
- (a) 0
 - (b) 1
 - (c) 2
 - (d) 3
 - (e) more than 3
22. The sum of the reciprocals of two numbers equals the quotient of the two reciprocals. The general solution to this problem determines
- (a) a straight line
 - (b) one pair of numbers only
 - (c) a circle
 - (d) a parabola
 - (e) an ellipse
23. The first time Randy assembled a puzzle it took him 128 minutes. Each time after that, he assembled the puzzle in one-half of his previous time until he could assemble it every time in eight minutes. From start to finish, he assembled the puzzle 12 consecutive times. How many minutes did he spend assembling the puzzle 12 times?
- (a) 304
 - (b) 176
 - (c) $255 \frac{15}{16}$
 - (d) 1536
 - (e) 316

24. How much copper and how much iron should be added to 100 kilograms of an alloy containing 25% copper and 40% iron in order to obtain an alloy containing 30% copper and 50% iron?
- (a) 5 kgs. of copper, 10 kgs. of iron
 - (b) 27.5 kgs. of copper, 47.5 kgs. of iron
 - (c) 47.5 kgs. of copper, 27.5 kgs. of iron
 - (d) 55 kgs. of copper, 90 kgs. of iron
 - (e) none of the above
25. Of 33 species of birds found in either the North American or South American continent, 12 species are found on the North American continent only while 8 species are found on both continents. How many of the 33 species are found on the South American continent?
- (a) 12
 - (b) 13
 - (c) 21
 - (d) 25
 - (e) 28
26. The equation $x^3 + x^2 + 23x + 23 = 0$ has
- (a) 3 real roots
 - (b) 3 imaginary roots
 - (c) 2 real roots and 1 imaginary root
 - (d) 2 imaginary roots and 1 real root
 - (e) no real or imaginary roots
27. One value of x that makes the inequality, $y - 2 < x - 3 < y < x$, true for every value of y , subject to $-1 < y < 3$, is
- (a) $x = \frac{3}{2}$
 - (b) $x = 5$
 - (c) $x = -3$
 - (d) $x = 0$
 - (e) No such value of x exists.

28. In the field of real numbers, the two field operations are addition and multiplication. Below are four statements concerning this field. Which of these statements is true?

- I. Additive inverses are unique.
- II. The field operation of multiplication is distributive over addition.
- III. All elements have a multiplicative inverse.
- IV. 1 is the multiplicative identity.

- (a) All are true.
- (b) None are true.
- (c) All but I are true.
- (d) All but II are true.
- (e) All but III are true.

29. Mary's father, John, will be 5 times Mary's age in 4 more years. At present, the difference in their ages is 32. Mary's age is

- (a) 2
- (b) 3
- (c) 4
- (d) 5
- (e) 6

30. Which of the following four equations is not a trigonometric identity?

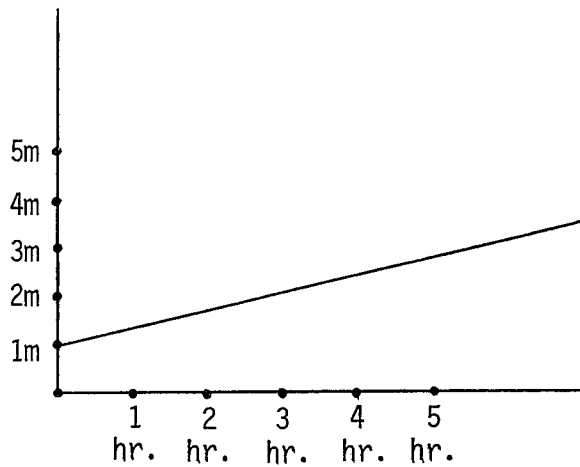
- (a) $\cos^2 x = \frac{1 + \cos 2x}{2}$
- (b) $\tan^2 x = 1 + \sec^2 x$
- (c) $\cos(x + y) = \cos x \cos y - \sin x \sin y$
- (d) $\sin x = 2 \sin\left(\frac{1}{2}x\right) \cos\left(\frac{1}{2}x\right)$
- (e) All of the above are trigonometric identities.

31. What is the length of the shortest side of a rectangle with an area of 1596 square meters, such that the difference of any two adjoining sides is 29 meters.

- (a) $\sqrt{784}$
- (b) 29
- (c) 49
- (d) 32
- (e) 221

32. Let $f(x) = x + 3$ and $g(x) = x^{-\frac{1}{2}}$. Let $h(x) = g(f(x))$. The set of all real numbers which may be included in the domain of h is
- (a) $\{x|x > 3\}$
 - (b) $\{x|x \geq -3\}$
 - (c) $\{x|x < -3\}$
 - (d) $\{x|x > -3\}$
 - (e) none of the above
33. The difference of the two roots to the equation $4x^2 + 8x + 13 = 0$ is
- (a) +1
 - (b) +2
 - (c) +3
 - (d) $3i$
 - (e) $\frac{+3i}{2}$
34. The speed of water flowing in a stream varies jointly as the drop in the stream bed and the slope of its banks. If water flows at 1.6 feet per second in a stream whose bed drops 1 foot in every thousand feet and whose banks slope at 20° , then what is the drop in every thousand feet of a stream whose banks slope at 30° and whose water speed is 0.8 feet per second.
- (a) $\frac{1}{3}$ foot
 - (b) 2 feet
 - (c) $\frac{2}{3}$ feet
 - (d) $\frac{4}{3}$ feet
 - (e) 1.28 feet
35. If $\tan x = \frac{4}{5}$, which of the following statements must be false?
- (a) $\sin x < 0$ and $\cos x < 0$
 - (b) $\csc x < 0$ and $\sec x < 0$
 - (c) $\sec x > 0$ and $\sin x < 0$
 - (d) $\sec x < 0$ and $\cos x < 0$
 - (e) none of the above

36.



Consider the time-distance graph above. Which one of the following statements about this graph is true?

- (a) Distance varies directly as time.
- (b) Time varies inversely as the distance.
- (c) The graph is representative of a second degree polynomial.
- (d) The product of each distance with its respective time is a constant.
- (e) The graph has a negative slope.

37. Given that $3 \times 27 = 81$, determine the value of $\log_9 27$.

- (a) 3
- (b) 2
- (c) $\frac{1}{3}$
- (d) .3333
- (e) $\frac{3}{2}$

38. In Alaska, the state license plates for automobiles have one letter followed by four numerals. How many possible different license plates can be made?

- (a) 100,000
- (b) 26,000,000
- (c) 10,000
- (d) 260,000
- (e) 170,586

39. If $-2i$ is one of the roots of $x^3 + 3x^2 + 4x + 12 = 0$, then a real root is
- (a) 4
 - (b) 3
 - (c) 1
 - (d) -1
 - (e) none of the above
40. Find the solution set, over the set of real numbers, of the inequality $x^2 < 12 - 4x$.
- (a) $\{x : -2 < x < 6\}$
 - (b) $\{x : x < -2 \text{ or } x > 6\}$
 - (c) $\{x : x < -6 \text{ or } x > 2\}$
 - (d) $\{x : -6 < x < 2\}$
 - (e) \emptyset