

TWENTY-FIRST ANNUAL MATHEMATICS CONTEST

Sponsored by  
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA II TEST

1977

Scoring Formula: 4R - W

EDITED BY:

Richard Detmer  
and  
Billy Edwards  
The University of Tennessee  
at Chattanooga,  
Chattanooga, Tennessee

This test was prepared from a list of Algebra II questions submitted by  
Roane State Community College.

---

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If  $2x = 3$ , then  $x$  equals
- |                    | A | B | C | D                                   | E |
|--------------------|---|---|---|-------------------------------------|---|
| (a). $2/3$ .       |   |   |   | <input checked="" type="checkbox"/> |   |
| (b). 3.            |   |   |   |                                     |   |
| (c). 6.            |   |   |   |                                     |   |
| (d). $3/2$         |   |   |   |                                     |   |
| (e). none of these |   |   |   |                                     |   |
- 1.

The correct answer for the sample problem is  $3/2$ , which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 1 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. If  $F(x) = 3x - 2$  and  $F(y) = x$ , then  $y =$ 
  - (a).  $3x - 2$
  - (b).  $(x + 2) / 3$
  - (c).  $-1/2$
  - (d).  $x$
  - (e). none of these
  
2. The solution set for  $2x^2 + 5x - 3 \geq 0$  is
  - (a).  $\{x: x \geq 3 \text{ or } x \leq -1/2\}$
  - (b).  $\{x: x \leq -3 \text{ or } x \geq 1/2\}$
  - (c).  $\{x: -3 \leq x \leq 1/2\}$
  - (d).  $\{x: -1/2 \leq x \leq 3\}$
  - (e).  $\phi$
  
3. If  $x = 1 - e^t$  and  $y = 1 + e^{-t}$ , find  $y$  in terms of  $x$ 
  - (a).  $y = x$
  - (b).  $y = \frac{1}{x-1}$
  - (c).  $y = \frac{x-2}{x-1}$
  - (d).  $y = \frac{x}{x-1}$
  - (e). none of these
  
4. The roots of the equation  $x^2 - 2x - 6 = 0$  are
  - (a). Complex
  - (b). real and unequal
  - (c). equal
  - (d). rational and unequal
  - (e). none of these

5. A golf ball is dropped from a height of 6 feet. On each rebound it rises  $\frac{2}{3}$  of the height from which it last fell. Approximately what distance has it traveled at the instant it strikes the ground for the 7th time?
- (a). 21.89 feet  
 (b). 27.89 feet  
 (c). 25.41 feet  
 (d). 32.71 feet  
 (e). 33.89 feet
6. If  $F(x) = 4x^5 - 2x^3 + x^2 - 1$  then  $F(-1)$  is
- (a). -2  
 (b). 0  
 (c). 2  
 (d). -4  
 (e). 1
7. The inverse  $F^{-1}(x)$  of a function  $F(x) = 2^x$  is
- (a).  $2^{-x}$   
 (b).  $2^{F(x)}$   
 (c).  $\log_x 2$   
 (d).  $\log_2 x$   
 (e). none of these
8. The solution set of  $x^2 - 3x - 7 = 0$  is
- (a).  $\left\{ \frac{3 + \sqrt{37}}{2}, \frac{3 - \sqrt{37}}{2} \right\}$   
 (b).  $\left\{ \frac{-3 + i\sqrt{19}}{2}, \frac{-3 - i\sqrt{19}}{2} \right\}$   
 (c).  $\{9/2, -3/2\}$   
 (d).  $\left\{ 3 + \frac{\sqrt{37}}{2}, 3 - \frac{\sqrt{37}}{2} \right\}$   
 (e).  $\left\{ 3 + \frac{i\sqrt{19}}{2}, 3 - \frac{i\sqrt{19}}{2} \right\}$

9. The determinant  $\begin{vmatrix} 1 & -2 & 8 \\ 1 & x & 3x^2 \\ 1 & 6 & 0 \end{vmatrix}$  is equal to  $-32$ .

Then the only values possible for  $x$  are

- (a). 3 and  $-2/5$
  - (b).  $-3$  and  $2/5$
  - (c).  $3/5$  and  $-2$
  - (d).  $5/3$  and  $-2$
  - (e). none of these
10. In order for the sum of two positive primes to be a prime, one of the primes must be odd and the other must be
- (a). 3
  - (b). 1
  - (c). 2
  - (d). the next consecutive prime
  - (e). odd
11. A central angle of  $3/4$  radian in a circle of radius 8 will intercept an arc of length
- (a). 6 units
  - (b). 8 units
  - (c).  $6/\pi$  units
  - (d).  $\pi/6$  units
  - (e). none of these
12. The graph of  $x^2 - y^2 = 25$  is
- (a). a circle
  - (b). an ellipse
  - (c). a line
  - (d). a parabola
  - (e). a hyperbola

13. If  $4^x = 2^5$ , then  $x$  is equal to
- (a). 2
  - (b). 10
  - (c).  $5/2$
  - (d).  $2/5$
  - (e). 2.4
14. If \$120 is invested at 6% annual rate for 6 months, and the new amount is reinvested at the same rate for an additional 6 months, find the amount at the end of the year.
- (a). \$126.20
  - (b). \$127.31
  - (c). \$130.00
  - (d). \$125.25
  - (e). \$126.71
15. The center of the circle described by  $x^2 + y^2 + 2x + 8y = 8$  is
- (a). ( 1, 4)
  - (b). (-1, 4)
  - (c). ( 2, 8)
  - (d). (-1, -4)
  - (e). (-2, -8)
16.  $x^6 - 64$  is factored completely into real factors as
- (a).  $(x^3 + 8)(x^3 - 8)$
  - (b).  $(x^3 + 8)(x - 2)(x^2 + 2x + 4)$
  - (c).  $(x + 2)(x^2 - 2x + 4)(x - 2)(x^2 + 2x + 4)$
  - (d).  $(x^3 - 8)(x + 2)(x^2 - 2x + 4)$
  - (e).  $(x + 2)(x^2 - 4x + 4)(x - 2)(x^2 + 4x + 4)$

17. The number  $2^n + 2^n + 2^n + 2^n$  can be written as
- (a).  $2^{4n}$
  - (b).  $16^n$
  - (c).  $24(4n^2 - 2)$
  - (d).  $2^{2n+2}$
  - (e).  $(16)^{\frac{n+2}{4}}$
18. The diameter of a circle in which a triangle with sides of length 3, 4, and 5 has been inscribed is
- (a). 5
  - (b).  $25/4$
  - (c).  $\sqrt{5}$
  - (d).  $\sqrt{50}$
  - (e). none of these
19. If the domain of  $x$  is  $\{1, 2, 10, 12\}$ , what is the solution set of  $x + 2 > 4$
- (a).  $\{1, 2\}$
  - (b).  $\{10, 12\}$
  - (c).  $\{x \mid x \geq 2\}$
  - (d).  $\phi$
  - (e). none of these
20. The solution set for  $\frac{2x}{x+1} + \frac{x}{1-x} - \frac{4}{x^2-1} = 0$  is
- (a).  $\phi$
  - (b).  $\{4, -1\}$
  - (c).  $\{4\}$
  - (d).  $\{0, 1\}$
  - (e).  $\{4, 1\}$

21. If the point  $(3, y)$  lies on the line joining  $(0, 3/2)$  and  $(9/4, 0)$ , then  $y =$
- (a).  $-7/2$
  - (b).  $-2$
  - (c).  $-1/2$
  - (d).  $1/2$
  - (e). none of these
22. A representative of the Lara See Co. reported the following data from a supermarket interview with 40 shoppers: 32 like the brownies, 23 liked the cheesecake, 19 liked the poundcake, 19 liked brownies and cheesecake, 7 liked cheesecake and poundcake, 14 liked brownies and poundcake, and 6 liked all three. How many did not like any of the Lara See products?
- (a). 2
  - (b). 5
  - (c). 1
  - (d). 10
  - (e). 0
23. Find the solution set for  $x$  in the following equation:  $2x + 3\sqrt{x} - 20 = 0$
- (a).  $\{16, 25/4\}$
  - (b).  $\{16\}$
  - (c).  $\{5/2, -4\}$
  - (d).  $\phi$
  - (e).  $\{25/4\}$
24. The slope of the perpendicular bisector of the line segment joining  $(2,1)$  and  $(6,9)$  is
- (a). 2
  - (b).  $-2$
  - (c).  $1/2$
  - (d).  $-1/2$
  - (e). none of these

25. If  $i = \sqrt{-1}$  then  $i^{371}$  equals
- $i$
  - $-1$
  - $1$
  - $-i^3$
  - $-i$
26. The equation  $x^3 - 2x^2 + x - 2 = 0$  has
- 3 real roots
  - 1 real root and 2 imaginary
  - 2 real roots and 1 imaginary
  - 3 imaginary roots
  - no roots
27. When simplified and expressed without negative exponents, the expression  $(x^{-2}y^{-2})^{-1}$  is equal to
- $x^2 - y^2$
  - $\frac{1}{x^2 - y^2}$
  - $\frac{x^2y^2}{y^2 - x^2}$
  - $\frac{1}{y^2 - x^2}$
  - $x^2 + y^2$
28. The simplified form for  $\frac{1}{1 - \frac{1}{1 - \frac{1}{1-x}}}$  is
- $x$
  - $\frac{1}{1-x}$
  - $1$
  - $x - 1$
  - $-x$



29. The product of  $(r^2 s^2 + t^2)(t^2 + r^2 s^2)$  is
- $r^4 s^4 + t^4$
  - $r^4 s^4 - t^4$
  - $r^4 s^4 + 4t^2 r^2 s^2 + t^4$
  - $r^4 s^4 + 2r^2 s^2 t^2 + t^4$
  - $r^4 s^4 t^4 + 2r^2 s^2 + t^4$
30. A value for  $k$  such that the equation  $x^2 + kx + 24 = 0$  has 2 consecutive even integers for roots is
- 4
  - 6
  - 12
  - 2
  - none of these
31. The solution for the system  $\begin{cases} x^2 - y^2 = 9 \\ x + y = 1 \end{cases}$  is
- (2, -1)
  - (4, -5)
  - (5, -4)
  - (-5, 4)
  - (3, -2)
32. The term containing  $x^4$  in the expansion of  $(2x - \frac{y}{4})^8$  is
- $\frac{35 x^4 y^4}{8}$
  - $-128 x^4 y^4$
  - $256 x^3 y^5$
  - $\frac{-256 x^5 y^3}{9}$
  - none of these

33. If matrix  $P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ , then a matrix  $Q$  such that  $QP = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$  is

(a).  $P$

(b).  $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

(c). nonexistent

(d).  $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

(e). none of these

34. The repeating decimal  $1.1\overline{66}$  may be expressed as

(a).  $6/7$

(b).  $7/6$

(c).  $5/6$

(d).  $6/5$

(e). none of these

35. Two boys on a boating trip could row 9 m.p.h. in still water. One day they rowed down a stream  $3 \frac{1}{3}$  hours before it ran into a river. They then rowed  $2 \frac{1}{2}$  hours down the river before making camp. They traveled 15 miles more in the stream than in the river. If the current in the stream was 2 m.p.h. faster than that of the river, find the speed of the current in the stream in miles per hour.

(a). 1

(b).  $2 \frac{1}{2}$

(c). 2

(d). 3

(e). none of these

36. The period of  $y = 2 \sin \left( \frac{x}{2} \right)$  is
- (a).  $2\pi$
  - (b).  $\pi$
  - (c).  $3\pi$
  - (d).  $4\pi$
  - (e).  $8\pi$
37. The equation  $\sqrt{x - 2} = x - 4$  has
- (a). no roots
  - (b). one root
  - (c). two roots
  - (d). three roots
  - (e). four roots
38. The solution set of  $|x + 3| \leq |x + 2|$  is
- (a).  $\{x | x \leq -5/2\}$
  - (b).  $\phi$ , the empty set
  - (c).  $\{x | -3 \leq x \leq -2\}$
  - (d).  $\{x | x \leq -3\}$
  - (e).  $\{x | x \leq -2\}$
39. Given that neither  $x$  nor  $m$  equals zero, the solution for  $x$  of the equation  $\log_m x^2 - \log_m \frac{x}{3} = 0$  is
- (a). 3 or 0
  - (b).  $1/3$
  - (c). 9 or 3
  - (d). 27
  - (e).  $1/9$

40. If  $\tan x = 1/3$ ,  $\sin 2x$  equals

(a).  $3/5$

(b).  $6/5$

(c).  $3/4$

(d).  $3/10$

(e). none of these