SIXTIETH ANNUAL MATHEMATICS CONTEST 2016

Algebra I

Prepared by:

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Scoring formula: 4 x (Number Right) - (Number Wrong) + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the <u>best</u> answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase <u>completely</u>. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

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- 1. Identify the next number in the pattern 1, 6, 15, 28, 45, 66, 91, ___
 - a. 110
 - b. 120
 - c. 130
 - d. 140
 - e. 150
- 2. Which of these expressions is closest in value to 9?
 - a. $(\sqrt{4} + \sqrt{5})^2$
 - b. $4\sqrt{5}$
 - c. $(\sqrt{14} \sqrt{5})^2$
 - d. $\sqrt{70}$
 - e. $3\sqrt{10}$
- 3. $8^5 \times 4^3 =$
 - a. 32¹⁵
 - b. 8⁸
 - c. 4^{15}
 - d. 2²¹
 - e. 12¹⁵
- 4. $(3.7 \times 10^{14}) \times (9.1 \times 10^{-13}) =$
 - a. 3.367×10^2
 - b. 3.367×10^{1}
 - c. 3.367×10^{-182}
 - d. 3.367×10^{28}
 - e. 3.367
- 5. A pair of pants is on sale for 30% off. I have a coupon that lets me take an additional 20% off the sale price of an item. What is the final discount of the pair of pants I buy on sale and with my coupon?
 - a. 50% less
 - b. 44% less
 - c. 56% less
 - d. 60% less
 - e. 0.6% less

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- 6. Using the simple interest formula $I=\operatorname{prt}$, determine the amount of interest (I) available after \$250 is placed into an account earning 7.25% annual interest for 18 months.
 - a. \$32,625
 - b. \$2718.75
 - c. \$36.25
 - d. \$326.25
 - e. \$27.19
- 7. If I flip a coin four times, what is the probability that I have the same number of heads as tails?

 - a. $\frac{1}{2}$ b. $\frac{3}{8}$
 - c.
 - d. $\frac{1}{16}$
 - e. 0
- 8. If I roll two regular six-sided dice, what is the probability of getting a sum that is a prime number?

 - a. $\frac{1}{2}$ b. $\frac{5}{11}$ c. $\frac{6}{11}$ d. $\frac{15}{36}$
- 9. In six days, I ran these distances in kilometers: 4, 6, 6, 2, 8, 4. Which of the following gives a number of kilometers that I can run on the seventh day so that my mean and median are equal for the week?
 - a. 4
 - b. 5
 - c. 6
 - d. 7
 - e. 8

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- 10. In a small bag are some dimes and quarters. In total, there are 13 coins in the bag worth a total of \$2.35. How many quarters are in the bag?
 - a. 5
 - b. 6
 - c. 7
 - d. 8
 - e. 9
- 11. I am 32 years older than my daughter. Five years from now, I will be five times older than she will be. How old is my daughter right now?
 - a. 3
 - b. 5
 - c. 8
 - d. 9
 - e. 10
- 12. Which of these coordinates could not be a vertex of a right triangle whose other vertices are located on the coordinates (24, 7) and (24, -4)?
 - a. (84,7)
 - b. (84, -4)
 - c. (-36,7)
 - d. (-36, -4)
 - e. (84,4)
- 13. Solve for x: $\frac{2}{3}x + \frac{5}{8}x = 10$ a. $x = \frac{110}{7}$ b. $x = \frac{310}{24}$ c. $x = \frac{240}{31}$ d. $x = \frac{70}{11}$ e. $x = \frac{209}{24}$
- 14. Solve the inequality: $-11x 9 \le 90$
 - a. $x \ge -9$
 - b. x < -9
 - c. $x \le -9$
 - d. $x \ge 9$
 - e. $x \leq 9$

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- 15. To convert a temperature from Fahrenheit to Celsius, you use the formula $C = \frac{5}{9}(F-32)$. What is the temperature in Celsius when it is 68 degrees Fahrenheit?
 - a. -25 degrees Celsius
 - b. 55 degrees Celsius
 - c. 20 degrees Celsius
 - d. 149 degrees Celsius
 - e. 85 degrees Celsius
- 16. The Acme Widget Company determined its cost function to be y = 3x + 500, where x = number of widgets produced and y = cost of producing x widgets. What does y = 3x + 500 tell us about the cost of making x widgets?
 - a. Acme spends \$3 to make an additional widget and \$500 no matter how many widgets they make.
 - b. Acme spends \$500 to make an additional widget and \$3 no matter how many widgets they make.
 - c. Acme spends \$1 to make 3 more widgets and \$500 no matter how many widgets they make.
 - d. Acme spends \$1 to make 500 more widgets and \$3 no matter how many widgets they make.
 - e. Acme spends \$503 to make an additional widget.
- $17. (3x^2 + y)(x + 2y^2) =$
 - a. $x^6 + 6y^4 + xy + y^5$
 - b. $4x^2 + x^6y^4 + xy + y^6$
 - c. $4x^3 + 5xy^4 + xy + 3y^3$
 - d. $3x^3 + 6x^2y^2 + xy + 2y^3$
 - e. $3x^2 + 6x^4 + xy + 2y^2$
- $18.\frac{x^2-4}{x^2+4} \div \frac{x^2-4x+4}{x^2-5x+6} =$
 - a. 0
 - b. undefined
 - c. $\frac{x^2-x-c}{x^2+4}$
 - $d. \quad \frac{x-3}{x+2}$
 - e. $\frac{3}{2}$

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19. Which of these coordinates is a solution to the system of equations

$$\begin{cases} y = 5x - 6 \\ y = x^2 - 5x + 10 \end{cases}$$

- a. (1,-1)
- b. (4,6)
- c. (3,9)
- d. (5, 10)
- e. (2,4)
- 20. Which coordinate is a solution to the system of inequalities $\begin{cases} y \le 3x 2 \\ y > 4x + 7 \end{cases}$
 - a. (-9, -29)
 - b. (-9, -30)
 - c. (-9, -32)
 - d. (-11, -34)
 - e. (-11, -36)
- 21. Which line will not intersect with the line whose equation is y = 4x + 1?
 - a. 8x 2y = -2
 - b. x 4y = 3
 - c. 4x + y = 5
 - d. 12x 3y = 5
 - e. 3x 12y = 8
- 22. For the given table of values, identify the equation that best models the data

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	Y	4	16	64	256

a.
$$y = x^2$$

b.
$$y = 2^x$$

c.
$$y = 2x$$

d.
$$y = x + 2$$

e.
$$y = 32x$$

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23. Evaluate $f(x) = (x-2)^{100} + (2x-7)^{99} + (x^2-8)^{98} + (-4x+11)^{97}$ when x = 3

- a. 0
- b. 2
- c. -2
- d. 4
- e. -4

24. Evaluate f(g(4)) when $f(x) = x^2$ and g(x) = 2x - 3

- a. 25
- b. 29
- c. 80
- d. 5
- e. 16

25. Which of these values can <u>not</u> be in the range of the function f(x) = 4|x-5| +

- 2?
- a. 2
- b. 4
- c. 0
- d. 5
- e. 6

26. Identify the slope of the line that passes through the points of the parabola given by $y = x^2 + 4x + 3$ when x = 0 and x = 4

- a. 0
- b. 3
- c. 4
- d. 8
- e. 32

27. A farmer has 300 feet of fencing. She wants to use this fencing to construct a rectangular corral and then divide the corral in half by constructing a fence across the width of the corral. What should the outer dimensions be to maximize the area of the corral?

- a. 150 feet by 150 feet
- b. 75 feet by 75 feet
- c. 200 feet by 100 feet
- d. 100 feet by 100 feet
- e. 50 feet by 75 feet

28. What is the sum of the roots of the quadratic equation $3x^2 - 18x + 15 = 0$?

- a. 18
- b. -18
- c. 6
- d. −6
- e. 3

29. If xy = 8 and $x^2 + y^2 = 20$, what is one possible value of x + y?

- a. 12
- b. 20
- c. 64
- d. 28
- e. 6

30. What is the maximum value of the function $f(x) = -2x^2 + 32x - 110$

- a. 110
- b. 8
- c. 18
- d. -8
- e. 32

31. For what value of *b* will $x^2 - bx + 49 = 0$ have exactly one solution?

- a. 0
- b. 7
- c. 14
- d. 49
- e. 98

32. A box with a square base has a volume of $16 in^3$. The height of the box is twice as long as the length of the sides of the square base. What is the height of the box?

- a. 2 in
- b. 4 in
- c. $2\sqrt{2}$ in
- d. $4\sqrt{2}$ in
- e. 8 in

- 33. The equation $x^3 + 5x^2 4x 20 = 0$ has 2 as one of its solutions. What is the product of the other two solutions?
 - a. -10
 - b. 10
 - c. 3
 - d. 7
 - e. -20
- 34. What value is **not** a solution to the equation $x^4 13x^2 + 36 = 0$?
 - a. -3
 - b. -2
 - c. 2
 - d. 3
 - e. 4
- 35. What is the domain of the function $(x) = \sqrt{x^2 + 7x + 10}$?
 - a. $x \ge 5$ or $x \le 2$
 - b. $x \le 5$ or $x \ge 2$
 - c. $x \ge -5$ or $x \le -2$
 - d. $x \le -5$ or $x \ge -2$
 - e. $-5 \le x \le -2$
- 36. The greatest common factor of $60x^3y^2z^5$ and $72x^2y^3z^2$ is
 - a. 2*xyz*
 - b. $12x^2y^2z^2$
 - c. $12x^3y^3z^5$
 - d. $6x^3y^2z^5$
 - e. $24x^3y^3z^5$
- 37. What is the solution to the equation $\frac{x^2 13x + 40}{x^2 10x + 25} = 0$?
 - a. x = 8 or 5
 - b. x = 8
 - c. x = 5
 - d. x = -5 or x = 5
 - e. no solution

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38. Solve for x:
$$\frac{1}{x+2} + \frac{5}{2x+5} = \frac{29}{2x^2+9x+10}$$

a.
$$x = 2$$

b.
$$x = 4$$

c.
$$x = 0$$

d.
$$x = \frac{22}{7}$$

e. $x = \frac{2}{11}$

e.
$$x = \frac{2}{11}$$

39. Determine the values of A and B that make the following equation true:

$$\frac{A}{x-5} + \frac{B}{x-2} = \frac{10x-41}{x^2-7x+10}$$

a.
$$A = 3$$
 and $B = 7$

b.
$$A = 7$$
 and $B = 3$

c.
$$A = 10$$
 and $B = -41$

d.
$$A = 4$$
 and $B = 6$

e.
$$A = 2$$
 and $B = 8$

40. For the function $f(x) = x^2 - x - 6$, simplify the expression $\frac{f(x) - f(3)}{x - 3}$

a.
$$x^2 + 1$$

c.
$$\frac{-2}{3}$$

d.
$$x - 3$$

e.
$$x + 2$$

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