

FIFTIETH ANNUAL MATHEMATICS CONTEST  
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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

**Algebra I 2006**

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Scoring formula:  $4R - W + 40$

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**DIRECTIONS:**

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

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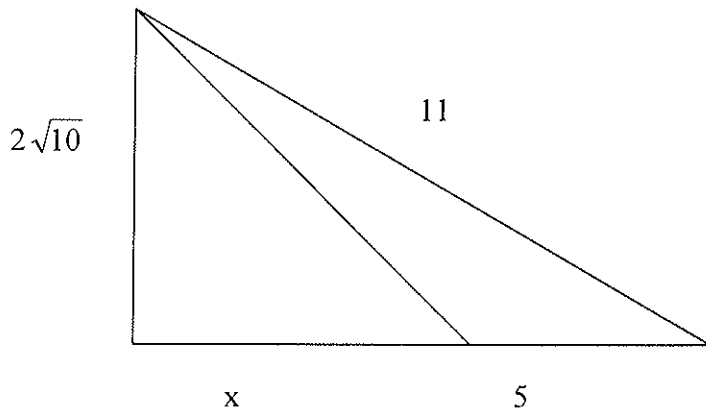
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## ALGEBRA I

- Perform the indicated operations:  $(6x^3 + 8) + (x^2 - 3x - 12)$ 
  - $7x^3 - 3x - 4$
  - $7x^3 - x^2 - 4$
  - $6x^3 + x^2 + 3x - 4$
  - $6x^3 + x^2 - 3x - 4$
  - $6x^3 + x^2 - 3x - 20$
- Divide:  $\frac{6x^2 - 13x + 6}{3x - 2}$ 
  - $2x + 3$
  - $3x^2 - 3$
  - $2x - 3$
  - $3x - 3$
  - $-11x - 3$
- Solve:  $4x - 2(1 + x) = 2x - 3$ 
  - $x = 0$
  - all real numbers
  - no solution
  - $x = -\frac{1}{3}$
  - $x = -\frac{1}{4}$
- The x-intercept is 4 and the y-intercept is 2. Find the equation for the line.
  - $y = \frac{1}{2}x + 2$
  - $y = \frac{1}{2}x - 2$
  - $y = -\frac{1}{2}x + 2$
  - $y = -\frac{1}{2}x - 2$
  - $y = -\frac{1}{2}x + 4$
- Simplify the following expression:  $\frac{x^2 - x - 6}{x^2 + 6x + 8} \div \frac{x^2 - 9}{x^2 - 16}$ 
  - $\frac{x^2 - 2x - 8}{x^2 - x - 6}$
  - $\frac{x - 3}{x + 4}$
  - $\frac{x - 4}{x + 3}$
  - $\frac{4}{3}$
  - $\frac{x + 3}{x - 4}$
- Find the coordinates of the vertex of the parabola representing the equation  $y = x^2 - 6x + 4$ .
  - (0, 4)
  - (1, -1)
  - (5, -3)
  - (3, -5)
  - (3, 3)
- The equation  $x^3 + x^3 = x^4$  is
  - true for all real numbers
  - false for all real numbers
  - true for exactly one real number
  - true for exactly two real numbers
  - true for exactly three real numbers

8. Find  $x$  in the right triangle (not drawn to scale):



- a) -14      b) 7      c) 8      d) 9      e) 4

9. Find the missing table value that will make this function linear:

x	y
a	-7
a + 5	3
a + 15	?

- a) 6      b) 13      c) 23      d) 2      e) 20

10. Simplify:  $\left(\frac{3x^2y^{-3}}{z^3}\right)^{-3}$

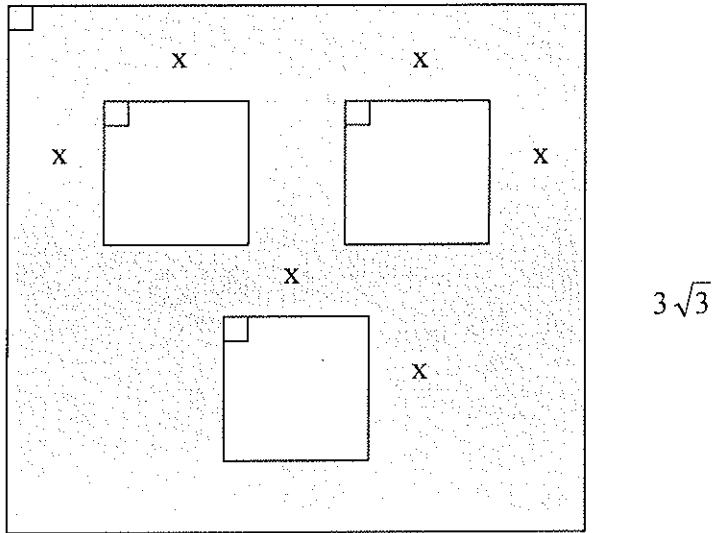
- a)  $-\frac{y^{27}z^{27}}{9x^8}$       b)  $\frac{z^6}{27xy^6}$       c)  $\frac{xy^6}{27}$
- d)  $\frac{y^9z^9}{27x^6}$       e)  $\sqrt[3]{\frac{3x^2}{y^3z^3}}$

11. Find the x-intercepts for the graph of the equation  $y = 8x^2 - 30x + 25$ .

- a)  $x = 5$  and  $x = 6$       b)  $x = \frac{5}{4}$  and  $x = \frac{5}{2}$       c)  $x = 3$  and  $x = 5$
- d)  $x = 5$  and  $x = 10$       e)  $x = 8$  and  $x = -30$

12. Express the area of the shaded region as a polynomial. If the polynomial is then factored completely, one of the factors is:

$$3\sqrt{3}$$



- a)  $3+x$       b)  $3\sqrt{3}-x$       c)  $27-6x$       d)  $5-x$       e)  $27$

13. Simplify:  $\frac{1+\frac{1}{x}}{1+\frac{1}{x}}$  ( $x \neq 0$  and  $x \neq -1$ )

- a)  $x$       b)  $\frac{x+1}{x}$       c)  $\frac{x}{x+1}$       d)  $\frac{1}{x}$       e)  $\frac{x^2+1}{x}$

14. Find the inverse function for the function defined by  $y = \frac{\sqrt{3x-5}}{2}$ .

- a)  $x = \frac{4y^2+5}{3}$       b)  $y = \frac{2}{\sqrt{3x-5}}$       c)  $y = \frac{4x^2+5}{3}, x \geq 0$   
d)  $y = \frac{3}{4x^2+5}$       e)  $y = \frac{\sqrt{3x+5}}{2}, x \geq -\frac{5}{3}$

15. The domain of the function  $f(x) = \frac{3x+2}{x^2-3x-10}$  includes all real numbers except which of the following?

- a)  $-\frac{2}{5}$       b)  $0$       c)  $3$  and  $-3$       d)  $2$  and  $-10$       e)  $-2$  and  $5$

16. Solve for  $x$ :  $16^{5x} = \frac{8^{3x}}{2}$

a)  $x = -\frac{1}{11}$     b)  $x = -\frac{4}{3}$     c)  $x = 3$     d)  $x = 0$     e)  $x = \frac{1}{5}$

17. What is the shortest distance between the line  $4x - 3y = 12$  and the origin?

a) 1.2    b)  $7\sqrt{6} + 3$     c) 2.4    d) 3.5    e) 3

18. Which of the following expressions is not a polynomial?

a)  $3x^2 + 5$     b)  $4x^3y^2 - 7$     c)  $7y^3$     d)  $\frac{x^4}{2y} + 3x$     e) 0

19. Find the values of  $m$  so that the distance between the roots of  $3x^2 + mx - 12 = 0$  is 5.

a) 0    b)  $\pm 4$     c)  $\pm 7$     d)  $\pm 9$     e)  $\pm 13$

20. Find the general expression of a quadratic function that passes through (0, 3) and (8, 3).

a)  $f(x) = ax^2 - 8x + 3$     b)  $f(x) = a(x^2 - 8x) + 3$     c)  $f(x) = a(x^2 - 8x + 3)$   
d)  $f(x) = x^2 - 8x + 3$     e)  $f(x) = 3x^2 + 5x + 8$

21. The distance from Nashville to Jackson is about 120 miles. If Joe has two hours to make the trip but drives 50 mph for the first 60 miles, how fast must Joe travel during the second 60 miles of the trip in order to reach Jackson in the two hour time period?

a) 25    b) 33.25    c) 30    d) 60    e) 75

22. Find the equation of the line perpendicular to  $4x - 3y = 12$  that passes through the point (2, 3).

a)  $y = 0.75x + 1.5$     b)  $y = -0.75x + 4.5$     c)  $y = -0.25x + 3.5$   
d)  $y = 1.33x - 4$     e)  $y = 4x + 3$

23. Find the vertex of the quadratic function that has " $a$ " for the  $x^2$  coefficient and  $x$ -intercepts (2, 0) and (10, 0).

a)  $(6, -16a)$     b)  $(6, a)$     c)  $(a, 0)$     d)  $\left(\frac{6}{a}, -16\right)$     e)  $(6, -16)$

24. For which value of  $c$  will the following equation have exactly one solution?

$$x(x - c) = 1 - c$$

- a) 0            b) 1            c) 2            d) 0.5            e)  $\pm 2$

25. Suppose a truck loaded with 400 computers weighs  $x$  lbs, and it weighs  $y$  lbs when loaded with 300 computers of the same type. How many lbs does the empty truck weigh?

- a)  $4y - 3x$     b)  $4x - 3y$     c)  $\left(\frac{4}{3}\right)(x - y)$     d)  $\left(\frac{3}{4}\right)(x - y)$     e)  $3y - 4x$

26. A composition teacher needs to choose 10 students out of 15 to serve as rough draft reviewers. A group of 10 seniors, 3 juniors, and 2 sophomores have volunteered. If the students are chosen randomly, what is the probability that 6 seniors, 2 juniors, and 2 sophomores will be selected?

- a)  $\frac{214}{3003}$     b)  $\frac{10}{429}$     c)  $\frac{30}{143}$     d)  $\frac{10}{143}$     e)  $\frac{10}{429}$

27. A bag contains 10 red marbles, 5 gray marbles, 12 black marbles, and 8 white marbles. Two marbles are randomly drawn from the bag without replacement. What is the probability of drawing a black marble followed by a red marble?

- a)  $\frac{276}{595}$     b)  $\frac{24}{245}$     c)  $\frac{22}{35}$     d)  $\frac{23}{49}$     e)  $\frac{12}{119}$

28. When Tom drives from his home to Chicago, he always arrives at 5:00 p.m. If he leaves home for Chicago at 1:00 p.m., he can average 60 miles per hour. What time should he leave home if he wants to average 75 miles per hour for the entire trip?

- a) 1:40 p.m.            b) 3:12 p.m.            c) 3:20 p.m.  
d) 1:48 p.m.            e) 1:15 p.m.

29. Considering the expression  $\frac{\frac{w}{x}}{\frac{y}{z}}$ , which of the variables can be 0 if it is true that  $\frac{\frac{w}{x}}{\frac{y}{z}} = \frac{wz}{xy}$ ?

- a)  $y$  only            b)  $w$  and  $y$             c)  $w$  only  
d)  $w$ ,  $x$  and  $y$             e) all of the variables could be 0

30. The sum of the reciprocals of the roots of the equation  $6x^2 + x - 15 = 0$  is:

- a)  $-\frac{1}{6}$       b)  $\frac{1}{6}$       c)  $\frac{1}{15}$       d)  $-\frac{1}{15}$       e)  $\frac{19}{15}$

31. The product of "two less than three times a number and five more than twice the same number" can be expressed as:

- a)  $6x^2 + 11x - 10$       b)  $6x^2 - 11x + 10$       c)  $6x^2 + 19x + 10$   
d)  $6x^2 - 11x - 10$       e)  $4x^2 + 4x - 15$

32. The solution of the inequality  $\left| \frac{5-2n}{5} \right| \geq 2$  is:

- a)  $\left( \frac{1}{2}, \frac{15}{2} \right)$       b)  $\left( \frac{15}{2}, \infty \right)$       c)  $\left( -\infty, -\frac{5}{2} \right) \cup \left( \frac{15}{2}, \infty \right)$   
d)  $\left( -\infty, -\frac{5}{2} \right] \cup \left[ \frac{15}{2}, \infty \right)$       e)  $\left( -\infty, \frac{1}{2} \right)$

33. The simplest form of the expression  $\frac{2m^2 - 5m - 3}{9 - m^2} \div \frac{4m + 2}{2m^2 + 2m - 12} \cdot \frac{2}{m - 2}$  is:

- a)  $\frac{2(2m-1)}{-(m-3)(2m+1)}$       b)  $\frac{4(m+3)}{-(m-3)}$       c) 2      d) -2      e)  $\frac{2m+1}{m+3}$

34. What is the equation of the perpendicular bisector of the line segment which has as its end points the points  $A(-2,3)$  and  $B(5,-4)$ ?

- a)  $x + y - 2 = 0$       b)  $-x + y + 2 = 0$       c)  $x + y + 1 = 0$   
d)  $x + y - 1 = 0$       e)  $-x + y - 2 = 0$

35. If the point  $P(3,-4)$  is on the graph of  $y = f(x)$ , then which of the following points is on the graph of  $G(x) = f(x-4) - 3$ ?

- a)  $(7,-4)$       b)  $(3,-7)$       c)  $(-1,-7)$       d)  $(7,-7)$       e)  $(0,-8)$

36. If  $f(x) = 3^x$  and  $g(x) = x - 4$ , what is the value of  $f(g(2))$ ?

- a) -6      b) -9      c) 5      d)  $\frac{1}{9}$       e) 9

37. Suppose  $y$  varies inversely as the square of  $x$  and directly as the cube of  $z$ . If the value of  $x$  is halved and the value of  $z$  is tripled, what happens to the value of  $y$ ?

- a)  $y$  is multiplied by 6      b)  $y$  is multiplied by  $\frac{1}{6}$       c)  $y$  is multiplied by 108  
d)  $y$  is multiplied by  $\frac{3}{2}$       e)  $y$  is unchanged

38. Rationalize the denominator and simplify:  $\frac{a}{\sqrt[3]{a+b}}$

- a)  $\frac{a\sqrt[3]{a^2} - ab\sqrt[3]{a} + ab^2}{a+b^3}$       b)  $\frac{a\sqrt[3]{a^2} + ab\sqrt[3]{a} + ab^2}{a+b^3}$       c)  $\frac{a\sqrt[3]{a} - ab}{a+b^3}$   
d)  $\frac{a\sqrt[3]{a} + ab}{a+b^3}$       e)  $\frac{a(\sqrt[3]{a} - b)}{a+b^3}$

39. Find all the values of  $k$  such that  $f(x) = kx^3 + x^2 + k^2x + 3k^2 + 11$  is divisible by the linear polynomial  $(x+2)$ .

- a) 3      b) 5      c) -3      d) -5      e) 3, 5

40. Three houses are sharing their electricity. One of the houses uses  $(12 + 5\sqrt{48})W$ , another uses  $(10 + 3\sqrt{27})W$ , and the final one uses  $(15 + 2\sqrt{108})W$ . How much total electricity are they using?

- a)  $(37 + 47\sqrt{3})W$       b)  $(37 + 41\sqrt{3})W$       c)  $(22\sqrt{3} + 10\sqrt{5})W$   
d)  $(9\sqrt{3} + 7\sqrt{5})W$       e)  $(37 + \sqrt{183})W$