THIRTY-SIXTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

NOTE: 1993 CONTEST DATE--APRIL 6

1. Evaluate: $-6^0 \cdot (-4)^2 \cdot (-2)^{-2} \cdot 8^{2/3}$

(a) −6   (b) 16   (c) −16   (d) 0   (e) 6

2. Evaluate: $((3^{-1})^2)^{-1} \cdot ((16)^{3/4})^{-2}$

(a) 81/16   (b) 7/12   (c) 12/7   (d) 9/64   (e) 16/81

3. Solve for b in terms of A, B, and h: $A = \dfrac{(b + B)h}{2}$

(a) $\dfrac{2A - B}{h}$ (b) $\dfrac{A}{2} - \dfrac{B}{h}$ (c) $\dfrac{2A - Bh}{h}$ (d) $\dfrac{A}{2h} - B$ (e) $\dfrac{2A - 2B}{h}$

4. The binomial, $x^8 - y^{16}$, has how many prime factors:

(a) 6   (b) 3   (c) 5   (d) 4   (e) 2

5. Simplify: $((x^{3/4}y^{-5/6})/(x^{-4/5}y^{2/3}))^{-2}$

(a) $\dfrac{y^3}{x^{31/10}}$ (b) $\dfrac{y^3}{x^{1/10}}$ (c) xy^3

(d) $\dfrac{1}{yx^{1/9}}$ (e) $\dfrac{x^{7/9}}{y^{7/9}}$

6. The closest approximation to the square root of 10 is:

(a) 3   (b) 3.1   (c) 3.2   (d) 3.3   (e) pi

7. Simplify:

$$\sqrt{(x-y)^2 + (x+y)^2}$$

(a) $2x$ (b) $\sqrt{2}(x+y)$ (c) $\sqrt{2x^2 + 2y^2}$ (d) $x+y$ (e) $x-y$

8. Simplify: $\dfrac{p^2 - q^2}{p^2 + 2pq + q^2}$

(a) $\dfrac{-1}{2pq}$ (b) -1 (c) $\dfrac{p-q}{p+q}$ (d) $p+q$ (e) $\dfrac{1}{2pq}$

9. Subtract: $(2x^2y^2 - 3x^2y + 4xy^2) - (3x^2y^2 + 7x^2y - 5xy^3)$

(a) $-x^2y^2 - 10x^2y + 9xy^2$ (b) $-x^2y^2 - 10x^2y + 4xy^2 + 5xy^3$ (c) $-x^2y^2 + 4x^2y + 4xy^2 - 5xy^3$

(d) $-x^2y^2 + 4x^2y - xy^3$ (e) $5x^2y^2 - 10x^2y + 4xy^2 - 5xy^3$

10. Multiply: $(4x^2+7y)(4x^2+2y+z)$

(a) $16x^4+36x^2y^2+14y^2$ (b) $16x^2+36xy+4x^2z+14y^2+7z$ (c) $16x^4+7yz$

   (d) $16x^4+36x^2y+14y^4+4x^2yz$ (e) $16x^4+36x^2y+4x^2z+14y^2+7yz$

11. Divide: $(-8x^2-42x-49)$ by $(-4x-7)$

(a) $x+7$ (b) $-8x+7$ (c) $7x+1$ (d) $2x+7$ (e) $2x-7$

12. Solve: $3^{(1+2x)} = 243$

(a) $\{6\}$ (b) $\{81\}$ (c) $\{-2\}$ (d) $\{2\}$ (e) $\{-6\}$

13. Factor completely: $18x^2-11xy-24y^2$

(a) $(6x-4y)(3x+6y)$ (b) $6(3x-2y)(x+2y)$ (c) $(9x-8y)(2x+3y)$

   (d) $2(3x+8y)(2x-y)$ (e) $(9x+8y)(2x-3y)$

14. The complete factorization of $2x^3+5x^2-18x-45$ is:

(a) $(x^2+9)(2x-5)$ (b) $(2x+5)^2(x+3)(x-3)$ (c) $(2x+5)(x-3)(x+3)$

   (d) $(2x-9)(x+5)$ (e) $(x-45)(2x+9)(x-2)$

15. Divide: $\dfrac{x^2+x}{y^2}$ by $\dfrac{x^2-1}{xy-y}$

(a) $(x-1)/y$ (b) y/x (c) x/y (d) y^2-1 (e) $y/(1-x)$

16. Simplify:

$$\sqrt[4]{81x^{12}y^4}$$

(a) $3x^3y$ (b) $3x^8y$ (c) $9x^3$ (d) $3x^3y^2$ (e) $9x^6y$

17. Solve: $5(x-4) = 4-(6-2x)$

(a) $\{3\}$ (b) $\{-6\}$ (c) $\{2/3\}$ (d) $\{-5\}$ (e) $\{6\}$

18. Solve: $\dfrac{8}{x^2-4} - \dfrac{1}{x-2} = \dfrac{3}{x+2}$

(a) $\{3\}$ (b) $\{6\}$ (c) $\{2/3\}$ (d) $\{0\}$ (e) $\{-3\}$

19. Solve for y: $1/x = 1/y + 1/z$

(a) $xz/(z-x)$ (b) $x-z$ (c) $xz/(z+x)$ (d) $1/(x-z)$ (e) $x + \dfrac{xy}{z}$

20. Solve:

$$\sqrt{5x+1} = 7-x$$

(a) {0,3}   (b) {3,16}   (c) {16}   (d) {-16,3}   (e) {3}

21. Solve: $x = \dfrac{-1}{2x} + \dfrac{3}{2}$

(a) {1/2}   (b) {1}   (c) {2}   (d) {1/2,1}   (e) {1,2}

22. Solve the inequality: $3x+2 \leq 7x-6$

(a)   x>2   (b)   x$\geq$2   (c)   x<2   (d)   x$\leq$2   (e)   x$\leq$1

23. The length of a rectangle is 4 inches more than its width. If 2 inches are taken from the length and added to the width, the figure becomes a square with an area or 192 square inches. What are the dimensions of the original figure?

(a) 10.9 by 14.9 in   (b) 12.9 by 16.9 in   (c) 119 by 159 in

   (d) 11.9 by 15.9 in   (e) 12.9 by 14.9 in

24. Find the values of x for which:

$$|3-x| < 5$$

(a) {-2,8}   (b) x $\leq$ -2 or x $\geq$ 8   (c) -2 < x < 8

   (d) -2 $\leq$ x $\leq$ 8   (e) x < -2 or x > 8

25. A train leaves Knoxville at 4:00 p.m. heading west at 60 mph. A second train leaves Memphis at 4:30 p.m. heading east at 90 mph. How many hours after the second train leaves will the two trains pass if Knoxville and Memphis are 480 miles apart?

(a) 132/15   (b) 69/30   (c) 222/15   (d) 3   (e) 4

26. What is the next number in the sequence 1,1,2,3,5,8,13,...?

(a) 18   (b) 21   (c) 17   (d) 15   (e) 20

27. In a race, runners recorded times (in minutes:seconds) of 29:16, 30:10, 29:29, and 29:05. What was the average time?

(a) 29:40   (b) 29:30   (c) 29:50   (d) 29:15   (e) 29:24

28. For $f(x) = -2x^2+x+10$, find $f(f(f(-2)))$.

(a) -180   (b) 200   (c) -160   (d) 140   (e) 60

29. The function $f(x) = ax^2-bx-4$ has $f(2) = 6$ and $f(-1) = 12$. Then a is:

(a) 2   (b) 7   (c) -2   (d) 1   (e) -3

30. When graphed, $f(x) = -2x^2+3x-7$ has how many zeroes?

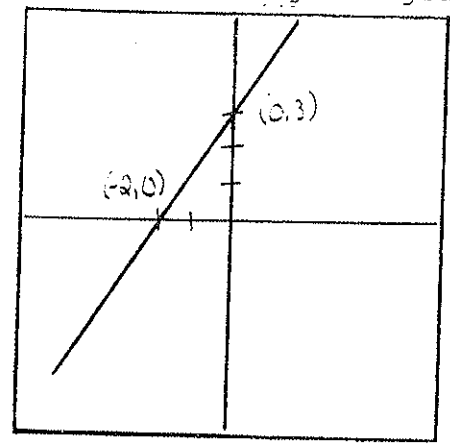(a) 0   (b) 1   (c) 2   (d) 3   (e) null set

31. If $f(x) = x+1$ and $g(x) = x-1$, then $f(g(x)) =$

(a) x^2-1 (b) $2x$ (c) $\dfrac{x+1}{x-1}$ (d) x   (e) 0

32. Find the slope of the line through the $(5,-2)$ and $(-4,2)$.

(a) $\dfrac{-4}{9}$ (b) $\dfrac{-9}{4}$ (c) $\dfrac{4}{9}$ (d) $\dfrac{9}{4}$ (e) 0

33. Identify the equation for the given graph:

(a) $3x-2y+6 = 0$ (b) $2x-3y = 0$ (c) $2x-3y = -6$

(d) $y = \dfrac{3x}{2} + 2$ (e) $y = \dfrac{-2x}{3} + 3$

34. Find the equation of the line through the point $(-1,-5)$ and parallel to the y-axis.

(a) $y = -5$ (b) $y = -1$ (c) $x = -5$ (d) $x = -1$ (e) $y = 5x$
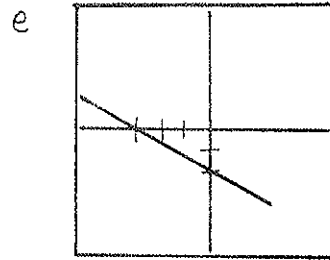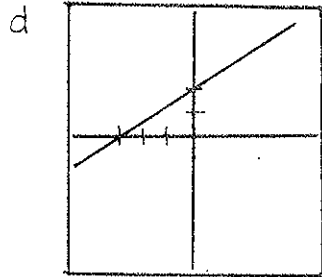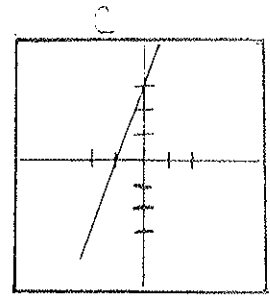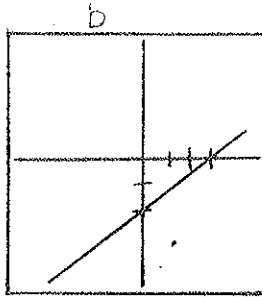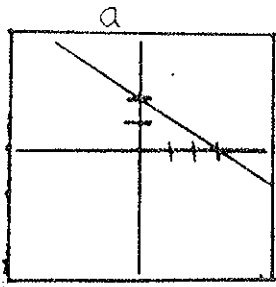
35. What would be the slope of a line perpendicular to $3x-5y = 15$?

(a) 5/3   (b) 3/5   (c) -5/3   (d) -3/5   (e) -1/3

36. The distance between $A(2,3)$ and $B(5,-6)$ is:

(a) $2\sqrt{17}$ (b) $3\sqrt{2}$ (c) $\sqrt{10}$ (d) $3\sqrt{10}$ (e) 6

37. Which of the following is the graph of 2x−3y+6 = 0?

a   b   c

d   e

38. Given the equations, 3x−2y = 1 and 2x+3y = 2, and the statements:

I.   The graphs of the equations intersect at (7/13,4/13).
II. The graphs of the equations are perpendicular to each other.
III. The graphs of the equations are both perpendicular to x+y=1.

Which of the above statements are true?

(a) I, II, and III   (b) I only   (c) III only

(d) I and II   (e) II only

39. In order for the determinants

$$A = \begin{vmatrix} x & -2 \\ 4 & -4 \end{vmatrix} \qquad B = \begin{vmatrix} 2x & -2 \\ 2x & 3x \end{vmatrix}$$

to be equal, x must be:

(a) 0   (b) 2/3 or −2   (c) 2 or −2/3   (d) 3/2 or 1/2   (e) −3/2 or 2

40. At a mall some teenagers (ages 14−17) were surveyed concerning their musical preferences, and the following data were collected:

    32 liked Extreme
    29 liked R. E. M.
    12 liked Amy Grant
    20 liked both Extreme and R. E. M.
     5 liked both Amy Grant and Extreme
     1 liked both Amy Grant and R. E. M.
     2 did not like any of the three
    no one liked all three

How many teenagers were surveyed?

(a) 101   (b) 73   (c) 49   (d) 47   (e) 75