

SEVENTEENTH ANNUAL MATHEMATICS CONTEST

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ALGEBRA I TEST

1973

Scoring Formula: 4R - W

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This test was prepared from a list of Algebra I questions submitted by U. T. Chattanooga and other mathematics teachers across Tennessee.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals
- | | a | b | c | d | e |
|--------------|---------------------|---------|-----------------------------|-------------------------------------|--------------------------|
| (a). $2/3$. | (b). 3. | (c). 6. | 1. <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| (d). $3/2$. | (e). none of these. | | <input type="checkbox"/> | <input checked="" type="checkbox"/> | <input type="checkbox"/> |

The correct answer for the sample problem is $3/2$, which is answer (d); so you would answer this problem by making a heavy black mark under space d as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statewide statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. Which one of the following sets of ordered pairs of real numbers is a function?

(a). $\{(2,4), (2,5), (2,6)\}$

(b). $\{(2,1), (1,2), (2,2)\}$

(c). $\{(x,y) | y^2 = x\}$

(d). $\{(x,y) | y = x\}$

(e). $\{(x,y) | x^2 + y^2 = 4\}$

2. Solve for x : $bx + cx = a - bx$

(a). $(2a)(b + c)$

(b). $(2b + c)/a$

(c). $a/(2b + c)$

(d). $(2a - b)/c$

(e). a/c

3. $\frac{8^{2n}}{2^{2n}}$ is equal to:

(a). 4

(b). 4^2

(c). 4^0

(d). 2^{4n}

(e). 4^n

4. Which one of the following is an irrational number?

(a). $\sqrt{5} / \sqrt{5}$

(b). $\sqrt{5} + \sqrt{5}$

(c). $(\sqrt{5})(\sqrt{5})$

(d). $(2\sqrt{5})^2$

(e). $\sqrt{5^2}$

5. Express $\sqrt{3/8}$ as a percentage of $\sqrt{6}$:
- (a). 75%
 - (b). 50%
 - (c). 25%
 - (d). 6.25%
 - (e). 400%
6. The factors over the real numbers of $3a(2x - y) - 2b(y - 2x)$ are:
- (a). $(2x + y)(3a + 2b)$
 - (b). $5ab(2x - y)$
 - (c). $(2x - y)(3a + 2b)$
 - (d). $(3a - 2b)(2x - y)$
 - (e). $6ax - 2by - 3ay + 4x$
7. If the roots of $x^2 - 4x + K = 0$ are equal, then K is equal to
- (a). 16.
 - (b). -16.
 - (c). 8.
 - (d). 4.
 - (e). -8.
8. The equation of the line through the two points whose coordinates are respectively $(-3,2)$ and $(5,-2)$ is:
- (a). $-3x + 2y = 6$
 - (b). $5x - 2y = 0$
 - (c). $y - 2 = (-1/2)(x + 3)$
 - (d). $y - 2 = -2(x + 3)$
 - (e). $y + 2 = 2(x - 5)$

9. If $f(x) = x^3 - 3x^2 + 5x - 1$ and $g(x) = (x - 2)/3$, then $f(g(5))$ equals:
- (a). 5
 - (b). 0
 - (c). 1
 - (d). 2
 - (e). $5/3$
10. The width of a rectangle is 3 centimeters less than the length, and the perimeter is 106 centimeters. What is the width of the rectangle?
- (a). 25
 - (b). 28
 - (c). 27
 - (d). 22
 - (e). 31
11. If $(a + b)/a = 7$, $a \neq 0$, find the value of a/b .
- (a). $1/6$
 - (b). 6
 - (c). 8
 - (d). $1/8$
 - (e). $6/7$
12. A box contains 9 balls numbered 1 through 9. Two balls are drawn at random. What is the probability that the numbers on the balls drawn will both be odd?
- (a). $5/18$
 - (b). $19/18$
 - (c). $5/9$
 - (d). $2/9$
 - (e). $2/5$

13. If -2 is a root of $3x^4 + p^2x^3 + 2px + 12 = 0$, the set of possible values of p is:
- (a). $\{5/2\}$
 - (b). $\{-3\}$
 - (c). $\{5/2, -3\}$
 - (d). ϕ
 - (e). $\{-5/2, 3\}$
14. If a is inversely proportional to the square root of b , and $a = 4$ when $b = 9$, what is b when $a = 6$?
- (a). 0
 - (b). 2
 - (c). 4
 - (d). 6
 - (e). -8
15. If the tens digit of a two-digit real number is increased by 3, the units digit is one-half the result. The sum of the number and the new number formed by reversing the digits is 165. Find the original number.
- (a). 33
 - (b). 12
 - (c). 76
 - (d). 69
 - (e). 96
16. In going to summer camp, Eric made a trip of 210 miles. He traveled by train for 3 hours and by bus for the rest of the trip. The average speed of the train was 15 miles per hour more than that of the bus. Find the speed of the bus.
- (a). 55 mph
 - (b). 45 mph
 - (c). 70 mph
 - (d). 30 mph
 - (e). impossible to determine

17. One factor of $\left(\frac{1}{ab} + \frac{1}{a^2}\right)$ is $\left(\frac{a}{b} - \frac{b}{a}\right)$. The other factor is:

(a). $\frac{1}{a^2} - \frac{1}{ab}$

(b). $a^2 - ab$

(c). $1/(a^2 + ab)$

(d). $1/(a^2 - ba)$

(e). some other expression

18. A box contains 3 red and 7 green balls. The probability of drawing exactly 2 red and 2 green balls in 4 draws without replacement is:

(a). $1/24$

(b). $4/10$

(c). $3/10$

(d). $189/1250$

(e). $4/21$

19. The quadratic equation having the solution set $\left\{\frac{1 \pm \sqrt{7}}{2}\right\}$ is:

(a). $2x^2 + 2x - 3 = 0$

(b). $2x^2 - 2x - 3 = 0$

(c). $2x^2 - x + \sqrt{7} = 0$

(d). $2x^2 - \sqrt{7}x + 1 = 0$

(e). $2x^2 + 2x + 3 = 0$

20. The solution set of $\sqrt{3\sqrt{x-1}} = \sqrt{x-1}$ is:

(a). $\{1\}$

(b). $\{0,1\}$

(c). $\{10\}$

(d). $\{1,10\}$

(e). ϕ

21. Reduce to lowest terms: $(x^3 - x^2 - x + 1)/(x^2 - 2x + 1)$

- (a). $x - 1$
- (b). $(x - 1)/(x + 1)$
- (c). $x + 1$
- (d). $(x + 1)/(x - 1)$
- (e). $(x^2 - 1)$

22. Simplify: $(4x^3y^{-3})/(2^3x^{-1}y^2)$

- (a). $x^4/(2y^5)$
- (b). $2x^2y^{-1}$
- (c). $(2x^4)/y$
- (d). $x^2/(2y^5)$
- (e). $(x^4y^5)/2$

23. Sam, the mechanic, can fix a car in three days and his assistant Harry can fix the same car in 4 days. How long will it take them working together, assuming that their work rates remain the same when working together as when working separately?

- (a). $1/7$ day
- (b). $12/7$ days
- (c). 2 days
- (d). $5/2$ days
- (e). $3/2$ days

24. The expression $(a^{-2} - b^{-2})/(a^{-1} + b^{-1})$ may be simplified to:

- (a). $a^{-1} + b^{-1}$
- (b). $a^{-1} - b^{-1}$
- (c). $(b + a)/ab$
- (d). $1/(a^{-1} - b^{-1})$
- (e). $1/(a^{-1} + b^{-1})$

25. The sum of the roots of $x^2 + 5x - 3 = 0$ is:
- (a). 5
 - (b). 10
 - (c). $\sqrt{37}$
 - (d). -5
 - (e). -3
26. A motorist can drive from city A to city B in 3 hours. By increasing his speed 10 mph he can cut his time down to $2\frac{1}{2}$ hours. How far is it from A to B?
- (a). 75 miles
 - (b). 100 miles
 - (c). 150 miles
 - (d). 50 miles
 - (e). 135 miles
27. For real numbers x , $|x + 2| > 3$ implies that:
- (a). $x = 1$
 - (b). $x > 1$
 - (c). $x < -5$
 - (d). $-5 > x$ or $x > 1$
 - (e). $x < -5$ or $x \geq 1$
28. If a side of a square inscribed in a circle is 2 inches long, the circumference of the circle is:
- (a). $\sqrt{2} \pi$ inches
 - (b). $4\sqrt{2} \pi$ inches
 - (c). $2\sqrt{2} \pi$ inches
 - (d). 2π inches
 - (e). 4π inches

29. For real numbers x , $\frac{x-4}{x-2} - \frac{3}{x+2} = 1 - \frac{8}{x^2-4}$ is an equation whose solution set is:
- (a). {5}
 - (b). {2}
 - (c). {-2}
 - (d). ϕ
 - (e). {4}
30. For what values of c will the following equation have real roots,
 $x^2 + 4x + c = 0$?
- (a). $4 \geq c$
 - (b). $4 < c$
 - (c). all values of c
 - (d). $4 \leq c$
 - (e). all positive values of c
31. Which one of the following is always true for any three real numbers a , b , and c ?
- (a). If $a < b$, then $ac < bc$
 - (b). If $a < b$, then $a^2 < b^2$
 - (c). If $a < b$, then $a^3 < b^3$
 - (d). If $ac < bc$, then $a < b$
 - (e). If $a > b$, then $\frac{1}{a} > \frac{1}{b}$
32. Junior is four times as old as his dog Spot. Spot is 22 years younger than Junior's mother. Junior's mother is three times as old as Junior. At a time in the future Junior will be one-half his mother's age. How old will he be at that time?
- (a). 8
 - (b). 12
 - (c). 16
 - (d). 18
 - (e). none of these

33. Let $M(a,b) = \frac{a+b}{2} + \frac{|a-b|}{2}$ and $m(a,b) = \frac{a+b}{2} - \frac{|a-b|}{2}$ for all real numbers a and b . Suppose $a \neq b$. Then which one of the following is true for all real numbers a and b ?

- (a). $m(a,b) = b$
- (b). $M(a,b) = b$
- (c). $M(a,b)$ is always smaller than $m(a,b)$
- (d). $m(a,b)$ is always smaller than $M(a,b)$
- (e). $M(a,b) = a$

34. Which step of the following "proof" that $1 = 2$ does not correctly follow from the previous step?

Given $a = b$

Step I. $a^2 = ab$

Step II. $a^2 - b^2 = ab - b^2$

Step III. $(a+b)(a-b) = b(a-b)$

Step IV. $(a+b) = b$

Step V. $(b+b) = b$

Step VI. $2b = b$

Step VII. $2 = 1$

- (a). Step II
- (b). Step III
- (c). Step IV
- (d). Step V
- (e). Step VI

35. If A and B are sets of real numbers defined by $A = \{x | (x \in R) \wedge (x \geq 2)\}$ and $B = \{x | (x \in R) \wedge (x < 8)\}$, where R refers to the real numbers, then $A \cap B$ is equal to:

- (a). $0 < x < 8$
- (b). $x < 8$
- (c). $2 \leq x < 8$
- (d). $3 < x \leq 7$
- (e). $x \geq 2$

36. Let set $A = \{1,2,3,4,5\}$. Define a binary operation \cdot on A by the table

\cdot	1	2	3	4	5
1	1	2	3	4	5
2	2	4	1	3	5
3	3	1	4	2	5
4	4	3	2	1	5
5	5	5	5	5	5

Define $a \div b = c$ to mean $a = b \cdot c$. The value of $2 \div 3$ is:

- (a). 1
- (b). 2
- (c). 3
- (d). 4
- (e). 5

37. Let set $S = \{1,2,3,4,5\}$. Define binary operations $+$ and \cdot on S by the tables

$+$	1	2	3	4	5
1	2	3	4	5	1
2	3	4	5	1	2
3	4	5	1	2	3
4	5	1	2	3	4
5	1	2	3	4	5

\cdot	1	2	3	4	5
1	1	2	3	4	5
2	2	4	1	3	5
3	3	1	4	2	5
4	4	3	2	1	5
5	5	5	5	5	5

What one of the following is false?

- (a). S is closed under $+$
- (b). The binary operation \cdot is commutative
- (c). The binary operation $+$ is commutative
- (d). There is an identity element for \cdot
- (e). There is not an identity element for $+$

38. How many solutions does the following system of equations have?

$$(x - 2)^2 + (y - 1)^2 = 4$$

$$\frac{y - 1}{x - 2} = -\frac{x}{y}$$

- (a). no solutions
 - (b). one solution
 - (c). two solutions
 - (d). three solutions
 - (e). four solutions
39. In the expression xy^2 , the values of x and y are each decreased 25%; the value of the expression is:
- (a). decreased 50%
 - (b). decreased 75%
 - (c). decreased $37/64$ of its value
 - (d). decreased $27/64$ of its value
 - (e). none of these
40. Suppose that F is a function from the real numbers to the positive real numbers such that $F(x + y) = F(x)F(y)$ for all real numbers x and y . Which of the following is true?
- (a). $F(xy) = F(x) + F(y)$
 - (b). $F(1) = F(-1)$
 - (c). $F(0) = 1$
 - (d). $F(1) = 0$
 - (e). None of these are true