



TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

SIXTY-FIRST ANNUAL MATHEMATICS CONTEST

2017

Algebra II

Prepared by:

Mathematics Faculty
Carson-Newman University
Jefferson City, TN
Coordinated by Thomas Bass

Reviewed by:

Mathematics Faculty
Walters State Community College
Morristown, TN

Scoring formula: $4 \times (\text{Number Right}) - (\text{Number Wrong}) + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work

- Find the value of $2^4 - 4^3 \div [-8(-3 + 2)]$
 - 7
 - 6
 - 8
 - 15
 - 16

- On a trip of 200 miles, you drive 50 miles per hour for the first 100 miles. How fast must you drive for the remainder of the trip to average 60 mph overall?
 - 60
 - 68
 - 70
 - 72
 - 75

- Using the given two-way frequency table, consider the events A : “the student is a freshman” and B : “the student is from Tennessee.”

Students at University of North Central South Dakota (West Campus)
Classified by Year and Home State

Home State	Year			
	FR	SO	JR	SR
Tennessee	26	16	12	6
North Carolina	10	11	7	2
Oregon	4	3	1	2

- Which of the following is incorrect?
- $P(A) = 0.4$
 - $P(B) = 0.6$
 - $P(A \cap B) = 0.26$
 - $P(A \cup B) = 0.74$
 - $P(A | B) = 0.65$
-
- A quadratic equation with real coefficients with the complex solutions $3 + i$ and $2 - i$
 - crosses the x -axis twice.
 - touches the x -axis but does not cross it.
 - crosses the x -axis once
 - crosses the y -axis but not the x -axis.
 - does not exist.

5. For which value(s) of k does the following system have infinitely many solutions?

$$3x + 2y = 1$$

$$kx + 6y = 3$$

- (a) $k \neq 9$ (b) $k = 3$ (c) $k = 9$ (d) $k = 9, -9$ (e) $k = -3$

6. Consider the following “proof” that $2 = 1$:

(1) Suppose $a = b \neq 0$

(2) $a^2 = ab$ (multiply both sides by a)

(3) $a^2 - b^2 = ab - b^2$ (subtract b^2 from both sides)

(4) $(a + b)(a - b) = b(a - b)$ (factoring)

(5) $(a + b) = b$ (cancelling $(a - b)$ on both sides)

(6) $b + b = b$ (substituting $a = b$ from (1))

(7) $2b = b$ (adding like terms)

(8) $2 = 1$ (cancelling b on both sides.) \square

There is obviously an error. On which line did it occur?

- (a) (1) (b) (2) (c) (5) (d) (6) (e) (8)

7. Simplify the expression $x^2 \left(x + \frac{1}{x}\right)^{-1}$

- (a) 1 (b) x (c) $\frac{1}{x^2 + 1}$ (d) $\frac{x^2 + 1}{x}$ (e) $\frac{x^3}{x^2 + 1}$

8. Suppose that the set $\{10, 25, 30, 35, x\}$ has the same number as its mean, median, and mode. Which of the following could be a value of x ?

- (a) 10 (b) 20 (c) 25 (d) 30 (e) 35

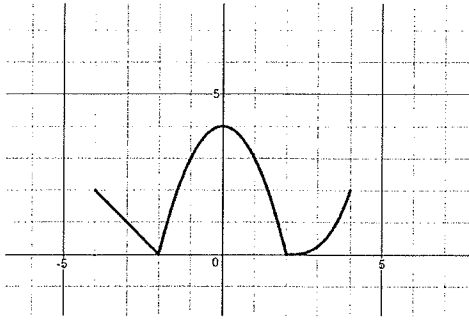
9. Solve $2e^{3t} = 2017$ for t to the nearest thousandth.

- (a) 1.001 (b) 1.268 (c) 2.305 (d) 6.916 (e) 5.818

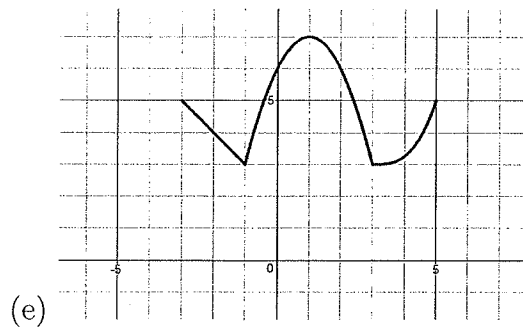
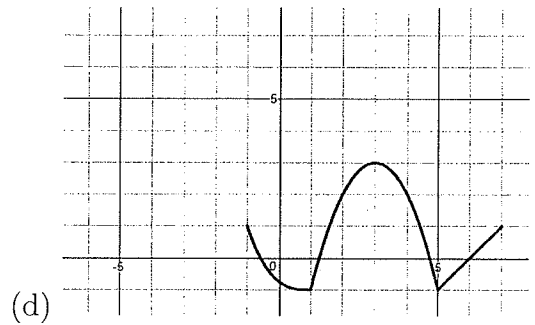
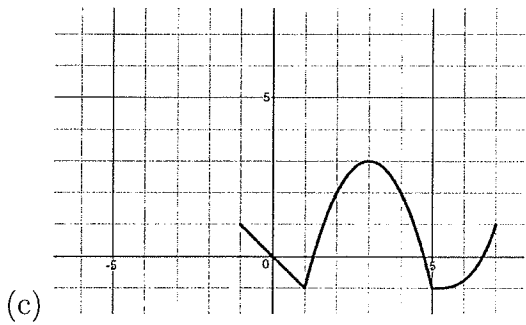
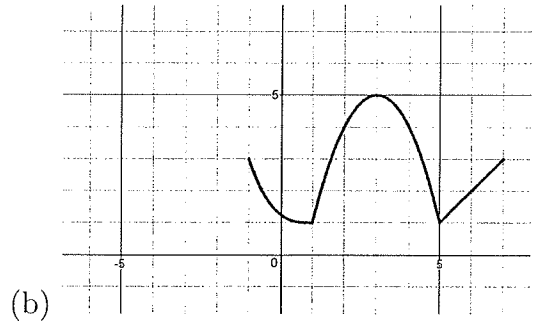
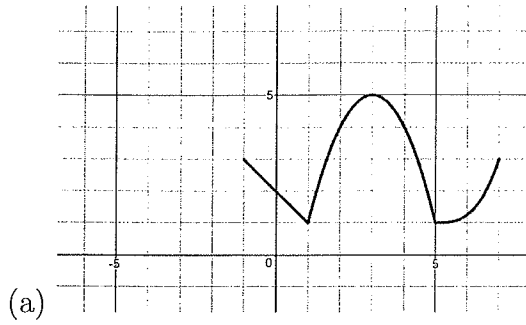
10. If $x + y = 5$ and $x^2 + y^2 = 97$, find $(|x| + |y|)^3$

- (a) 1279 (b) 1297 (c) 2179 (d) 2197 (e) 125

11. Suppose the graph pictured here is the graph of $y = f(x)$



Which of the following is the graph of $y = f(3 - x) + 1$?



12. Suppose that $f(x) = \frac{x - 2}{x + 3}$. Which of the following is equal to $f^{-1}(x)$?

(a) $\frac{3x + 2}{-x + 1}$

(b) $\frac{x + 3}{x - 1}$

(c) $\frac{x + 1}{x - 3}$

(d) $\frac{-x + 1}{-x - 3}$

(e) $\frac{3x - 1}{x + 1}$

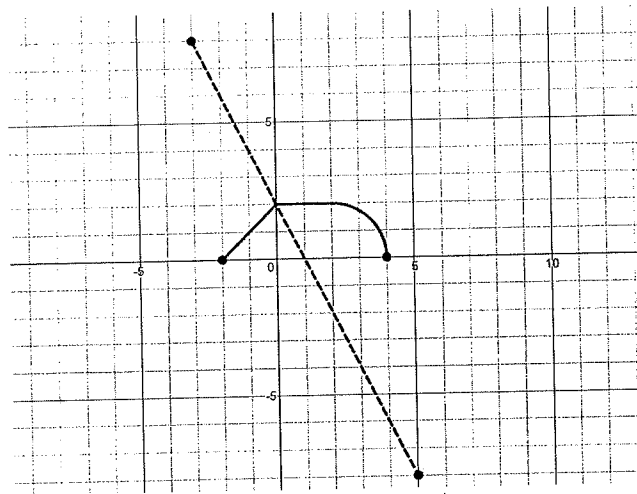
13. How many times do the graphs of $y = 0.58x^3 + 1.3x^2 + 0.42$ and $y = 0.29x^4 + 1.58x$ intersect?

- (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

14. It is a well-known fact that $\sin 2\theta = 2 \sin \theta \cos \theta$. If $\tan \theta = \frac{3}{5}$, find $\sin 2\theta$.

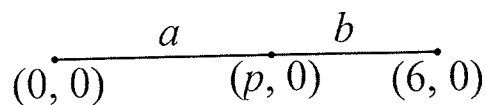
- (a) $\frac{10}{\sqrt{28}}$ (b) $\frac{15}{28}$ (c) $\frac{15}{16}$ (d) $\frac{15}{17}$ (e) $\frac{3}{2}$

15. Pictured are the graphs of $y = f(x)$ (solid) and $y = g(x)$ (dashed). What is the domain of $f \circ g$?



- (a) $[-2, 4]$ (b) $[-1, 2]$ (c) $[0, 2]$ (d) $[-3, 5]$ (e) $[-8, 8]$

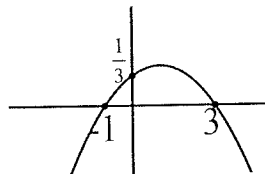
16. Using the given figure, find p so that $\frac{a+b}{a} = \frac{a}{b}$



- (a) $p = 3\sqrt{5} - 3$
 (b) $p = -3 \pm \sqrt{45}$
 (c) $p = 9 - 3\sqrt{5}$
 (d) $p = 3$
 (e) $p = 4$

17. To the nearest integer, find the distance between the x - and y -intercepts of the graph of $y = \frac{24 - 3x}{x - 6}$.
- (a) 9 (b) 11 (c) 12 (d) 15 (e) 17
18. Find the point(s) of intersection of the circles centered at the origin and at $(1, 0)$, both of which have radius 1.
- (a) $\left(\frac{1}{2}, \frac{1}{2}\right)$
- (b) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
- (c) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ and $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
- (d) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
- (e) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
19. Let $u(x) = x^2 + 1$, $v(x) = 2x + 1$, and $w(x) = 2^x + 1$. Find $v(w^{-1}(u(\sqrt{8})))$.
- (a) 1 (b) 3 (c) 4 (d) 7 (e) 9
20. Consider the sequence $S = \{5, 11, 23, 47, 95, \dots\}$. Which of the following recurrence relations would generate the sequence?
- (a) $a_{n+1} = a_n + 6; a_0 = 5$
- (b) $a_{n+1} = 3a_n - 4; a_0 = 5$
- (c) $a_{n+1} = 2a_n + 1; a_0 = 5$
- (d) $a_{n+1} = 3a_n - 4; a_0 = 3$
- (e) $a_{n+1} = a_n + 6; a_0 = -1$

21. Find the formula for the quadratic function whose graph is pictured below.



- (a) $f(x) = \frac{1}{3}(x-1)(x+3)$
(b) $f(x) = -\frac{1}{3}(x-1)(x+3)$
(c) $f(x) = \frac{1}{3}(x+1)(x-3)$
(d) $f(x) = -\frac{1}{9}(x+1)(x-3)$
(e) $f(x) = \frac{1}{9}(x-1)(x+3)$
22. Consider two points $P(2, 9)$ and $Q(14, 1)$ in the Cartesian plane. Let PQ be the line segment between P and Q . Find the equation of the line that passes through the midpoint of PQ that is perpendicular to PQ .
- (a) $y = 0.75x - 1$ (b) $y = 1.25x - 5$ (c) $y = 1.5x - 7$ (d) $y = 1.75x - 9$ (e) $y = 2.5x - 15$
23. What is the value of $1 - 2 + 4 - 8 + 16 - 32 + \dots - 2^{99} + 2^{100}$?
- (a) $\frac{2^{101} + 1}{3}$ (b) $\frac{2^{101} - 1}{3}$ (c) $\frac{2^{100} + 1}{3}$ (d) $\frac{2^{101} - 1}{3}$ (e) $\frac{2^{101} - 1}{2}$
24. Suppose that a quadratic polynomial with real coefficients has leading coefficient 1 and has a root $x = 3 + i$. What is the polynomial?
- (a) $x^2 - 3i + 1$
(b) $x^2 - 6x + 10$
(c) $x^2 + 6x + 10$
(d) $10x^2 - 6x + 1$
(e) There is not enough information to find the polynomial.
25. Suppose that $c_1 = 1 - i$ and that $c_n = c_{n-1}^2 + c_1$. Find c_3
- (a) $1 - 3i$ (b) $-7 - 7i$ (c) $-7 - 9i$ (d) $-2i$ (e) $1 - i$
26. Which of the following is the solution set for the equation $\sqrt{t^2} = \sqrt{2t + 8}$?
- (a) $\{4, -2\}$ (b) $\{4\}$ (c) $\{-2\}$ (d) $\{4, -4, 2, -2\}$ (e) $\{\}$

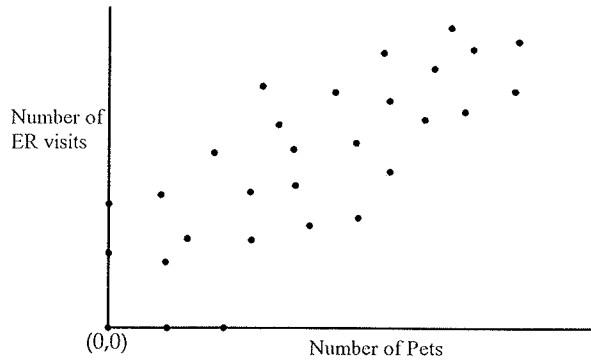
27. Suppose that

$$\frac{5}{x^2 + x - 6} = \frac{1}{x - 2} + \frac{B}{x + 3}.$$

What is the value of B ?

- (a) -2 (b) -1 (c) 0 (d) 1 (e) 2

28. The pictured scatterplot shows the relationship between the number of pets a person owns and the number of times a person visits the emergency room (ER) each year. Which of the following conclusions cannot be made based on the graph?



- (a) There is a positive association between number of pets and number of ER visits.
(b) Owning pets causes people to visit the ER more often.
(c) The line of best fit between these two variables has a positive slope.
(d) On average, the more pets a person owns, the more s/he visits the ER.
(e) Some people have no pets.

29. What is the solution set of the equation $\sqrt{2x} = \sqrt[3]{3x}$?

- (a) $\{0, \frac{27}{4}\}$ (b) $\{0\}$ (c) $\{0, -\frac{9}{8}\}$ (d) $\{0, \frac{9}{8}\}$ (e) $\{\}$

30. What are the center and radius of the circle whose equation is

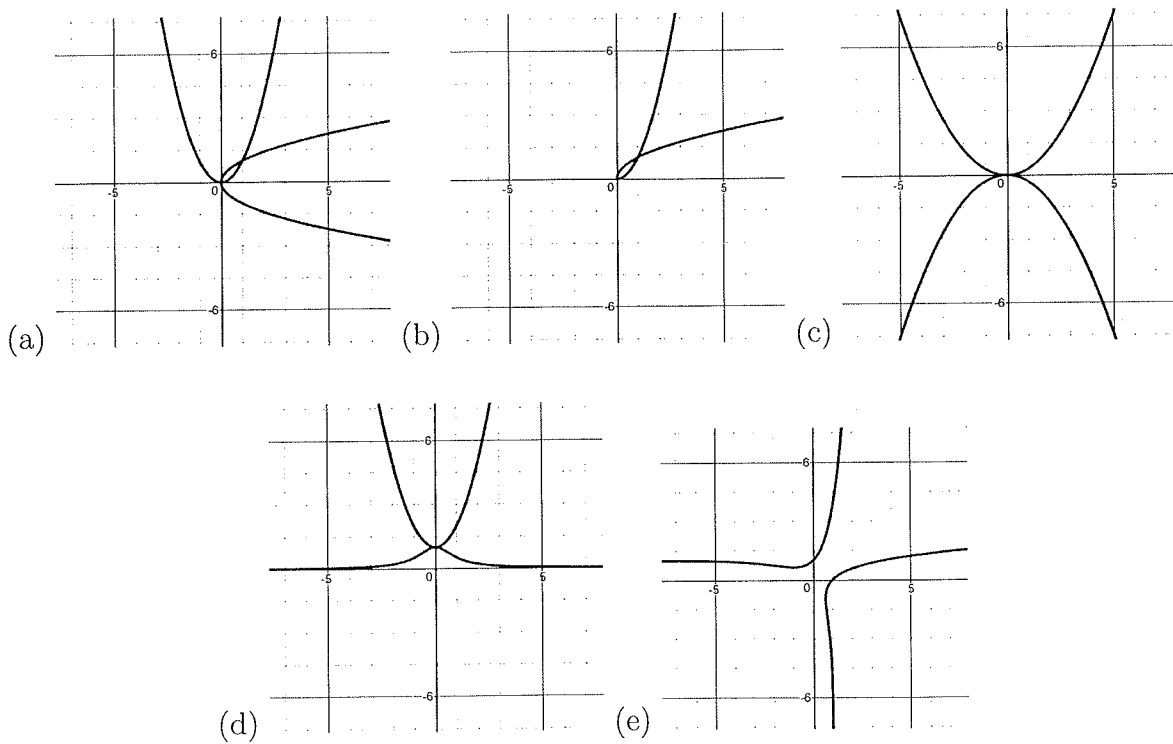
$$x^2 + y^2 - 2x + 6y = -1?$$

- (a) Center $(2, -6)$; $r = 1$
(b) Center $(-2, 6)$; $r = 1$
(c) Center $(1, -3)$; $r = 1$
(d) Center $(-1, 3)$; $r = 3$
(e) Center $(1, -3)$; $r = 3$

31. Find a value of b and c so that the graph of the equation $y = x^2 + bx + c$ has its vertex at $(-1, 1)$.

- (a) $b = 2; c = 2$
- (b) $b = 2; c = -1$
- (c) $b = 2; c = 1$
- (d) $b = -2; c = -1$
- (e) $b = -2; c = 1$

32. Which of the following is the graph of a function and its inverse function?



33. Suppose that the heights of the freshman class at the University of North Central South Dakota (East Campus) are normally distributed with mean 175cm and standard deviation 7.5cm. Which of the following ranges contains approximately 68% of the freshman heights in centimeters?

- (a) $[152.5, 167.5]$
- (b) $[160, 175]$
- (c) $[167.5, 182.5]$
- (d) $[175, 190]$
- (e) $[182.5, 197.5]$

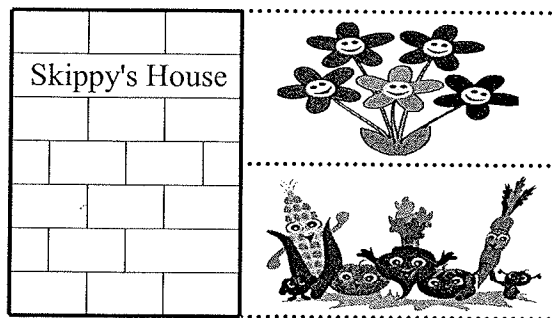
34. Which of the following is the equation of the parabola whose focus is $P(2, -1)$ and whose directrix is the line $x = 4$?

- (a) $x = \frac{1}{4}(y + 1)^2 + 3$
- (b) $y = \frac{1}{4}(x - 3)^2 - 1$
- (c) $x = \frac{1}{4}(y + 1)^2 + 2$
- (d) $y = \frac{1}{4}(x - 2)^2 - 1$
- (e) $y^2 = \frac{1}{4}(x - 2)^2 - 1$

35. The points $P(-1, a)$ and $Q(3, b)$ are both on the graph of a parabola whose vertex is at the origin. If the slope of the line joining P and Q is -2 , which of the following could be the equation whose graph is this parabola?

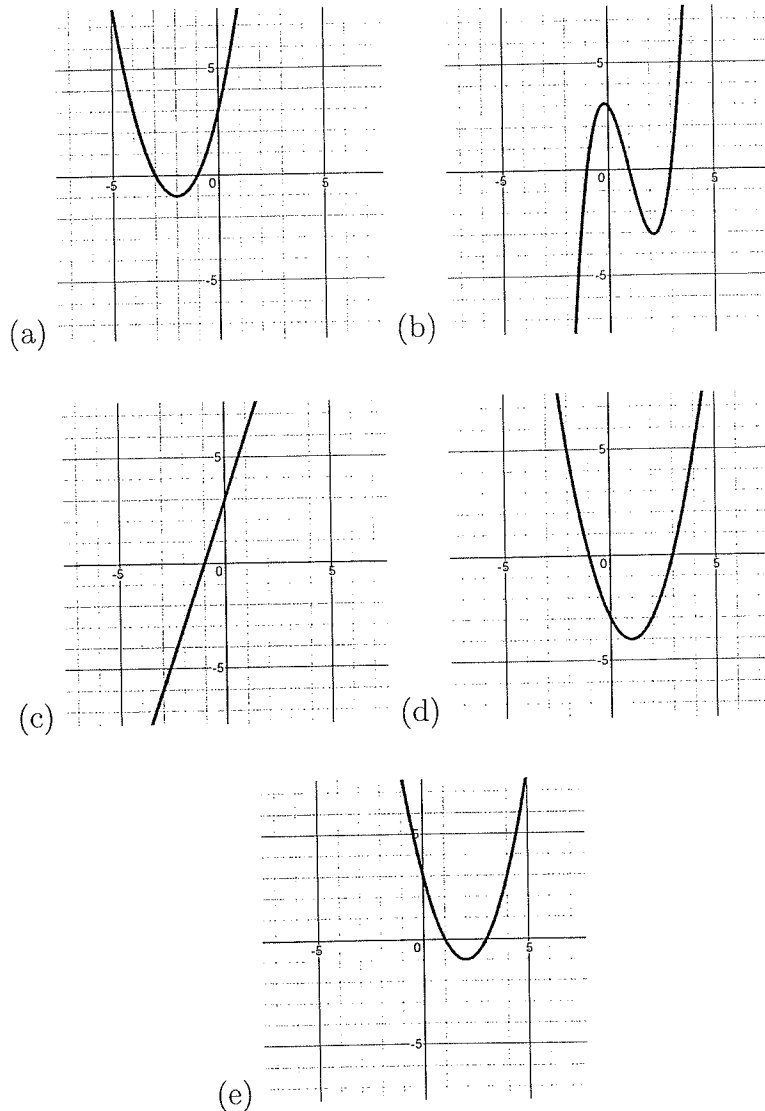
- (a) $(y - 3)^2 = -2(x + 1)$
- (b) $y = -2x^2$
- (c) $y = -x^2$
- (d) $(y - 1) = 3x^2$
- (e) $y^2 - 1 = 3 + x$

36. Skippy wants to build a rectangular garden by fencing in a portion of land next to his house. He also wants to divide his garden into two separate plots (one for vegetables, one for flowers) by building a dividing fence perpendicular to his house, as pictured. If he has 600 feet of fencing to use, and if he isn't going to build fence next to his house, what is the maximum area of the garden he can construct this way?



- (a) 100 ft^2
- (b) 300 ft^2
- (c) $15,000 \text{ ft}^2$
- (d) $22,500 \text{ ft}^2$
- (e) $30,000 \text{ ft}^2$

37. The graph of a polynomial $P(x)$ has y -intercept 3 and one of its x -intercepts is -1 . If the remainder when $P(x)$ is divided by $x - 2$ is -3 , which of the following could be the graph of $y = P(x)$?



38. The swimming pool at University of North Central South Dakota (East Campus) has two fill valves. Johanna turns on the slower valve, which takes 20 hours to fill the pool, but she cannot find the wrench to turn on the faster valve. Finally, after 6 hours of searching, she finds the tool and turns on the faster valve. After 6 more hours, the pool is full. To the nearest hour, how long would it have taken the fill the pool if she had been able to turn on the fast valve at the same time as the slow valve?

- (a) 6 (b) 7 (c) 8 (d) 9 (e) 10

39. Suppose that the graph of $y = f(x)$ is a parabola with y -intercept 2 that has a root at $x = 3$. On a suitably restricted domain, we find that $f^{-1}(-38) = 6$. Which of the following is the formula for $f(x)$?
- (a) $f(x) = 6x^2 + 30x - 36$
 - (b) $f(x) = 6x^2 - 30x + 36$
 - (c) $f(x) = -2x^2 + \frac{16}{3}x - 2$
 - (d) $f(x) = -2x^2 + \frac{16}{3}x + 2$
 - (e) $f(x) = x^2 - \frac{8}{3}x + 2$
40. Skippy and Johanna can both walk at a rate of 5mph and row at a rate of 3mph. They need to get to a point B on the shore of a large circular lake ($d = 6$ miles) that is diametrically opposite the point at which they are now standing. Skippy begins walking around the circumference of the lake toward point B . At the same time, Johanna gets in a boat and begins rowing toward a point on the shore that is exactly 5 miles from their starting point. She then gets out of her boat and continues her trek by walking around the circumference of the lake to point B . Who arrives at point B first, and by approximately what time margin?
- (a) Skippy, by 48 minutes
 - (b) Skippy, by 29 minutes
 - (c) Skippy, by 19 minutes
 - (d) Johanna, by 48 minutes
 - (e) Johanna, by 29 minutes