

TWENTY-EIGHTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

COMPREHENSIVE 1984

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Scoring Formula: $4R - W + 40$

This test was prepared from a list of Comprehensive questions submitted by the Kingsport University Center of East Tennessee State University.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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1. If $a, b, c,$ and e are integers, none of which is zero and

$a > b > c$ then

a) $ae > be > ce$

d) $a + e \geq b + c$

b) $\frac{1}{a} > \frac{1}{b} > \frac{1}{c}$

e) None of the above

c) $\frac{1}{c} > \frac{1}{b} > \frac{1}{a}$

2. If $\log_{10} N = 3$, then N is

a) 1,000

d) 3

b) 300

e) None of the above

c) 30

3. The roots of a quadratic equation have as their sum $\frac{7}{2}$ and their product 3. Such an equation could be

a) $2x^2 + 7x + 6 = 0$

d) $2x^2 - 7x + 6 = 0$

b) $2x^2 - 7x - 6 = 0$

e) $7x^2 - 6x + 2 = 0$

c) $2x^2 + 7x - 6 = 0$

4. Which of the following equations does not have a real root?

a) $3x - 27 = 0$

d) $3x^4 - 27 = 0$

b) $3x^2 - 27 = 0$

e) None of the above

c) $3x^3 - 27 = 0$

5. If three sides of a triangle are respectively 7, 24 and 25 units long, the triangle must have

a) an obtuse angle

d) an area of 168 square units

b) a right angle

e) None of the above

c) a pair of equal angles

6. A pyramid has a base which is a square 3 cm on an edge and an altitude of 9 cm. The volume of the pyramid is
- a) 27
b) $\frac{81}{2}$
c) 81
d) 243
e) None of the above
7. Which of the following is possible?
- a) $\cos \theta = \frac{11}{10}$
b) $\cot \theta$ is undefined
c) $\sec \theta = \frac{\pi}{4}$
d) $\tan \theta = \frac{1}{2}$ and $\cot \theta = -2$
e) None of the above
8. If the adjacent sides of a parallelogram are 4 cm and 6 cm in length and the adjacent sides of a similar parallelogram are 10 cm and 15 cm in length, the ratio of the areas of the similar parallelograms is
- a) $A_1:A_2::4:6$
b) $A_1:A_2::4:10$
c) $A_1:A_2::4:15$
d) $A_1:A_2::4:25$
e) None of the above
9. What is the equation of a line with a slope of -3 and an x-intercept of +3?
- a) $y = -3x - 9$
b) $y = -3x + 9$
c) $y = -3x$
d) $y = -3x + 3$
e) None of the above
10. If a, b, c and d are real numbers such that $a = \frac{1}{2}b$ and $c = \frac{1}{2}d$, then
- a) $a = c$
b) $c > d$
c) $a < b$
d) $a^2 > b^2$
e) None of the above

11. A 30° arc of a circle has a length of π . What is the area of the circle?
- a) $\frac{\pi}{2}$
 b) $2\pi^3$
 c) $4\sqrt{3}\pi$
 d) $\frac{36}{\pi}$
 e) 36π
12. If one root of a real quadratic equation is $2 - \sqrt{2}i$, what is the equation?
- a) $x - 2 + 2\sqrt{i} = 1$
 b) $x^2 - 4x + 6 = 0$
 c) $x^2 + 2ix + 6 = 0$
 d) $x^2 + 2(2-2\sqrt{2})ix + 6 = 0$
 e) None of the above
13. The equation $x^2 + y^2 - 2x + 4y + 5 = 0$ is best described as the equation of a(n)
- a) Hyperbola
 b) Ellipse
 c) Parabola
 d) Point
 e) Line
14. The sum of two adjacent sides of a rectangle is 14. If its area is 48 square units, what is the length of its diagonal?
- a) $7\sqrt{2}$
 b) 10
 c) $14\sqrt{2}$
 d) 28
 e) None of the above
15. The solution set of $x^2 - 2x \leq 0$ is
- a) $0 \leq x \leq 2$
 b) $-2 \leq x \leq 0$
 c) $-2 \leq x \leq 2$
 d) $x \leq 0 \cup x \geq 2$
 e) None of the above

16. If $\tan x = \frac{3}{4}$ and $\sec x$ is negative, the value of $\sin x$ is

a) $\frac{3}{5}$

d) $-\frac{4}{5}$

b) $\frac{4}{5}$

e) $-\frac{3}{5}$

c) 1

17. If the product of the roots of a cubic equation is -4 and one of the roots is $2i$, the equation is

a) $x^3 - 4 = 0$

d) $x^3 + x^2 + 4x + 4 = 0$

b) $x^3 + 4 = 0$

e) None of the above

c) $x^3 - x^2 + 4x - 4 = 0$

18. What is the sum of the infinite series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \dots$

a) 1

d) 0

b) $\frac{3}{4}$

e) $\frac{3}{2}$

c) $\frac{1}{2}$

19. If the third term of an arithmetic series is -7 and the twenty-second term is 63 , what is the tenth term?

a) 6

d) 21

b) $\frac{15}{2}$

e) 27

c) 18

20. What is the sum of the first 100 positive odd integers?

a) 5050

d) 10,000

b) 7500

e) None of the above

c) 9950

21. Which is largest?

a) 2^3^4

d) 4^2^3

b) 2^4^3

e) 3^2^4

c) 4^3^2

22. If θ is a 60° angle in a triangle and has adjacent sides with lengths of 5 and 8 respectively, then the length of the third side is

a) 3

d) $13\sqrt{2}$

b) 7

d) None of the above

c) $\sqrt{89}$

23. If $f(x) = \frac{1-x}{1+x}$ then $f(f(x))$ is

a) x

d) 1

b) $\frac{1-x^2}{1+x^2}$

e) None of the above

c) $\frac{1-2x+x^2}{1+2x+x^2}$

24. What is the remainder when 7^7 is divided by 5?

a) 0

d) 3

b) 1

e) 4

c) 2

25. What is the largest possible value of $3 \sin \theta + 4 \cos \theta$?

a) 3

d) 6

b) 4

e) 7

c) 5

26. Which of the following is equal to $1 + \sin \theta$?
- a) $\frac{\cos \theta}{1 + \sin^2 \theta}$ d) $\frac{\cos^2 \theta}{1 - \sin \theta}$
- b) $\frac{\cos \theta}{\sin 2\theta}$ e) $\sec \theta \tan \theta$
- c) $\frac{\cos^2 \theta}{1 + \sin \theta}$
27. If a triangle with sides of 3, 4, and 5 units is inscribed in a circle then
- a) The perimeter of the circle is 5π .
- b) The three medians of the triangle meet at the center of the circle.
- c) The area of the triangle is at least half that of the circle.
- d) The arc of the circle which has the side of 4 as its chord equals 120° .
- e) The area of the triangle is 12 units.
28. If a family has four children, the probability that exactly two will be boys is
- a) 0 d) $\frac{3}{8}$
- b) $\frac{1}{2}$ e) None of the above
- c) $\frac{3}{16}$
29. A closet is 3 feet deep, 4 feet wide and 12 feet high. What is the length of the largest pole that can fit in the room?
- a) 12 d) 16
- b) 13 e) 19
- c) 15

30. At a reception attended by 25 people, 10 people drank tea, 12 drank coffee and 3 drank both coffee and tea. How many drank neither coffee nor tea?
- a) 0 d) 6
 b) 1 e) 8
 c) 3
31. In how many ways can a committee of three be chosen from a group of 7?
- a) 6 d) 210
 b) 21 e) None of the above
 c) 35
32. What is the solution set of $\sqrt{\frac{x-1}{x+1}} \geq 0$?
- a) $\{x > -1\}$ d) $\{x \geq 1\} \cup \{x \leq -1\}$
 b) $\{x \geq 1\}$ e) None of the above
 c) $\{x \geq 1\} \cup \{x < -1\}$
33. If a line segment AB is divided by a point C such that $AC:CB::CB:AB$ then the ratio of $AB:CB$ is
- a) $\frac{1 + \sqrt{5}}{2}$ d) $\log \pi$
 b) $\frac{1 - \sqrt{5}}{2}$ e) None of the above
 c) $\frac{1 \pm \sqrt{5}}{2}$
34. With how many zeros does the number 100! end?
- a) 10 d) 30
 b) 24 e) 100
 c) 25

35. What is the value of x in a solution to the system of equations?

$$x + 2y = 0$$

$$2x + z = 0$$

$$4y - z = 0$$

- a) -2
 b) -1
 c) 0
 d) All of the above are possible.
 e) None of the above

36. What is the equation of the perpendicular bisector of the line segment whose end points are $(1, -5)$ and $(5, -3)$?

- a) $2x + y + 3 = 0$
 b) $2x + y - 2 = 0$
 c) $2x + y - 7 = 0$
 d) $x - 2y - 11 = 0$
 e) $x - 2y - 2 = 0$

37. If the domain of the function $y = 2x^2$ is $\{-1 < x \leq 2\}$ then the range of y is

- a) $-1 < y \leq 2$
 b) $0 < y \leq 2$
 c) $0 < y \leq 4$
 d) $0 \leq y \leq 4$
 e) None of the above

38. What is $(1 - i) \div (1 + i)$?

- a) i
 b) $-i$
 c) $1 - i$
 d) 0
 e) None of the above

39. If $\theta = \arcsin \frac{1}{2}$ then $\tan \theta$ is

- a) 0
 b) $\frac{1}{2}$
 c) $\frac{\sqrt{2}}{2}$
 d) $\frac{\sqrt{3}}{2}$
 e) $\frac{\sqrt{3}}{3}$

40. What is the volume of the largest sized cube which can be enclosed inside a sphere of radius $\sqrt{3}$?

- a) 1
 b) $3\sqrt{3}$
 c) 8
 d) $\pi^{3/2}$
 e) 3π