

TWENTY-SEVENTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

COMPREHENSIVE TEST

Edited by: William N. Anderson, Jr.
East Tennessee State University
Johnson City, Tennessee

Scoring Formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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1. A circle has area 4π . What is its circumference?
 - a) 2π
 - b) 4π
 - c) 8π
 - d) $\pi\sqrt{2}$
 - e) $2\sqrt{\pi}$

2. A party of six people is seated randomly at a circular table. Included in the party are two enemies. What is the probability that the two enemies are seated next to each other?
 - a) $1/30$
 - b) $1/15$
 - c) $1/3$
 - d) $2/5$
 - e) $3/5$

3. A triangle has vertices $(1,1)$, $(2,3)$, and $(4,-1)$. What is its area?
 - a) 2
 - b) 4
 - c) 8
 - d) $\sqrt{10}$
 - e) $2\sqrt{10}$

4. The equation $x^2 + 8xy + 4y^2 = 0$ can best be described as the equation of a(n)
 - a) circle
 - b) ellipse
 - c) hyperbola
 - d) parabola
 - e) point

5. If x , y and z satisfy the equations
- $$\begin{aligned}x + 2y + 3z &= 2 \\x + 3y + 5z &= 2 \\2x + 3y + 6z &= 2\end{aligned}$$

what is the value of y ?

- a) -2
b) -1
c) 0
d) 1
e) 2
6. The real solutions of $2^x - 2^{-x} = 2^2$ are
- a) $\log_2(2 \pm \sqrt{5})$
b) $\log_2(2 + \sqrt{5})$
c) $2 \pm \sqrt{5}$
d) $2 + \sqrt{5}$
e) 1
7. If $i^2 = -1$, which of the following represents $(1/2 - \sqrt{3}/2i)^{3/2}$?
- a) i
b) -1
c) $\sqrt{2}/2 + \sqrt{2}/2i$
d) $\sqrt{2}/2 - \sqrt{2}/2i$
e) -1
8. The equation $\sin(t) - \cos(t) = 1$ has how many roots in the interval $0 \leq t < 2\pi$.
- a) 0
b) 1
c) 2
d) 3
e) 4

9. How many distinct permutations can be made from the letters of the word "letters?"
- a) 720
 - b) 1260
 - c) 2520
 - d) 5040
 - e) 10080
10. The sum of the roots (real and complex) of the polynomial $x^4 - 2x^3 + 3x^2 - 4x + 5$ is
- a) -5
 - b) -2
 - c) 4
 - d) 2
 - e) 5
11. If the radius of a circle is 4, what is the area of the circumscribed equilateral triangle?
- a) $12\sqrt{3}$
 - b) $12\pi\sqrt{3}$
 - c) $24\sqrt{3}$
 - d) $48\sqrt{3}$
 - e) $16\pi\sqrt{3}$
12. What is the solution set of $|(x + 1)| < 2|(x-1)|$?
- a) $x < 0$
 - b) $x < 1/3$
 - c) $1/3 < x < 3$
 - d) $x < 1/3 \cup x > 3$
 - e) $x > 3$

13. How many real roots are there for the equation $x^3 - 5x^2 + 9x - 5 = 0$.
- a) 0
 - b) 1
 - c) 2
 - d) 3
 - d) 4
14. If $20x^2 + 9x - 18 = (ax + b)(cx - d)$ where a, b, c and $d > 0$, then $a =$
- a) 1
 - b) 2
 - c) 4
 - d) 5
 - e) 10
15. Given that $f(x) = 1 + 2x + x^2$ on the interval $-1 \leq x < \infty$, the inverse function $f^{-1}(x)$ is
- a) $-1 + \sqrt{x}$
 - b) $1 + \sqrt{x}$
 - c) $-1 + \sqrt{x^2 - 2x}$
 - d) $(1 + x)^2$
 - e) $1 + x$
16. For a set A , let $|A|$ denote the number of members of A . If $|A| = 10$, $|B| = 7$, $|A \cap B| = 5$, $|B \cap C| = 4$, $|A \cap B \cap C| = 3$, $|A \cap C| = 3$, $|A \cup B \cup C| = 20$, what is $|C|$?
- a) 1
 - b) 3
 - c) 6
 - d) 12
 - e) 13

17. If 6 is a root of the equation $x^2 - 12x + 30 = 0$, what is the base of the number system?
- a) 6
 - b) 8
 - c) 10
 - d) 12
 - e) 16
18. A man drives up a hill at 30 mph. How fast must he drive back down to average 40 mph for the entire trip?
- a) 30 mph
 - b) 50 mph
 - c) 60 mph
 - d) 120 mph
 - e) not possible
19. In the expansion of $(2x^2 - xy^2)^6$ what is the coefficient of x^9y^6 ?
- a) -160
 - b) -32
 - c) 15
 - d) 60
 - e) 160
20. Let ABC be a triangle. If AB = 9 inches, BC = 4 inches and AC = 3 inches, which angle is largest?
- a) $\angle A$
 - b) $\angle B$
 - c) $\angle C$
 - d) Not enough information
 - e) There is no such triangle.

21. In triangle ABC, $\angle A = 30^\circ$, $AB = 4$ and $BC = 4$. What is the area of triangle ABC?
- a) $2\sqrt{3}$
 - b) $8\sqrt{3}/3$
 - c) $4\sqrt{3}$
 - d) 8
 - e) 16
22. For real numbers A and B let $A \textcircled{H} B$ denote the harmonic mean $A \textcircled{H} B = 2AB/(A+B)$. The inequality $A \textcircled{H} (B + C) \leq A \textcircled{H} B + A \textcircled{H} C$
- a) is never true
 - b) is always true
 - c) is true if and only if $A = B = C$
 - d) is true if and only if $B = C$
 - e) is true if and only if $A \leq B$ and $A \leq C$.
23. The base of a pyramid is an equilateral triangle with side of length $2\sqrt{3}/3$. The other three edges of the pyramid have lengths $2\sqrt{3}/3$, $2\sqrt{3}/3$, and 1. What is the height of the pyramid?
- a) $\sqrt{3}/2$
 - b) 1
 - c) $2\sqrt{3}/3$
 - d) $\sqrt{3}$
 - e) $\sqrt{6}$
24. When 1983^{1066} is divided by 5, the remainder is
- a) 0
 - b) 1
 - c) 2
 - d) 3
 - e) 4

25. One die from a pair of dice is renumbered so that each of the numbers 1, 2, and 3 appears twice. When the pair of dice is thrown, the possible sums are thus 2, 3, 4, 5, 6, 7, 8, 9. What is the probability of obtaining the sum of 6 when the dice are thrown?
- a) $1/9$
 - b) $5/36$
 - c) $1/6$
 - d) $7/36$
 - e) $2/9$
26. A line passes through the point $(0,13)$ and is tangent to the conic $x^2 + y^2 - 2y = 24$. If the point of tangency is B, what is the length of the line segment from $(0,13)$ to B?
- a) 5
 - b) $\sqrt{119}$
 - c) 12
 - d) 13
 - e) $\sqrt{221}$
27. The polynomial $p(x)$ has the values $p(1) = 4$ and $p(-1) = 2$. What is the remainder when $p(x)$ is divided by $x^2 - 1$?
- a) $x + 3$
 - b) $4x + 2$
 - c) $3x + 1$
 - d) 0
 - e) not enough information
28. Let AB and CD be chords of a circle, intersecting in the point E. If $AE = 2$, $BE = 4$, and $CE = 6$, what is DE?
- a) 0
 - b) 2
 - c) 3
 - d) 4
 - e) 6

29. The sides of a triangle are in the ratio 2:3:4. The formula for the area A in terms of the perimeter P is

- a) $A = 12P$
- b) $A = 24P$
- c) $A = \sqrt{15}/216 P^2$
- d) $A = \sqrt{15}/108 P^2$
- e) $A = 12 P^2$

30. If $f(x) = 2x$ and $g(x) = 1 + x^2$, then $f(g(x)) =$

- a) $1 + 4x^2$
- b) $1 + (4x)^2$
- c) $2(1 + x^2)$
- d) $2x + 2x^3$
- e) $1 + 2x + 2x^3$

31. The algebraic expression

$$\frac{\frac{2}{2x+1} - \frac{1}{x+2}}{\frac{1}{2x+1} - \frac{2}{x+2}}$$

equals

- a) $1/x$
- b) $-1/x$
- c) 1
- d) -1
- e) $-(2x - 1)^{-2} (x + 2)^{-2}$

32. Let $i^2 = -1$. Then a possible value of i^i is

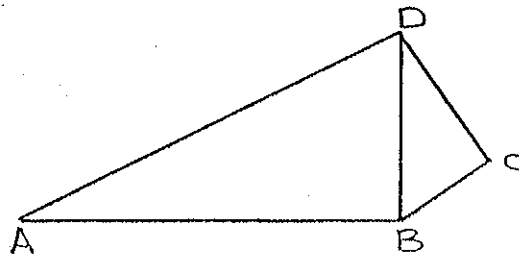
- a) $e^{\pi/2}$
- b) $e^{-\pi/2}$
- c) -1
- d) $-i$
- e) i

33. Let a and b be positive real numbers. Then $\log_a b \log_b a =$
- $\log(a^b + b^a)$
 - $\log(ab)^{ab}$
 - 1
 - e
 - 10

34. If $\sin(2t) + \sqrt{3} \cos(2t) = D \sin(2t + s)$, then $s =$
- $-\pi/3$
 - $-\pi/6$
 - 0
 - $\pi/6$
 - $\pi/3$

35. In the figure ABCD, $\angle DAB$ and $\angle BDC = 15^\circ$. The angles $\angle ABD$ and $\angle BCD$ are right angles. Segment AB has length $2 + \sqrt{3}$. What is the length of segment BC?

- $\sqrt{2} - \sqrt{3}/2$
- $\sqrt{2} + \sqrt{3}/2$
- $2 + \sqrt{3}$
- $2 - \sqrt{3}$
- 1



36. The equation $x^4 + 4x^3 + x^2 - 6x + 2$ has how many rational roots?
- 0
 - 1
 - 2
 - 3
 - 4

37. If $\tan x = 3/4$, then $\sin x =$
- a) $3/4$
 - b) $4/3$
 - c) $3/5$
 - d) $3/5$ or $-3/5$ depending on x
 - e) $4/5$ or $-4/5$ depending on x
38. The expression $\cos^4 x - \sin^4 x$ is equal to
- a) $\sin 2x$
 - b) $\cos 2x$
 - c) $-\sin 2x$
 - d) $-\cos 2x$
 - e) $\cos 4x - \sin 4x$
39. If $5/(6x^2 + 13x + 6) = A/(3x + 2) + B/(2x + 3)$ then $A =$
- a) -3
 - b) -2
 - c) 1
 - d) 2
 - e) 3
40. In the figure $AB = AC$, angle $BAD = 30^\circ$, and $AE = AD$. Then angle EDC equals

- a) $7\ 1/2^\circ$
- b) 10°
- c) $12\ 1/2^\circ$
- d) 15°
- e) 20°

