

THIRTY-THIRD ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA II 1989

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Scoring formula: $4R - W + 40$

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

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1. Assuming that x is the independent variable, which of the following is not a function of x ?

a. $y = x^2 - 3$

b. $xy = 1$

c. $x + 3 = y^2$

d. $\frac{x^2}{y} = 5$

e. $x\sqrt{x+5} = y + 7$

2. Rank the three students from highest to lowest relative to the way they compare with others in their own class.

	<u>Jim</u>	<u>Mary</u>	<u>Bob</u>
Raw score (x)	82	79	80
Mean (\bar{x})	80	75	83
Standard deviation (σ)	15	35	20

- a. Jim, Bob, Mary
- b. Jim, Mary, Bob
- c. Bob, Jim, Mary
- d. Mary, Bob, Jim
- e. Mary, Jim, Bob
3. The inverse function of $f(x) = 3 - \sqrt{3 - x}$ is:
- a. $\sqrt{3 - x} - 3$
- b. $x, x \geq 3$
- c. $-x^2 + 6x - 6, x \geq 3$
- d. $x^2 - 6x + 9, x \leq 3$
- e. $-x^2 + 6x - 6, x \leq 3$

4. The graph of the following quadratic equations intersect each other in how many points?

$$\begin{aligned}x^2 + y^2 &= 4 \\2x^2 - y^2 &= 8\end{aligned}$$

- a. 0
 - b. 1
 - c. 2
 - d. 3
 - e. 4
5. Tammie, Beth, and Jennifer are cousins. The sum of their ages is 55. In four years, Jennifer will be twice as old as Tammie is now. Ten years ago, Jennifer was twice as old as Beth was then. How much older is Beth than Tammie?
- a. 2 years
 - b. 3 years
 - c. 7 years
 - d. 10 years
 - e. 14 years
6. Given $f(x) = 3 - x^2$ and $g(x) = 3x - 7$, $f[g(-2)]$ is:
- a. -10
 - b. 166
 - c. 172
 - d. -166
 - e. 10

7. What is the value of c such that $x + 2$ divides $x^3 - cx^2 + 3x + 7c$ and gives a remainder of zero?
- a. 0
 - b. -2
 - c. 6
 - d. $3/4$
 - e. $\frac{14}{3}$
8. The center and radius of the circle whose equation is $x^2 + y^2 - 4x - 10y = -4$ are:
- a. $(-2, -5), 5$
 - b. $(-4, -10), -4$
 - c. $(2, 5), 5$
 - d. $(-2, -5), \sqrt{29}$
 - e. $(2, 5), \sqrt{-4}$
9. The simplified form of $2i^{34} - 3i^{24}$ is:
- a. 5
 - b. -5
 - c. $-5i$
 - d. $5i$
 - e. -1

10. Which of the following pairs of lines are perpendicular?

(1) $2x + 3y = 5$

(2) $2y - 4x = 6$

(3) $2y + 12 = x$

(4) $-3x + 2y = 4$

a. 1 and 2

b. 2 and 3

c. 1 and 3

d. 3 and 4

e. 1 and 4

11. If $f(x) = \sqrt{x}$, then what is $\frac{f(2+h) - f(2)}{h}$?

a. 0

b. 1

c. $\frac{1}{\sqrt{h}}$

d. $\frac{1}{\sqrt{2+h} - \sqrt{2}}$

e. $\frac{1}{\sqrt{2+h} + \sqrt{2}}$

12. Given: $2x + 17y = 162$

$$17x + 2y = 275$$

then: $x + y =$

a. $\frac{113}{15}$

b. 32

c. 23

d. $\frac{2}{17}$

e. $\frac{17}{2}$

13. The roots of a quadratic equation whose discriminant is 84 may best be described as:
- unequal and rational
 - unequal and irrational
 - imaginary
 - equal
 - additive inverses of each other
14. Simplify: $\frac{1 - \frac{1}{v+1}}{1 + \frac{1}{v-1}}$
- 1
 - 1
 - $\frac{v-1}{v+1}$
 - $\frac{1-v}{1+v}$
 - 2
15. All the values of x that satisfy the inequality $-8 \leq 1 - 3(x - 2) < 13$ are:
- $-2 < x \leq 5$
 - $-6 < x \leq 1$
 - $5 \leq x < -2$
 - $1 \leq x < -6$
 - $-4 \frac{1}{2} < x \leq 6$

16. A survey is made of house pets in a given community containing 70 houses. The number of houses and pets owned was as follows:

Cats - 19 houses
 Dogs - 38 houses
 Birds - 10 houses
 Cat and bird only - 2 houses
 Cat and dog only - 3 houses
 Dog and bird only - 1 house
 Cat, dog and bird - 2 houses

How many houses had no pets?

- a. 38
 b. 13
 c. 57
 d. 5
 e. 0
17. The solution of the inequality $3m^2 < -10 - 13m$ using interval notation is?
- a. $(\frac{10}{3}, -1)$
 b. $(-\infty, -\frac{10}{3}) \cup (-1, \infty)$
 c. $(-\frac{10}{3}, 1)$
 d. $(-\frac{10}{3}, -1]$
 e. $(-\frac{10}{3}, -1)$

18. Multiply: $\frac{x^3 - 27}{5x - 15} \cdot \frac{2x^2 - 18}{3x^2 + 9x + 27}$

- a. $\frac{2x + 6}{15}$
 b. $\frac{x^2 + 6x + 9}{x - 3}$
 c. $\frac{2x^2 - 18}{15}$
 d. $\frac{2}{15}$
 e. $\frac{2x - 6}{5x + 15}$

19. All the values of x that satisfy the inequality $|2x - 5| > 7$ are:
- $-1 < x < 6$
 - $x > 6$
 - $x < -1$
 - $x < -1$ or $x > 6$
 - $x > -1$
20. $-2 + 2i$ is a zero of $x^3 + x^2 - 4x - 24$. The other two zeros are:
- -3 and $-2 - 2i$
 - 3 and $2 + 2i$
 - 3 and $-2 - 2i$
 - -3 and $1 + 2i$
 - 3 and $1 + 2i$
21. P dollars is invested at a simple interest rate of r . The balance in the account after t years is given by $A = P + Prt$. In order for an investment of \$1,000 to grow to more than \$1,250 in two years, what must the interest rate be?
- $r = 10\%$
 - $r = 12.5\%$
 - $r = 25\%$
 - $r > 25\%$
 - $r > 12.5\%$
22. A group of college students attended a party. In the group there were 15 foreign students, 43 American males, 35 more American females than foreign males, as many males as females. How many students attended the party?
- 100
 - 93
 - 106
 - 141
 - 143

Algebra II

23. The solutions of the equation $4x^4 - 8x^2 + 3 = 0$ are:

- a. $3\sqrt{2/2}, \sqrt{2/2}$
- b. $\frac{3}{2}, \frac{1}{2}$
- c. $\frac{\sqrt{6}}{2}, \frac{-\sqrt{6}}{2}, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}$
- d. $\pm \frac{3}{2}, \pm \frac{1}{2}$
- e. $\frac{3\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$

24. Subtract: $\frac{a + 3}{2a^2 + 5a - 12} - \frac{a - 1}{2a^2 + 7a - 4}$

The numerator of the simplified answer is:

- a. $10a - 6$
- b. $4a^2$
- c. $6a - 6$
- d. $4a^2 + 6a - 6$
- e. $4a^2 + 6a + 6$

25. The complex number $\frac{2 + 3i}{4 - 2i}$ written in $a+bi$ form is:

- a. $\frac{7}{6} + \frac{2}{3}i$
- b. $\frac{7}{6} - \frac{2}{3}i$
- c. $\frac{7}{6} + \frac{4}{3}i$
- d. $\frac{1}{10} - \frac{4}{5}i$
- e. $\frac{1}{10} + \frac{4}{5}i$

26. An urn contains six red balls and four blue balls. If two balls are drawn at random, with replacement, what is the probability that exactly one of each color is drawn?
- $\frac{1}{10}$
 - $\frac{9}{25}$
 - $\frac{12}{25}$
 - $\frac{24}{45}$
 - 0
27. $f(x) = 3x^3 - 2x^2 - 150$ Which one of the following is true?
- There is at least one real root between $x = 1$ and $x = 5$
 - There is no real root between $x = 1$ and $x = 5$
 - There are 4 real roots between $x = 1$ and $x = 5$
 - $x - 1$ is a factor of $3x^3 - 2x^2 - 150$
 - $x + 1$ is a factor of $3x^3 - 2x^2 - 150$
28. The complex number $\frac{1}{i^7}$ is equal to:
- 1
 - i
 - 1
 - $-i$
 - 2
29. The probability a certain door is locked is one-half. The key which unlocks the door is one of ten keys hanging on a rack. If two of the keys are selected at random, what is the probability that the person will be able to enter the room without returning to select another key?
- 0.2
 - 0.6
 - 0.5
 - 0.7
 - 1.0

30. Simplify $\frac{(x^{2/3}y^{4/5}z^{1/3})^2}{(x^{1/3}y^{2/3}z)^3}$

a. $\frac{x^{1/3}}{y^{7/3}z^{2/5}}$

b. $\frac{x^{1/3}}{y^{2/5}z^{7/3}}$

c. $\frac{x^{2/3}}{y^{1/3}z^{3/2}}$

d. $x^{2/3}y^{2/5}z^{7/3}$

e. $\frac{z^{7/3}}{x^{2/3}y^{1/3}}$

31. Determine the value of "a" so that the solution set is the set of all real numbers.

$$\frac{1}{a} + \frac{4ax + 1}{2a} + \frac{1}{3a} = \frac{60x + 11}{30}$$

a. -2

b. -1

c. 0

d. 1

e. 5

32. Which of the following is the equation for the given graph?

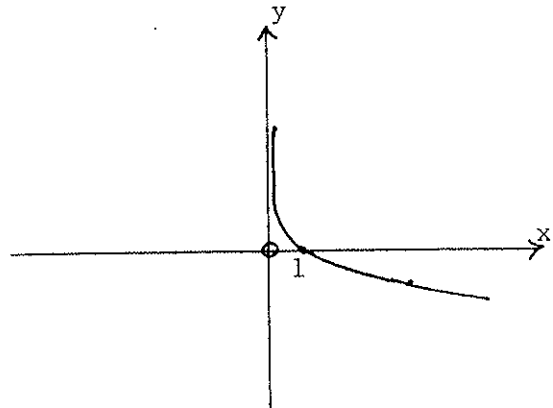
a. $y = 3^x$

b. $y = \log_3 x$

c. $y = (\frac{1}{3})^x$

d. $y = \log_{-3} x$

e. $y = \log_{1/3} x$



33. Solving for M in the equation $\frac{M}{C} = \frac{M}{A} + \frac{1}{D}$ leads to the conclusion that M =

a. $\frac{AC}{D - C}$

b. $\frac{C - A}{D}$

c. $\frac{AC}{AD - DC}$

d. $\frac{AD}{DC - CA}$

e. $\frac{AC}{DC}$

34. The value of the determinant $\begin{vmatrix} i & 1 \\ -1 & -i \end{vmatrix}$ is

a. 0

b. 1

c. -1

d. -2

e. 2

35. Consider the graph of $\frac{x^2 - x - 2}{2x^2 - 18}$. Which of the following is not true?

a. $x = 3$ is a vertical asymptote

b. $x = -3$ is a vertical asymptote

c. $y = 1$ is a horizontal asymptote

d. $x = 2$ is a zero of the function

e. The graph intercepts the y - axis only once.

36. If $\tan A = -1$ and $\sin A$ is positive, then the measurement of A is

a. 45°

d. -45°

b. 135°

e. 225°

c. 180°

37. A box contains ten ball point pens, two of which are defective. If three pens are chosen at random, without replacement, what is the probability that at least one will work properly?

- a. 1
- b. $\frac{3}{10}$
- c. $\frac{7}{15}$
- d. $\frac{1}{45}$
- e. $\frac{1}{125}$

38. Use the factor theorem to find which one of the following is true:

- a. $x - 5$ and $x + 2$ are factors of $3x^3 - 12x^2 - 11x - 20$
- b. $x + 5$ is a factor of $3x^3 - 12x^2 - 11x - 20$
- c. $x - 5$ is a factor of $3x^3 - 12x^2 - 11x - 20$
- d. $x + 1$ is a factor of $3x^3 - 12x^2 - 11x - 20$
- e. $x + 10$ is a factor of $3x^3 - 12x^2 - 11x - 20$

39. y varies directly with r and the square of x and inversely with the cube of p . If x is tripled and p is halved, what is the effect on y ?

- a. it is $\frac{3}{2}$ as much as it was originally
- b. it is $\frac{8}{9}$ as much as it was originally
- c. it is $\frac{9}{8}$ as much as it was originally
- d. it is 108 times as much as it was originally
- e. it is 72 times as much as it was originally

40. Which of the following is the equation for the given graph?

- a. $y = -\frac{3}{4}x + 3$
- b. $y = \frac{4}{3}x - 4$
- c. $y = \frac{3}{4}x + 3$
- d. $y = -\frac{4}{3}x + 3$
- e. $y = \frac{3}{4}x - 4$

