

THIRTY-SECOND ANNUAL MATHEMATICS CONTEST  
sponsored by  
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA II 1988

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Scoring formula:  $4R - W + 40$

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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Algebra II

1. The complex number  $(i^6 - i^4)/i$  is equal to:
  - a)  $i$
  - b)  $2$
  - c)  $-2i$
  - d)  $2i$
  - e)  $1$
2.  $\sqrt{-105}$  is equal to:
  - a)  $i\sqrt{-105}$
  - b)  $5i\sqrt{21}$
  - c)  $i\sqrt{105}$
  - d)  $3i\sqrt{35}$
  - e)  $105i$
3. The complex number  $(1 - i)/(3i + 1)$  written in  $a + bi$  form is:
  - a)  $1/5 + (2/5)i$
  - b)  $-1/5 + (-2/5)i$
  - c)  $1/2 + (1/2)i$
  - d)  $-(1/4) + (-3/8)i$
  - e)  $2/5 + (2/5)i$
4. The line  $mx - ny = 1$  passes through the two points  $(2, -3)$  and  $(-4, -1)$ . The values of  $m$  and  $n$  are:
  - a)  $m = 3/7, n = -1/7$
  - b)  $m = 1/7, n = 3/7$
  - c)  $m = 1/7, n = -3/7$
  - d)  $m = -1, n = 3$
  - e)  $m = -1/7, n = 3/7$
5. Given  $f(x) = 5x - 3$  and  $g(x) = -x^2 + 4x$ ,  $g[f(-1)]$  is:
  - a)  $96$
  - b)  $40$
  - c)  $32$
  - d)  $-96$
  - e)  $-40$
6. All the values of  $x$  that satisfy the inequality
$$\frac{1}{x} + \frac{3}{4x} > \frac{7}{8}$$
are:
  - a)  $x > 1/2$
  - b)  $0 < x < 2$
  - c)  $x < 2, x \neq 0$
  - d)  $x > 2$
  - e)  $x < 1/2, x \neq 0$

7. The solution set of the absolute value equation

$$| 4x + 7 | = | x - 3 |$$

is:

- a)  $\{-10/3\}$
- b)  $\{10/3\}$
- c)  $\{-10/3, -4/5\}$
- d)  $\{4/5\}$
- e)  $\{x:x < -16/3\}$

8. All the values of x that satisfy the absolute value inequality

$$| 3x + 7 | > 9$$

are:

- a)  $x > 2/3$
- b)  $x > -16/3$
- c)  $-16/3 < x < 2/3$
- d)  $x < -16/3$  or  $x > 2/3$
- e)  $x < -16/3$

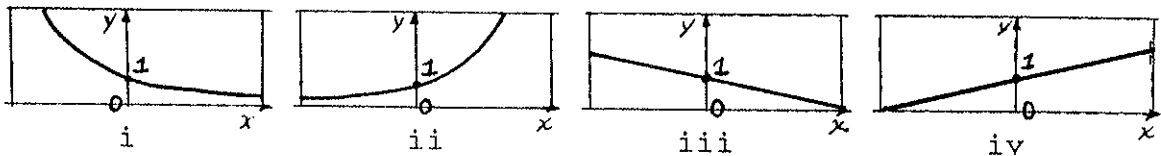
9. Laura drives 10 miles per hour faster than Steve. Both start at the same time for Alta from Springdale. The distance between the two cities is 100 miles. It takes Steve 1/3 of an hour longer than Laura to make the trip. What is Steve's average speed?

- a) 60 mph
- b) 55 mph
- c) 45 mph
- d) 50 mph
- e) 65 mph

10. Given  $R(x) = (3x - 1)/(4x + 2)$ , what is  $R[R(x)]$ ?

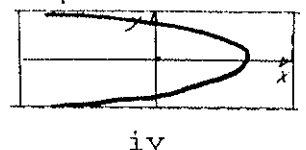
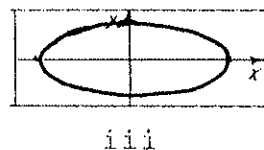
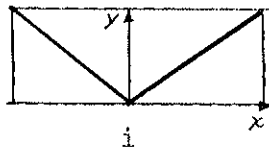
- a)  $(3x - 1)^2/(4x + 2)^2$
- b)  $(x - 1)/4x$
- c)  $(5x - 1)/2x$
- d)  $(3x - 1)/(4x + 2)$
- e)  $(4x + 2)/(3x - 1)$

11. Which of the following is the graph of  $y = 5^{-x}$  ?



- a) i
- b) ii
- c) iii
- d) iv
- e) i and iii

12. Which of the following can be the graph of a function  $y = f(x)$ ?



- |             |             |
|-------------|-------------|
| a) i only   | d) iv only  |
| b) ii only  | e) i and ii |
| c) iii only |             |

13. The expression  $\frac{x^{-1}y^2 + x^2y^{-1}}{xy^{-1} - x^{-1}y}$  can be written without negative exponents as:

- |                                   |                                  |
|-----------------------------------|----------------------------------|
| a) $\frac{x^2 + xy + y^2}{x + y}$ | d) $\frac{x^3 + y^3}{x^2 + y^2}$ |
| b) $\frac{x^2 + xy + y^2}{x - y}$ | e) $\frac{y^3 - x^3}{x^2 - y^2}$ |
| c) $\frac{x^2 - xy + y^2}{x - y}$ |                                  |

14. If  $\theta = \pi/4$  radians, then  $\sin^2\theta + \cos^2\theta - \tan\theta =$

- |                |                 |
|----------------|-----------------|
| a) 1           | d) -1           |
| b) $2\sqrt{2}$ | e) $2/\sqrt{2}$ |
| c) 0           |                 |

15. At the instant that the angle of elevation of the sun is  $60^\circ$ , a monument casts a shadow 10 feet long. How tall is the monument?

- |           |                     |
|-----------|---------------------|
| a) 20 ft. | d) $10\sqrt{3}$ ft. |
| b) 10 ft. | e) $5\sqrt{3}$ ft.  |
| c) 5 ft.  |                     |

16. If  $\log x$  denotes the common logarithm, then the solution set for  $\log x = 1 - \log(x - 3)$  is:

- |                |             |
|----------------|-------------|
| a) $\{10\}$    | d) $\{1\}$  |
| b) $\{-2, 5\}$ | e) $\{20\}$ |
| c) $\{5\}$     |             |

17. A window has the shape of a square surmounted by an equilateral triangle. The area of the window expressed as a function of  $s$ , the side of the square, would be  $A = f(s) =$

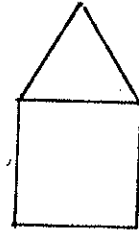
a)  $3s^2/2$

d)  $\frac{3s^2\sqrt{3}}{2}$

b)  $\frac{5s^2\sqrt{3}}{4}$

e)  $5s^2/2$

c)  $\frac{5s^2\sqrt{2}}{2}$



18. If the polynomial  $7x^{55} - 10x^{32} - 12x^{17}$  is divided by  $x + 1$ , the remainder is:

a)  $-12$

d)  $12$

b)  $-2$

e)  $-5$

c)  $8$

19. If  $x - 3$  is a factor of  $x^3 - 7x - 2b$ , then  $b$  is equal to:

a)  $6$

d)  $-3$

b)  $-7/2$

e)  $24$

c)  $3$

20. Determine the nature of the roots of  $x^6 + 3x^4 - 2x^2 - 4$ :

a)  $6$  complex

b)  $2$  positive,  $2$  negative,  $2$  complex

c)  $1$  positive,  $1$  negative,  $4$  complex

d)  $2$  positive,  $4$  complex

e)  $2$  negative,  $4$  complex

21. The equation of the line perpendicular to  $3x + 2y = 8$  at the point  $(0, 4)$  is:

a)  $2x + 3y = 8$

d)  $2x - 3y = 4$

b)  $2x - 3y = 8$

e)  $3y - 2x = 12$

c)  $2x - 3y = 4$

22. The determinant of the matrix  $\begin{bmatrix} 1 & 3 \\ -4 & -2 \end{bmatrix}$  is:

- a) 10
- b) -14
- c) -10
- d) -2
- e) 0

23. The system of linear equations  $6x + 3y = 9$  and  $2x = 3 - y$  has

- a) no solution
- b) exactly one solution
- c) exactly two solutions
- d) more than two but a finite number of solutions
- e) infinitely many solutions

24. A plane flies 1320 miles in 3 hours with a head wind and 1200 miles in 2 hours with the same wind as a tail wind. If the rate at which the plane flies in still air is constant, what is the speed of the wind?

- a) 80 mph
- b) 530 mph
- c) 60 mph
- d) 520 mph
- e) 130 mph

25. Mr. Pecan sells three types of nuts. Peanuts sell for \$1.00 a pound. Pecans sell for \$1.50 a pound, and cashews sell for \$2.50 a pound. He wants to mix up 27 pounds of nuts which cost \$2.00 a pound. In this mixture he wants there to be twice as many cashews as there are peanuts. How many pounds of cashews will be in the mixture?

- a) 12
- b) 18
- c) 9
- d) 10.8
- e) 6

26. Given the matrix  $A = \begin{bmatrix} 6 & 4 \\ 3 & -2 \end{bmatrix}$ , the inverse  $A^{-1}$  is:

- a)  $\begin{bmatrix} 1/6 & 1/4 \\ 1/3 & -1/2 \end{bmatrix}$
- b)  $1 \div \begin{bmatrix} 6 & 4 \\ 3 & -2 \end{bmatrix}$
- c)  $\begin{bmatrix} -6 & -4 \\ -3 & 2 \end{bmatrix}$
- d)  $\begin{bmatrix} 4 & 6 \\ -2 & 3 \end{bmatrix}$
- e)  $\begin{bmatrix} 1/12 & 1/6 \\ 1/8 & -1/4 \end{bmatrix}$

27. The determinant  $\begin{vmatrix} 1 & 4 & 3 \\ -2 & 6 & -1 \\ 3 & 2 & 1 \end{vmatrix}$  is equal to:
- a)  $\begin{vmatrix} 1 & 3 & 4 \\ -2 & 1 & 6 \\ 3 & 1 & 2 \end{vmatrix}$                       d) -54
- b)  $6 \begin{vmatrix} 1 & 2 & 3 \\ -2 & 3 & -1 \\ 3 & 1 & 1 \end{vmatrix}$                       e) 26
- c)  $\begin{vmatrix} 1 & 4 & 3 \\ -1 & 10 & 2 \\ 3 & 2 & 1 \end{vmatrix}$
28. The sum of the roots of the equation  $3x^2 + 4x - 1 = 0$  is:
- a)  $-4/3$                                       d)  $\sqrt{7}$
- b) 0    e) 6
- c) -8
29. A rectangular swimming pool is surrounded by a concrete walkway of uniform width. The outside dimensions of the walkway are 20 feet by 15 feet. If the water surface area is 126 square feet, then the perimeter of the pool is
- a) 23 ft                                      d) 65 ft
- b) 46 ft                                      e) 80 ft
- c) 50 ft
30. If the quadratic equation  $ax^2 + bx = 2$  has  $-1/3$  and 2 as roots, then the sum  $a+b$  is
- a) -2    d) 1
- b) -1    e) 2
- c) 0
31. The vertex of the parabola  $2x^2 - y - 8x + 5 = 0$  is found at
- a) (0,2)                                      d) (-2,-3)
- b) (1,-3)                                      e) (2,-3)
- c) (-2,5)



Algebra II

32. The center of the circle  $x^2 + y^2 + 4x - 8y = 5$  is

- a) (2,4)
- b) (2,-4)
- c) (-2,-4)
- d) (-2,4)
- e) (4,2)

33. The graph of  $x^2 - 6x - 3y^2 = 0$  is

- a) a line
- b) a parabola
- c) a circle
- d) an ellipse
- e) a hyperbola

34. The illumination from a light source is directly proportional to the strength of the source and inversely proportional to the square of the distance from the source. What happens to the illumination when both the strength and the distance are cut in half?

- a) It remains the same.
- b) It is cut in half.
- c) It is divided by 4.
- d) It is quadrupled.
- e) It is doubled.

35. Leighton is 4 years older than Liesl. The difference in the reciprocal of their ages is 4 times the reciprocal of Leighton's age. How old is Leighton?

- a) 25
- b) 21
- c) 14
- d) 8
- e) 5

36. The rational function  $f(x) = \frac{8x^2}{4x^2 + 8x - 5}$  has the horizontal asymptote:

- a)  $y = -5/2$
- b)  $x = 2$
- c)  $x = 1/2$
- d)  $y = 1/2$
- e)  $y = 2$

37. If a state has license plates imprinted with three letters followed by a three digit number, how many different license plates are possible?

- a) 17,576,000
- b) 11,232,000
- c) 108
- d) 1,000,000
- e) 78,000

38. If three coins are tossed in the air, what is the probability that exactly two heads or exactly two tails appear?

- a)  $1/4$
- b)  $3/4$
- c)  $3/8$
- d)  $1/2$
- e)  $1/8$

39. Sarah, editor of the literary journal, has received 20 applications for a staff position. Of these 20 people, 8 are enrolled in Creative Writing, 9 are straight A students, and 5 are both straight A students and are enrolled in Creative Writing. If Sarah selects an applicant at random, what is the probability that she chooses an A student or one enrolled in Creative Writing?

- a)  $17/20$
- b)  $8/9$
- c)  $19/20$
- d)  $1/4$
- e)  $3/5$

40. Suppose a box of 4 light bulbs contains one bad bulb and 3 good ones. If two bulbs are selected from the box, what is the probability of selecting 2 good bulbs?

- a)  $1/3$
- b)  $1/2$
- c)  $2/7$
- d)  $2/3$
- e)  $3/8$



