

TWENTY-NINTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA II 1985

Prepared by: Walters State Community College
Mathematics Department
Morristown, Tennessee
Judy Hector, Coordinator

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Community College

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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1. If $\begin{cases} 3x - 5y = -13 \\ 4x + 3y = 2 \end{cases}$ then $y = ?$
- Handwritten work:

$$\begin{array}{r} 12x - 15y = -39 \\ -12x - 9y = -6 \\ \hline -24y = -45 \\ y = 1.875 \end{array}$$
- Options:
 a) 1
 b) 2
 c) 0
 d) -1
 e) -2

2. If $\theta = 45^\circ$, then $\sin \theta + \tan \theta =$
- Handwritten work:
 $\frac{\sqrt{2}}{2} + 1$
- Options:
 a) $1 + \sqrt{2}$
 b) $1 + \sqrt{3}$
 c) $\frac{\sqrt{2} + 2}{2}$
 d) $\frac{1}{\sqrt{3}} + 1$
 e) Does not exist

3. The curves $y = \frac{1}{x^2}$ and $x^2 + y^2 = 4$ intersect in how many points?
- Handwritten work:

 $4x^2 = 1$
- Options:
 a) 0
 b) 1
 c) 2
 d) 3
 e) 4

4. The larger root of $x^2 + 3x - 40 = 0$ is
- Handwritten work:
 $(x + 8)(x - 5) = 0$
- Options:
 a) 4
 b) 5
 c) 8
 d) 10
 e) 20

5. The numbers 32 and 64 are written in base 9. Their product written in base 9 is
- Options:
 a) 2725
 b) 1068
 c) 2048
 d) 2248
 e) 2268

Handwritten multiplication in base 9:

$$\begin{array}{r} 32_9 \\ \times 64_9 \\ \hline 138 \\ 213 \\ \hline 2268 \end{array}$$

Algebra II

6. The slope of the straight line perpendicular to $2x - 3y = 1$ at the point $(1/2, 0)$ is

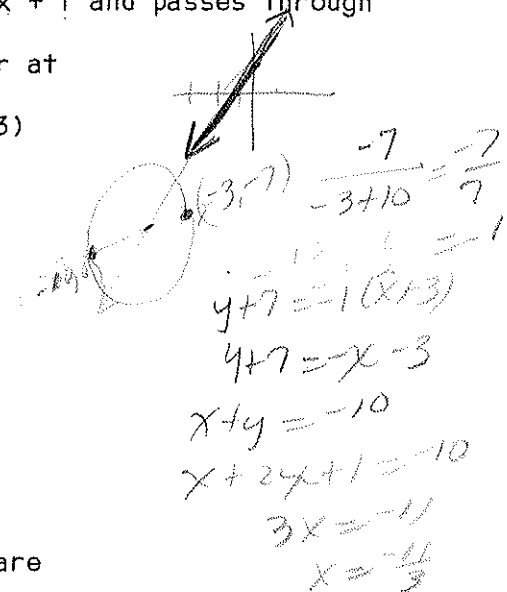
- a) 3
 b) $1/3$
 c) $2/3$
 d) $-2/3$
 e) $-3/2$

$$-3y = -2x + 1$$

$$y = \frac{2}{3}x - \frac{1}{3}$$

7. A circle whose center lies on the line $y = 2x + 1$ and passes through the points $(-10, 0)$ and $(-3, -7)$ has its center at

- a) $(1, 3)$
 b) $(2, 5)$
 c) $(-1, -1)$
 d) $(-2, -3)$
 e) $(0, 1)$



8. The radius of the circle of problem 7 is

- a) $\sqrt{5}$
 b) 5
 c) 7
 d) 13
 e) $\sqrt{82}$

9. The solutions to the inequality $x^2 - 3x > 18$ are

- a) $x < -3$ and $x < 6$
 b) $x < -3$ or $x > 6$
 c) $x > -3$ and $x < 6$
 d) $x < -2$ or $x > 9$
 e) $x < 3$ and $x > -6$

$$x^2 - 3x - 18 > 0$$

$$(x - 6)(x + 3) > 0$$

10. If $x + 2y - 2z = 2$ then $x + y + z = ?$

$$3x + y - 3z = -6$$

$$x - 2y + z = -7$$

- a) -1
 b) 0
 c) 1
 d) -2
 e) 2

Algebra II

15. The length of a rectangular rock garden is 2 meters more than its width. If each dimension were increased by 3 meters, the area would be increased by 33 square meters. Find the length of the original garden.

- a) 8
- b) 6
- c) 5
- d) 3
- e) 9

16. The fourth term in the expansion of $(x^2 - 2y)^8$ is:

- a) $-56x^5y^3$
- b) $-448x^{10}y^3$
- c) $56x^{10}y^3$
- d) $-448x^5y^3$
- e) $-56x^7y^3$

17. The equation $x^4 + x^2 - 5x - 2 = 0$ has how many rational roots?

- a) 0
- b) 1
- c) 2
- d) 3
- e) 4

18. The equation $\sqrt{x^2 + 3} = x^2 + 1$ has how many real roots?

- a) 0
- b) 1
- c) 2
- d) 3
- e) 4

19. If 3 coins are tossed in the air, what is the probability that at least one will turn up heads?

- a) $1/8$
- b) $3/8$
- c) $3/4$
- d) $7/8$
- e) $1/2$

Algebra II

24. Let $g(x) = 2 - 3x$. What is $g(g(x))$?
- a) $-(9x + 4)$
 - b) $9x + 8$
 - c) $2 - 3x$
 - d) $9x - 4$
 - e) $4 - 12x + 9x^2$
25. The graphs of the functions $y = 3^x$ and $y = \log_3 x$
- a) intersect when x is infinitely large
 - b) intersect in a single point
 - c) do not intersect
 - d) intersect in exactly two points
 - e) never intersect the x -axis
26. Since the area of the surface of a sphere varies as the square of its radius, what effect does multiplying the radius by 3 have on the area of the surface?
- a) multiplies it by 3
 - b) divides it by 3
 - c) multiplies it by 9
 - d) divides it by 9
 - e) multiplies it by 6
27. If $\log_9 1/2 + \log_9 x = 3/2$, then x is equal to
- a) 3
 - b) 9
 - c) 27
 - d) 1
 - e) 54
28. If $\log_{10} a = 1/2$, then $\log_a 100 =$
- a) $1/4$
 - b) 1
 - c) 2
 - d) $1/2$
 - e) 4

29. If $\log 2 = .301$ and $\log 3 = .477$, what is $\log 12$?

- a) .602
- b) .778
- c) .954
- d) 1.255
- e) 1.079

30. What is the coefficient of x^6y^4 in $(2x + 3xy^2)^6$?

- a) 144
- b) 240
- c) 270
- d) 2160
- e) 576

31. Evaluate $\begin{vmatrix} 3 & -10 \\ 2 & -7 \end{vmatrix}$

- a) -41
- b) 41
- c) 1
- d) -1
- e) -12

32. For what value of a is the determinant equal to 12?

$$\begin{vmatrix} 0 & 0 & 2 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & a \\ 0 & 4 & 0 & 0 \end{vmatrix}$$

- a) $-1/2$
- b) $1/2$
- c) 6
- d) 4
- e) -4

33. The maximum value of $-x^2 + 5x - 4$ is

- a) $3/2$
- b) $7/4$
- c) $-5/2$
- d) $5/2$
- e) $9/4$

Algebra II

34. A club of 3 men and 4 women selects different officers (a president and a secretary) every month at random. What is the probability of the slate of officers having a woman as president?

- a) $9/21$
- b) $4/42$
- c) $4/21$
- d) $12/21$
- e) none of these

35. If $f(x)$ is a real-valued function, the domain of $f(x) = \frac{\sqrt{x}}{x^2 - 1}$ is

- a) $x \geq 0$
- b) $-1 < x < 1$
- c) $0 \leq x < 1$ or $x > 1$
- d) all real numbers
- e) all real numbers except ± 1

36. If matrix $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ and matrix $B = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

then $A \times B =$

- a) $\begin{bmatrix} -2 & -1 \\ -6 & -8 \end{bmatrix}$
- b) $\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$
- c) $\begin{bmatrix} -4 \\ -11 \end{bmatrix}$
- d) $B \times A$
- e) does not exist

37. The greatest common divisor of the polynomials $x^3 + 2x^2 - 9x - 18$ and $x^4 - 16x^3 + 86x^2 - 176x + 105$ is

- a) $x + 3$
- b) $x - 3$
- c) $x - 2$
- d) $x - 5$
- e) $x + 2$

38. The operation $a \{.\} b$ is defined by $a \{.\} b = a^3 - b^3$. The operation is
- a) Associative and commutative
 - b) Associative, but not commutative
 - c) Commutative, but not associative
 - d) Neither commutative, nor associative
 - e) Closed in the set of irrational numbers
39. How many imaginary roots does $2x^5 + x^4 + x^2 + 1 = 0$ have?
- a) 1
 - b) 2
 - c) 3
 - d) 4
 - e) 5
40. The system of linear equations $4x - 3y = 8$ has a unique solution (x, y)
- $$ax + 2y = 5$$
- except when a is
- a) $-8/3$
 - b) -2
 - c) $5/2$
 - d) $8/3$
 - e) 0

