

TWENTY-SIXTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA II TEST 1982

Edited by: The University of
Tennessee at Martin

Scoring Formula: $4R - W + 40$

This test was prepared from a list of Algebra II questions submitted by Middle Tennessee State University.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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5. The simplified form for $\frac{1 - \frac{2}{x+1}}{x - \frac{1}{x}}$ is

a) $\frac{x}{(x+1)^2}$

b) $\frac{x}{x+1}$

c) $\frac{-x}{x^2 - 1}$

d) $\frac{(x-1)^2}{x}$

e) none of the above

6. Give the solution for the system of linear equations suggested by the equations:

$$\begin{cases} \log_3(3)^4 + \log_7(7)^x = \log_{10}100 \\ \log_2(2)^x + \log_4(4)^3 = \log_y(y)^y \end{cases}$$

a) $(4x, 100y)$

b) $(4, 100)$

c) $(2, 3)$

d) $(-2, 1)$

e) $(3x, 2y)$

7. If $x^2 + x + 1 = 0$, then $x =$

a) $-1 \pm \frac{\sqrt{3}}{2}$

b) $1 \pm \frac{\sqrt{3}}{2}$

c) $-1 \pm \frac{i\sqrt{3}}{2}$

d) $-\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$

e) $\frac{-1 \pm \sqrt{3}i}{2}$

12. The number $\left(\frac{2^{2n}}{2^{n2}3^n}\right)^{\frac{1}{n}}$ can be written as
- a) $\frac{1}{2}$
 - b) $\frac{1}{3}$
 - c) $\frac{1}{4}$
 - d) $\frac{1}{2^{4n}}$
 - e) $\frac{1}{2^{4n-2}}$
13. y varies directly as the square root of m and inversely as p^2 . If $y = 15$ when $m = 25$ and $p = 2$, what is the value of y when $m = 9$ and $p = 4$?
- a) $\frac{9}{8}$
 - b) $\frac{9}{4}$
 - c) 9
 - d) 18
 - e) $\frac{27}{4}$
14. If $i = \sqrt{-1}$ then i^{-371} equals
- a) $-i$
 - b) -1
 - c) i
 - d) 1
 - e) none of the above
15. In how many ways can 4 boys and 3 girls be seated in 7 seats if the end seats are to be occupied by boys?
- a) 144
 - b) 14400
 - c) 720
 - d) 1440
 - e) none of the above

20. Express $\frac{3}{1 + \frac{1}{i}}$ in the form $a + bi$.

a) $\frac{3}{i + 2}$

b) $\frac{\frac{3}{i + 1}}{i}$

c) $\frac{3i}{i + 1}$

d) $\frac{3}{2} + \frac{3}{2}i$

e) $\frac{2}{3} + \frac{2}{3}i$

21. A bag contains 6 white and 4 red marbles. Two marbles are drawn from the bag and replaced. Two more marbles are then drawn from the bag. What is the probability of drawing 4 white marbles?

a) $\frac{1}{3}$

b) $\frac{1}{9}$

c) $\frac{1}{27}$

d) $\frac{1}{81}$

e) none of the above

22. The inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ is

a) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

b) $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$

d) $\begin{bmatrix} -1 & -2 \\ -2 & -3 \end{bmatrix}$

e) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

27. The curves $x^2 + y^2 = 40$ and $xy = 12$ intersect in
- a) no points
 - b) exactly 1 point
 - c) exactly 2 points
 - d) exactly 3 points
 - e) exactly 4 points
28. Let $2 + 3i$ be a root of the equation $ax^2 + bx + c = 0$, where a , b , and c are real. Then the other root
- a) must be complex
 - b) must be negative
 - c) must be positive
 - d) must not be complex
 - e) cannot be determined
29. In drilling a well, the cost is 55¢ for the first foot; for each succeeding foot the cost is 10¢ more than that for the preceding foot. How deep a well may be drilled at a cost of \$100?
- a) 30 ft.
 - b) 35 ft.
 - c) 40 ft.
 - d) 45 ft.
 - e) none of the above.
30. A stack of bricks has 80 bricks in the first row, 77 in the second row and so on until there are 2 bricks in the top row. How many bricks are in the stack?
- a) 2214
 - b) 504
 - c) 1107
 - d) 1200
 - e) 1308

34. The solution for the system $\begin{cases} x^2 - y^2 = 9 \\ x + y = 1 \end{cases}$ is
- a) (2, -1)
 - b) (4, -5)
 - c) (5, -4)
 - d) (-5, 4)
 - e) (3, -2)
35. The coordinates of the mid-point of the line segment between $P_1(-5, 3)$ and $P_2(-3, 4)$ are
- a) $-1, \frac{1}{2}$
 - b) (2, -1)
 - c) $(-\frac{7}{2}, 4)$
 - d) $(-4, \frac{7}{2})$
 - e) $(-1, \frac{7}{2})$
36. Six of the apples in a box containing twenty apples are wormy. What is the probability if three apples are selected at random that all three will be wormy?
- a) $\frac{6}{20}$
 - b) $\frac{3}{20}$
 - c) $\frac{2}{47}$
 - d) $\frac{1}{57}$
 - e) none of the above
37. A boat which goes 12 mph in still water travels on a river where the current is 3 mph. What is the average speed for a round trip, 60 miles downstream and 60 miles return, in miles per hour?
- a) $12 \frac{3}{4}$
 - b) 12
 - c) 11
 - d) $11 \frac{1}{4}$
 - e) insufficient data to solve